



BRITISH LEARNING CENTRE

KEY STAGE 3 · YEAR 9

Mathematics

Year 9

Year 9 mathematics for the English National Curriculum and Cambridge Lower Secondary Stage 9 (Checkpoint year), building a strong bridge to IGCSE.

Name:

Key knowledge · Practice · Exam-style questions · Mark schemes

How to use this workbook

1

Key knowledge

Read the notes, worked examples and common mistakes.

2

Practice

Build the skill on Sheet A.

3

Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [].

4

Check & reflect

Mark your work with the mark scheme, then colour the tracker.

My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

#	Curriculum ref	Topic · I can...	R	A	G
1	Number	Indices and standard form I can use the index laws and calculate with numbers in standard form.			
2	Number	Surds, rounding and bounds I can round to significant figures, find bounds and simplify simple surds.			
3	Ratio, proportion and rates of change	Percentages, ratio and proportion I can solve compound percentage, ratio and proportion problems.			
4	Algebra	Expanding and factorising I can expand double brackets and factorise simple quadratic expressions.			
5	Algebra	Formulae and simultaneous equations I can rearrange formulae and solve a pair of linear simultaneous equations.			
6	Algebra	Linear and quadratic sequences I can find and use the n th term of linear and simple quadratic sequences.			
7	Algebra	Linear and quadratic graphs I can use $y = mx + c$ and draw and interpret quadratic graphs.			
8	Geometry and measures	Pythagoras' theorem and bearings I can use Pythagoras' theorem and bearings to solve problems.			
9	Geometry and measures	Trigonometry with SOH CAH TOA I can use sine, cosine and tangent to find sides and angles in right-angled triangles.			
10	Geometry and measures	Similarity and congruence I can use congruence conditions and scale factors in similar shapes.			
11	Geometry and measures	Circles, sectors and volume I can find arc lengths, sector areas and volumes of cylinders, cones and spheres.			
12	Probability	Probability and tree diagrams I can use tree diagrams and relative frequency to find probabilities.			
13	Statistics	Cumulative frequency and comparing data I can draw cumulative frequency graphs and compare two distributions.			
14	All strands	Checkpoint-style mixed revision I can choose and combine methods from the whole year to solve problems.			



CHECK

Starting point check

Starting point check

Name _____ Date _____ Score _____ / 12

Answer as many questions as you can. This shows what you already know.

1

Write $2^3 \times 2^4$ as a power of 2

2

Write 52 000 in standard form

3

Round 3.4762 to 2 significant figures

4

Increase £80 by 15%

5

Share 45 in the ratio 4 : 5

6

Expand $(x + 3)(x + 4)$

7

Solve $3x - 7 = 11$

8

Find the n th term: 4, 7, 10, 13, ...

9

Gradient of $y = 5x - 2$

10

Shorter sides 6 cm and 8 cm. Find the hypotenuse.

11

Area of a circle of radius 3 cm (1 d.p.)

12

 $P(\text{rain}) = 0.35$. Find $P(\text{not rain})$.

1

UNIT 1 · KEY KNOWLEDGE

Indices and standard form

I can... use the index laws and calculate with numbers in standard form.

Curriculum ref: Number

An **index** (power) tells you how many times a number is multiplied by itself. The index laws let you simplify powers of the **same base** quickly.

Standard form writes very large or very small numbers as $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Key words

index

base

power

reciprocal

standard form

Key points

- $a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$ and $a^{-n} = 1/a^n$, e.g. $5^{-2} = 1/25$

Worked example 1

Simplify $x^5 \times x^3 \div x^{-2}$

$$x^5 \times x^3 = x^8 \text{ (add the indices)}$$

$$x^8 \div x^{-2} = x^{8-(-2)} = x^{10}$$

Worked example 2

Work out $(3 \times 10^4) \times (5 \times 10^{-7})$ in standard form.

Multiply the numbers: $3 \times 5 = 15$. Add the powers: $10^{4+(-7)} = 10^{-3}$

15×10^{-3} is not in standard form, so write $15 = 1.5 \times 10^1$

Answer: 1.5×10^{-2}

Common mistakes

- Multiplying the indices instead of adding them: $x^3 \times x^4$ is x^7 , not x^{12} .
- Thinking a negative index gives a negative number: $2^{-3} = 1/8$, not -8 .

Exam tip: After a standard form calculation, check that the first number is at least 1 and less than 10.

Answer each question in the box. Show working on spare paper.

1 Simplify $5^3 \times 5^4$

2 Simplify $x^9 \div x^4$

3 Simplify $(y^3)^4$

4 Work out 7^0

5 Work out 2^{-3} as a fraction

6 Write 10^{-2} as a decimal

7 Work out 4^{-1}

8 Work out $(2/3)^{-2}$

9 Simplify $3a^2 \times 4a^5$

10 Simplify $20b^8 \div 5b^2$

11 Simplify $(2x^3)^3$

12 Simplify $x^{-2} \times x^5$

13 Write 45 000 000 in standard form

14 Write 0.000 62 in standard form

15 Write 3.08×10^5 as an ordinary number

16 Write 7.1×10^{-3} as an ordinary number

17 Work out $(2 \times 10^3) \times (4 \times 10^5)$

18 Work out $(9 \times 10^7) \div (3 \times 10^2)$

19 Work out $(6 \times 10^4) \times (5 \times 10^3)$

20 Work out $(4.8 \times 10^6) + (3 \times 10^5)$

21 Which is larger: 3.2×10^{-4} or 4.1×10^{-5} ?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].**1** Simplify fully.

(a) $p^4 \times p^6 \div p^3$ [1]

Answer

(b) $(3x^2y)^3$ [2]

Answer

(c) $12a^5b^2 \div 4a^{-1}b^2$ [2]

Answer

(d) Find n when $2^n = 1/32$ [1]

Answer $n =$ **2** A grain of sand has a mass of 6.5×10^{-5} g.

(a) Write 6.5×10^{-5} as an ordinary number. [1]

Answer g

(b) Work out the mass of 4×10^6 grains of sand. Give your answer in kilograms, in standard form. [3]

Answer kg**3** The distance from the Earth to the Sun is about 1.5×10^8 km. Light travels at about 3×10^5 km per second.

(a) Work out how many seconds light takes to travel from the Sun to the Earth. [2]

Answer s

(b) Write your answer to (a) in minutes and seconds. [1]

Answer min s

4 Work out these without a calculator.

(a) $8^0 + 3^{-2}$. Give your answer as a fraction. [2]

Answer

(b) $(1/2)^{-3}$ [1]

Answer

(c) Write $4^3 \times 8^2$ as a single power of 2. [2]

Answer 2^{....}

5 The population of country A is 2.4×10^7 . The population of country B is 8×10^5 .

(a) How many times greater is the population of A than the population of B? [2]

Answer

(b) Work out the total population of A and B. Give your answer in standard form. [2]

Answer

6 (a) Show that $(5 \times 10^{-3})^2 = 2.5 \times 10^{-5}$ [2]

(b) Which is larger, 2.5×10^{-5} or 0.00003? Show how you decide. [1]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Surds, rounding and bounds

I can... round to significant figures, find bounds and simplify simple surds.

Curriculum ref: Number

To round to **significant figures**, start counting at the first non-zero digit. Rounding each number to 1 s.f. gives a quick **estimate**.

A rounded measurement has an **error interval**. The **bounds** are half a unit either side. A **surd** is a root that cannot be written exactly, such as $\sqrt{2}$.

Key words

significant figure

estimate

error interval

upper bound

lower bound

surd

Key points

- 0.040 739 to 3 s.f. is 0.0407 (the first zeros do not count).
- 7.4 cm to 1 d.p. gives $7.35 \leq l < 7.45$ (half of 0.1 either side).
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, so $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$
- $\sqrt{a} \times \sqrt{a} = a$; only like surds can be added: $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

Worked example 1

A length is 6.3 m to 1 d.p. A width is 2.8 m to 1 d.p. Find the upper bound of the area.

Upper bounds: 6.35 m and 2.85 m

$$6.35 \times 2.85 = \mathbf{18.0975 \text{ m}^2}$$

Worked example 2

Simplify $\sqrt{12} + \sqrt{27}$

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \text{ and } \sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$

$$2\sqrt{3} + 3\sqrt{3} = \mathbf{5\sqrt{3}}$$

Common mistakes

- Writing 63 451 to 2 s.f. as 63. The answer must keep its size: 63 000.
- Using 8.4 as the upper bound of 8 cm to the nearest cm. It is 8.5.
- Thinking $\sqrt{9} + \sqrt{16} = \sqrt{25}$. Roots cannot be added like this: $3 + 4 = 7$.

Exam tip: For the largest value of a division, use the upper bound on top and the lower bound underneath.

Answer each question in the box. Show working on spare paper.

1 Round 4872 to 1 significant figure

2 Round 0.0386 to 1 significant figure

3 Round 63 451 to 2 significant figures

4 Round 0.007 049 to 2 significant figures

5 Round 2.0362 to 3 significant figures

6 Round 199 960 to 3 significant figures

7 Estimate $39.6 \times 5.12 \div 0.49$

8 Estimate $\sqrt{98.7} \times 4.1$

9 A length is 8 cm to the nearest cm. Write the error interval.

10 A mass is 3.6 kg to 1 decimal place. Write the error interval.

11 A crowd is 240 to the nearest 10. Write the error interval.

12 A time is 70 s to the nearest 5 s. What is the upper bound?

13 Simplify $\sqrt{18}$

14 Simplify $\sqrt{48}$

15 Simplify $\sqrt{75}$

16 Work out $\sqrt{3} \times \sqrt{12}$

17 Work out $2\sqrt{7} \times 3\sqrt{7}$

18 Simplify $\sqrt{2} + 3\sqrt{2}$

19 Simplify $\sqrt{20} + \sqrt{45}$

20 Work out $\sqrt{(4/9)}$ as a fraction

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1** Use your calculator to work out $(\sqrt{41.6} + 2.3^2) \div 1.8$

(a) Write down all the figures on your calculator display. [2]

Answer

(b) Write your answer to (a) correct to 3 significant figures. [1]

Answer

- 2** A rectangle measures 12 cm by 5 cm. Both lengths are correct to the nearest centimetre.

(a) Write down the error interval for the length, l . [2]

Answer $\leq l <$

(b) Work out the upper bound of the area of the rectangle. [2]

Answer cm^2

(c) Work out the lower bound of the perimeter of the rectangle. [2]

Answer cm

- 3** Do not use a calculator in this question.

(a) Simplify $\sqrt{72}$ [2]

Answer

(b) Show that $\sqrt{8} + \sqrt{50} = 7\sqrt{2}$ [2]

(c) Expand and simplify $\sqrt{3}(\sqrt{3} + 4)$ [2]

Answer

4

(a) By rounding each number to 1 significant figure, estimate the value of $(4.92 \times 31.7) \div 0.198$ [3]

Answer

(b) Use a calculator to work out the exact value. Give your answer to 3 significant figures. [1]

Answer

5

A car travels 120 km, correct to the nearest 10 km. The journey takes 1.5 hours, correct to the nearest 0.1 hour.

Work out the upper bound for the average speed of the car. Give your answer to 1 decimal place. [3]

Answer km/h

6

A square has an area of 50 cm^2 .

(a) Find the side length of the square. Give your answer as a surd in its simplest form. [2]

Answer cm

(b) Find the perimeter of the square in surd form. [1]

Answer cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Percentages, ratio and proportion

I can... solve compound percentage, ratio and proportion problems.

Curriculum ref: Ratio, proportion and rates of change

A **multiplier** makes percentage change quick. To increase by 6% multiply by 1.06; to decrease by 6% multiply by 0.94.

Compound change repeats the multiplier each year.

In **direct proportion** both quantities grow at the same rate ($y = kx$). In **inverse proportion**, as one doubles the other halves ($y = k/x$).

Key words

multiplier

compound interest

depreciation

ratio

direct proportion

inverse proportion

Key points

- Compound change: final amount = original $\times$ multiplierⁿ
- Reverse percentage: original = new amount $\div$ multiplier
- Percentage change = change $\div$ original $\times$ 100
- To share in a ratio, find the value of one part first.

Worked example 1

£4000 is invested at 3% compound interest per year. Find its value after 5 years.

Multiplier = 1.03

$$4000 \times 1.03^5 = 4637.0963...$$

Answer: **£4637.10**

Worked example 2

Share 360 in the ratio 2 : 3 : 4.

$$2 + 3 + 4 = 9 \text{ parts, so } 1 \text{ part} = 360 \div 9 = 40$$

80, 120 and 160

Common mistakes

- Working out 3% of the original and multiplying by 5. That is simple interest, not compound.
- Finding 20% of the new price for a reverse percentage. Divide by 1.2 instead.
- Dividing by the wrong total when sharing: 2 : 3 has 5 parts, not 3.

Exam tip: Write the multiplier down before you start. It shows the examiner your method.

Answer each question in the box. Show working on spare paper.

1 Multiplier for an increase of 15%

2 Multiplier for a decrease of 8%

3 Multiplier for an increase of 2.5%

4 Increase 240 by 35%

5 Decrease 84 by 12.5%

6 £500 is invested at 4% compound interest. Value after 2 years?

7 A car worth £12 000 loses 15% of its value each year. Value after 3 years?

8 After a 20% increase a price is £96. Original price?

9 After a 30% decrease a price is £42. Original price?

10 Percentage increase from 50 to 63

11 Simplify the ratio 45 : 60

12 Share £84 in the ratio 3 : 4

13 Ratio 2 : 5. The smaller part is 18. Find the larger part.

14 Write 4 : 18 in the form 1 : n

15 5 pens cost £3.40. Cost of 8 pens?

16 6 workers take 10 days to build a wall. How long for 4 workers?

17 $y \propto x$ and $y = 21$ when $x = 3$. Find y when $x = 10$.

18 $y \propto 1/x$ and $y = 8$ when $x = 5$. Find y when $x = 2$.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 Ana invests \$6000 in a savings account. The account pays 3.5% compound interest per year.

(a) Work out the value of her investment after 4 years. Give your answer to the nearest cent. [3]

Answer \$

(b) After how many complete years will the value first be more than \$8000? [2]

Answer years

- 2 In a sale, all prices are reduced by 25%. A phone costs £540 in the sale.

(a) Work out the price of the phone before the sale. [2]

Answer £

(b) After the sale, the shop increases the sale price by 25%. Explain why the price does not return to the original price. [2]

(c) Work out the overall percentage change from the original price. [2]

Answer %

3 In a club, the ratio of boys to girls is 4 : 5. There are 12 more girls than boys.

(a) How many members are in the club?

[3]

Answer

(b) 6 more boys join the club. Write the new ratio of boys to girls in its simplest form.

[2]

Answer :

4 It takes 8 pumps 9 hours to empty a tank. All pumps work at the same rate.

(a) How long would it take 6 pumps to empty the tank?

[2]

Answer hours

(b) How many pumps are needed to empty the tank in 4 hours?

[2]

Answer

5 y is directly proportional to x^2 . When $x = 3$, $y = 45$.

(a) Find a formula for y in terms of x .

[2]

Answer $y = \dots\dots\dots$

(b) Find y when $x = 4$.

[1]

Answer $y = \dots\dots\dots$

(c) Find the positive value of x when $y = 245$.

[1]

Answer $x = \dots\dots\dots$

6 A population of bacteria starts at 2000. It increases by 12% every hour.

Work out the number of bacteria after 6 hours. Give your answer to the nearest whole number.

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Expanding and factorising

I can... expand double brackets and factorise simple quadratic expressions.

Curriculum ref: Algebra

To **expand** double brackets, multiply every term in the first bracket by every term in the second, then collect like terms. A grid helps you not miss a term.

Factorising is the reverse. For $x^2 + bx + c$, find two numbers that **multiply** to c and **add** to b .

Key words

expand

factorise

quadratic

coefficient

difference of two squares

Key points

- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + a)^2 = x^2 + 2ax + a^2$
- Difference of two squares: $x^2 - a^2 = (x + a)(x - a)$
- Always take out a common factor first: $3x^2 + 6x = 3x(x + 2)$

Worked example 1

Expand and simplify $(x + 5)(x - 3)$.

$$x \times x = x^2, x \times (-3) = -3x, 5 \times x = 5x, 5 \times (-3) = -15$$

$$x^2 + 2x - 15$$

Worked example 2

Factorise $x^2 - 7x + 12$.

Two numbers that multiply to +12 and add to -7: -3 and -4

$(x - 3)(x - 4)$. Check by expanding.

Common mistakes

- Writing $(x + 5)^2 = x^2 + 25$. The middle term $10x$ is missing.
- Sign errors: $(-3) \times (-4) = +12$, not -12 .
- Not checking a factorisation by expanding it again.

Exam tip: If a question says "factorise fully", check whether there is a common factor left inside the brackets.

Answer each question in the box. Show working on spare paper.

1 Expand $3(2x - 5)$

2 Expand $x(x + 7)$

3 Expand $(x + 2)(x + 6)$

4 Expand $(x + 4)(x - 9)$

5 Expand $(x - 3)(x - 8)$

6 Expand $(x + 5)^2$

7 Expand $(x - 6)^2$

8 Expand $(2x + 1)(x + 3)$

9 Expand $(x + 7)(x - 7)$

10 Expand $(3x - 2)(2x + 5)$

11 Factorise $8x + 12$

12 Factorise $x^2 - 9x$

13 Factorise $6y^2 + 15y$

14 Factorise $x^2 + 7x + 10$

15 Factorise $x^2 + x - 12$

16 Factorise $x^2 - 10x + 21$

17 Factorise $x^2 - 25$

18 Factorise $x^2 - 2x - 15$

19 Factorise $x^2 + 12x + 36$

20 Factorise $x^2 - 100$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].**1** Expand and simplify.

(a) $(x + 6)(x - 2)$

[2]**Answer**

(b) $(2x - 3)^2$

[2]**Answer**

(c) $3(x + 2) - (x - 4)(x + 1)$

[3]**Answer****2** Factorise fully.

(a) $12a^2b - 18ab^2$

[2]**Answer**

(b) $x^2 - 3x - 28$

[2]**Answer**

(c) $x^2 - 64$

[1]**Answer****3** A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm.

(a) Show that the area of the rectangle is $(x^2 + 3x - 10)$ cm².

[2]

(b) The area is 44 cm². Show that $x^2 + 3x - 54 = 0$.

[1]

(c) Solve the equation to find the length and width of the rectangle.

[3]**Answer** length cm, width cm

4 Solve $x^2 - 5x - 14 = 0$ by factorising.

[3]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

5 (a) Show that $(n + 3)^2 - (n - 3)^2 = 12n$.

[3]

(b) Hence work out $103^2 - 97^2$ without a calculator.

[1]

Answer $\dots\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Formulae and simultaneous equations

I can... rearrange formulae and solve a pair of linear simultaneous equations.

Curriculum ref: Algebra

To **change the subject** of a formula, use inverse operations in the reverse order, doing the same to both sides, just like solving an equation.

Simultaneous equations are two equations that are true at the same time. Make the coefficients of one letter match, then add or subtract to **eliminate** it.

Key words

subject

rearrange

inverse operation

simultaneous

eliminate

substitute

Key points

- Same signs → subtract the equations; different signs → add them.
- You may need to multiply one or both equations first.
- Substitute your first answer back to find the other letter.
- Check both answers in the equation you did not use.

Worked example 1

Make r the subject of $A = \pi r^2$.

Divide by π : $r^2 = A/\pi$

Square root: $r = \sqrt{A/\pi}$

Worked example 2

Solve $3x + 2y = 18$ and $5x - 2y = 14$.

Different signs on $2y$, so add: $8x = 32$, $x = 4$

$3(4) + 2y = 18$, so $2y = 6$ and $x = 4$, $y = 3$

Common mistakes

- Only multiplying one term when scaling an equation: every term must be multiplied.
- Subtracting when the signs are different, e.g. $+2y - (-2y) = 4y$, not 0.
- Forgetting to find the second unknown.

Exam tip: Number your equations (1) and (2) and say what you did, e.g. (1) + (2).

Answer each question in the box. Show working on spare paper.

1 Make x the subject: $y = x + 7$

2 Make x the subject: $y = 4x$

3 Make x the subject: $y = 3x - 5$

4 Make x the subject: $y = x/2 + 1$

5 Make x the subject: $y = (x - 4)/3$

6 Make x the subject: $p = 5(x + q)$

7 Make a the subject: $v = u + at$

8 Make h the subject: $A = \frac{1}{2}bh$

9 Make x the subject: $y = x^2 - 3$

10 Make r the subject: $C = 2\pi r$

11 Make t the subject: $s = (u + v)t/2$

12 $v = u + at$. Find v when $u = 4$, $a = 3$, $t = 5$

13 Solve: $x + y = 10$, $x - y = 4$

14 Solve: $2x + y = 11$, $x + y = 7$

15 Solve: $3x + 2y = 12$, $x - 2y = -4$

16 Solve: $4x + 3y = 25$, $2x + 3y = 17$

17 Solve: $5x + 2y = 1$, $3x + 4y = 9$

18 Solve: $2x - 3y = 7$, $x + y = 1$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 The formula $F = 9C/5 + 32$ changes a temperature in $^{\circ}\text{C}$ to $^{\circ}\text{F}$.

(a) Work out F when $C = 35$.

[1]

Answer $F = \dots\dots\dots$

(b) Make C the subject of the formula.

[2]

Answer $C = \dots\dots\dots$

(c) Work out C when $F = 14$.

[1]

Answer $C = \dots\dots\dots$

- 2 Rearrange each formula.

(a) Make t the subject of $k = (3t - a)/4$

[2]

Answer $t = \dots\dots\dots$

(b) Make r the subject of $V = (1/3)\pi r^2 h$

[2]

Answer $r = \dots\dots\dots$

(c) Make x the subject of $ax + b = cx + d$

[3]

Answer $x = \dots\dots\dots$

- 3 Solve the simultaneous equations.

$$4x + 3y = 6$$

$$2x - y = 8$$

[3]

Answer $x = \dots\dots\dots y = \dots\dots\dots$

- 4 Adult tickets cost £ a and child tickets cost £ c . 2 adult and 3 child tickets cost £41. 1 adult and 4 child tickets cost £33.

(a) Write two equations in a and c .

[2]

(b) Solve your equations to find the cost of each ticket.

[3]

Answer adult £ child £

- 5 The lines $y = 2x + 1$ and $x + y = 10$ cross at the point P.

Find the coordinates of P.

[3]

Answer (..... ,)

- 6 Solve the simultaneous equations.

$$3x + 4y = 5$$

$$2x - 3y = 9$$

[4]

Answer $x = \dots\dots\dots$ $y = \dots\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Linear and quadratic sequences

I can... find and use the n th term of linear and simple quadratic sequences.

Curriculum ref: Algebra

A **linear** sequence goes up or down by the same amount each time. Its n th term is $an + b$, where a is the difference.

In a **quadratic** sequence the **second differences** are the same. Its n th term contains n^2 .

Key points

- Linear: 5, 9, 13, 17 → difference 4 → $4n + 1$
- Quadratic: coefficient of n^2 = second difference $\div 2$
- Subtract the n^2 part, then find the linear part of what is left.
- To test if a number is a term, solve n th term = number. n must be a positive whole number.

Key words

term

n th term

difference

second difference

quadratic sequence

Worked example

Find the n th term of 4, 11, 22, 37, 56, ...

First differences: 7, 11, 15, 19. Second differences: 4, so start with $2n^2$

$2n^2$: 2, 8, 18, 32, 50. Sequence – $2n^2$: 2, 3, 4, 5, 6 = $n + 1$

n th term = $2n^2 + n + 1$. Check: $n = 1$ gives $2 + 1 + 1 = 4$ ✓

Common mistakes

- Using the second difference as the coefficient of n^2 . Halve it first.
- Writing $n + 4$ for 5, 9, 13, ... (the difference is the coefficient: $4n + 1$).
- Squaring before multiplying: $2n^2$ with $n = 3$ is $2 \times 9 = 18$, not 36.

Exam tip: Always check your n th term by substituting $n = 1$ and $n = 2$.

Answer each question in the box. Show working on spare paper.

1 Write the next two terms: 3, 7, 11, 15, ...

2 Find the n th term: 6, 11, 16, 21, ...

3 Find the n th term: 20, 17, 14, 11, ...

4 Find the 30th term of $4n - 7$

5 Is 100 a term of $3n + 2$? (yes/no)

6 Which term of $7n + 4$ is equal to 151?

7 Write the first four terms of $n^2 + 3$

8 Write the first four terms of $2n^2 - 1$

9 Find the 5th term of $n^2 - 2n$

10 Find the second difference: 2, 5, 10, 17, 26, ...

11 Find the n th term: 2, 5, 10, 17, ...

12 Find the n th term: 0, 3, 8, 15, ...

13 Find the n th term: 3, 12, 27, 48, ...

14 Find the n th term: 2, 6, 12, 20, 30, ...

15 Find the n th term: 5, 12, 23, 38, ...

16 Write the next term: 1, 1, 2, 3, 5, 8, ...

17 Write the next term: 3, 6, 12, 24, ...

18 Write the term-to-term rule: 2, 6, 18, 54, ...

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 Here are the numbers of tiles in a sequence of patterns: 5, 8, 11, 14, ...

(a) Find an expression for the number of tiles in pattern n . [2]

Answer

(b) Which pattern uses 80 tiles? [2]

Answer

(c) Explain why no pattern uses exactly 100 tiles. [1]

- 2 Here is a sequence: 7, 16, 29, 46, 67, ...

(a) Show that the sequence is quadratic. [1]

(b) Find the n th term of the sequence. [3]

Answer

(c) Work out the 10th term. [1]

Answer

- 3 The n th term of a sequence is $n^2 + 4n$.

(a) Write down the first three terms. [1]

Answer

(b) Show that 140 is a term in the sequence. [2]

(c) Is 200 a term in the sequence? Give a reason. [2]

- 4 Sequence A has n th term $4n + 3$. Sequence B has n th term $n^2 - 2$.

Find the first value of n for which the term in B is greater than the term in A.

[3]

Answer $n = \dots\dots$

- 5 The triangular numbers are 1, 3, 6, 10, ...

(a) Write down the next two triangular numbers.

[1]

Answer $\dots\dots$

(b) Show that the n th term is $\frac{1}{2}n(n + 1)$.

[3]

(c) Work out the 50th triangular number.

[1]

Answer $\dots\dots$

- 6 The n th term of a sequence is $5 - 2n^2$.

Write down the first three terms.

[2]

Answer $\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Linear and quadratic graphs

I can... use $y = mx + c$ and draw and interpret quadratic graphs.

Curriculum ref: Algebra

A straight line has equation $y = mx + c$. The **gradient** m tells you how steep it is; c is where it crosses the y -axis.

A **quadratic** graph such as $y = x^2 - 2x - 3$ is a smooth U-shaped curve called a **parabola**. It is symmetrical about a vertical line through its **turning point**.

Key points

- Gradient = change in $y \div$ change in x
- Parallel lines have the same gradient.
- The **roots** of a quadratic are where its graph crosses the x -axis ($y = 0$).
- Plot quadratic graphs with a table of values and join the points with a smooth curve, not straight lines.

Key words

gradient

intercept

parallel

parabola

root

turning point

Worked example 1

Find the equation of the line through (1, 5) and (4, 11).

Gradient = $(11 - 5) \div (4 - 1) = 6 \div 3 = 2$

Substitute (1, 5) into $y = 2x + c$: $5 = 2 + c$, so $c = 3$

$y = 2x + 3$

Worked example 2

Complete a table for $y = x^2 - 2x - 3$ for $x = -2$ to 4.

Substitute carefully: when $x = -2$, $y = 4 + 4 - 3 = 5$

y values: 5, 0, -3, -4, -3, 0, 5. Roots: **$x = -1$ and 3** ; turning point **(1, -4)**

Common mistakes

- Writing $(-2)^2 = -4$. A negative number squared is positive.
- Joining quadratic points with a ruler, or drawing a pointed V at the bottom.

Exam tip: Read values from a graph to the nearest small square, and draw lines on the graph to show your method.

Answer each question in the box. Show working on spare paper.

1 Gradient and y-intercept of $y = 3x - 4$

2 Gradient and y-intercept of $y = 7 - 2x$

3 Gradient and y-intercept of $2y = 6x + 8$

4 Gradient of $x + y = 5$

5 Gradient of the line through (2, 3) and (6, 11)

6 Gradient of the line through (-1, 4) and (3, -4)

7 Gradient of the line through (0, 1) and (4, 3)

8 Equation of the line with gradient 5 through (0, -2)

9 Equation of the line with gradient 3 through (2, 7)

10 Equation of the line through (1, 2) and (3, 8)

11 Line parallel to $y = 4x + 1$ through (0, -3)

12 Does (3, 10) lie on $y = 3x + 1$? (yes/no)

13 Where does $y = 2x - 6$ cross the x-axis?

14 $y = x^2 - 4$. Find y when $x = -3$

15 Roots of $y = x^2 - 9$

16 y-intercept of $y = x^2 + 2x - 8$

17 Roots of $y = x^2 + 2x - 8$

18 Line of symmetry of $y = x^2 - 6x + 5$

19 Turning point of $y = x^2 - 6x + 5$

20 Which is a curve: $y = 2x + 1$ or $y = x^2 + 1$?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

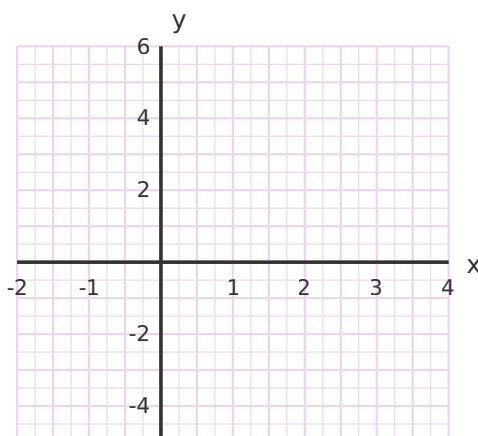
Answer all questions. Show your working. Marks are shown in [].

- 1 Here is a table of values for $y = x^2 - 2x - 3$.

x	-2	-1	0	1	2	3	4
y	5		-3		-3		5

(a) Complete the table. [2]

(b) Draw the graph of $y = x^2 - 2x - 3$ on the grid for $-2 \leq x \leq 4$. [2]



(c) Write down the roots of $x^2 - 2x - 3 = 0$. [1]

Answer $x = \dots\dots$ Or $\dots\dots$

(d) Write down the coordinates of the turning point. [1]

Answer ($\dots\dots$, $\dots\dots$)

(e) Use your graph to solve $x^2 - 2x - 3 = 2$. [2]

Answer $x = \dots\dots$ Or $\dots\dots$

- 2 The line L has equation $3x + 2y = 12$.

(a) Find the coordinates where L crosses each axis. [2]

Answer ($\dots\dots$, $\dots\dots$) and ($\dots\dots$, $\dots\dots$)

(b) Work out the gradient of L. [2]

Answer $\dots\dots$

(c) Does the point (2, 3) lie on L? Show how you decide. [1]

3 A line passes through A(-1, -1) and B(3, 7).

(a) Work out the gradient of the line AB.

[2]

Answer

(b) Find the equation of the line AB.

[2]

Answer $y = \dots\dots$

(c) Write down the equation of the line parallel to AB that passes through (0, 5).

[1]

Answer $y = \dots\dots$

(d) Find the midpoint of AB.

[1]

Answer (..... ,)

(e) Work out the length of AB. Give your answer to 3 significant figures.

[2]

Answer

4 A plumber uses the formula $C = 30h + 45$ to work out the charge, £C, for a job lasting h hours.

(a) What does the gradient, 30, represent?

[1]

(b) What does the 45 represent?

[1]

(c) Work out the charge for a job lasting 3.5 hours.

[1]

Answer £

(d) Another plumber charges $C = 40h + 20$. For how many hours do the two plumbers charge the same?

[2]

Answer hours

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Pythagoras' theorem and bearings

I can... use Pythagoras' theorem and bearings to solve problems.

Curriculum ref: Geometry and measures

In a right-angled triangle, the longest side is the **hypotenuse**. It is opposite the right angle. **Pythagoras' theorem:** $a^2 + b^2 = c^2$.

A **bearing** is an angle measured **clockwise from North**, written with three figures, e.g. 045°.

Key points

- Longest side: square, square, **add**, square root.
- Shorter side: square, square, **subtract**, square root.
- Back bearing: add 180° (or subtract 180° if the bearing is over 180°).

Key words

hypotenuse

Pythagoras' theorem

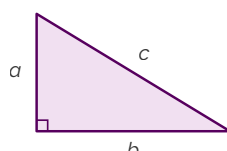
right angle

bearing

clockwise

Worked example 1

Find c when $a = 6$ cm and $b = 8$ cm.



$$c^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$c = \sqrt{100} = \mathbf{10 \text{ cm}}$$

Worked example 2

The hypotenuse is 13 cm and one side is 5 cm. Find the other side.

$$13^2 - 5^2 = 169 - 25 = 144$$

$$\sqrt{144} = \mathbf{12 \text{ cm}}$$

Common mistakes

- Adding when you need a shorter side. If you know the hypotenuse, subtract.
- Writing a bearing as 45° instead of 045°, or measuring anticlockwise.

Exam tip: Check your answer: the hypotenuse must be the longest side.

Answer each question in the box. Show working on spare paper.

1 Shorter sides 9 cm and 12 cm. Find the hypotenuse.

2 Shorter sides 5 cm and 12 cm. Find the hypotenuse.

3 Shorter sides 7 cm and 10 cm. Find the hypotenuse (1 d.p.).

4 Shorter sides 2.5 m and 6 m. Find the hypotenuse.

5 Hypotenuse 17 cm, one side 8 cm. Find the other side.

6 Hypotenuse 10 cm, one side 7 cm. Find the other side (1 d.p.).

7 Hypotenuse 25 cm, one side 24 cm. Find the other side.

8 Is a triangle with sides 6, 8 and 11 right-angled?

9 Is a triangle with sides 20, 21 and 29 right-angled?

10 Find the diagonal of a rectangle 8 cm by 15 cm.

11 Find the diagonal of a square of side 5 cm (2 d.p.).

12 Distance between (1, 2) and (7, 10)

13 Distance between (-3, 1) and (2, 13)

14 An isosceles triangle has sides 10, 10 and 12 cm. Find its height.

15 Write the bearing of due East.

16 Write the bearing of South-West.

17 The bearing of A from B is 070° . Find the bearing of B from A.

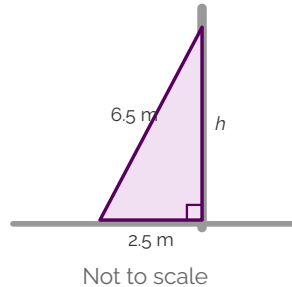
18 The bearing of B from A is 300° . Find the bearing of A from B.

19 A ship sails 6 km North then 8 km East. How far is it from its start?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 A ladder 6.5 m long leans against a vertical wall. The foot of the ladder is 2.5 m from the wall on horizontal ground.



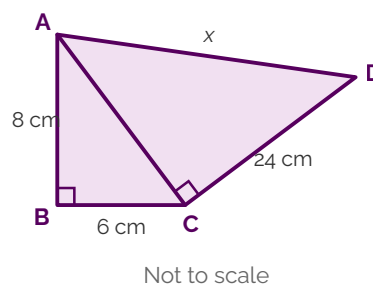
- (a) Work out how far up the wall the ladder reaches. [3]

Answer m

- (b) The foot of the ladder slips so it is now 3 m from the wall. How far does the top of the ladder slide down the wall? Give your answer to 2 decimal places. [2]

Answer m

- 2 The diagram shows two right-angled triangles, ABC and ACD.



- Work out the length x . [4]

Answer $x =$ cm

- 3 A ship leaves port P. It sails 12 km on a bearing of 090° and then 5 km on a bearing of 180° to reach S.

(a) Work out the distance PS.

[3]

Answer km

(b) Is the bearing of S from P more or less than 135° ? Explain your answer.

[2]

- 4 A is the point (2, -1) and B is the point (8, 7).

(a) Work out the length of AB.

[3]

Answer

(b) Find the midpoint of AB.

[1]

Answer (..... ,)

- 5 The bearing of Q from P is 065° .

(a) Work out the bearing of P from Q.

[2]

Answer $^\circ$

(b) R is due South of Q. The bearing of R from P is 115° . Show that angle $PRQ = 65^\circ$.

[2]

(c) Show that triangle PQR is isosceles.

[2]

- 6 A rectangular TV screen is 48 cm wide. Its diagonal is 55 cm.

Work out the height of the screen. Give your answer to 1 decimal place.

[2]

Answer cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Trigonometry with SOH CAH TOA

I can... use sine, cosine and tangent to find sides and angles in right-angled triangles.

Curriculum ref: Geometry and measures

Trigonometry links the angles and sides of a right-angled triangle. Label the sides from the angle θ : the **hypotenuse** (longest), the **opposite** (facing θ) and the **adjacent** (next to θ).

Remember **SOH CAH TOA**. Use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ to find an angle.

Key points

- $\sin \theta = \text{opposite} \div \text{hypotenuse}$
- $\cos \theta = \text{adjacent} \div \text{hypotenuse}$
- $\tan \theta = \text{opposite} \div \text{adjacent}$

Key words

sine

cosine

tangent

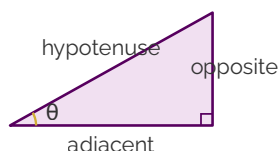
opposite

adjacent

hypotenuse

Worked example 1

Angle $\theta = 35^\circ$ and the hypotenuse is 10 cm. Find the opposite side.



Opposite and hypotenuse $\rightarrow$ SOH: $\text{opp} = 10 \times \sin 35^\circ$
 $= 5.7358\dots = \mathbf{5.74 \text{ cm}}$ (3 s.f.)

Worked example 2

The opposite side is 5 cm and the adjacent side is 8 cm. Find θ .

TOA: $\tan \theta = 5 \div 8 = 0.625$

$\theta = \tan^{-1}(0.625) = \mathbf{32.0^\circ}$

Common mistakes

- Labelling the sides before choosing the angle. The opposite side depends on which angle you use.
- Using a calculator in radians, which gives very strange answers.

Exam tip: Write the ratio you are using (e.g. SOH) before you substitute. It can earn a method mark.

All triangles are right-angled. Give lengths to 3 significant figures and angles to 1 decimal place.

1 Write down $\sin 30^\circ$

2 Write down $\cos 60^\circ$

3 Write down $\tan 45^\circ$

4 Find $\sin 52^\circ$ (3 d.p.)

5 Find $\cos 18^\circ$ (3 d.p.)

6 Find $\tan 70^\circ$ (3 d.p.)

7 $\sin x = 0.6$. Find x (1 d.p.)

8 $\cos x = 0.25$. Find x (1 d.p.)

9 $\tan x = 2$. Find x (1 d.p.)

10 Angle 40° , hypotenuse 12 cm. Find the opposite side.

11 Angle 25° , hypotenuse 8 cm. Find the adjacent side.

12 Angle 60° , adjacent 5 cm. Find the opposite side.

13 Angle 35° , opposite 9 cm. Find the hypotenuse.

14 Angle 50° , adjacent 6 cm. Find the hypotenuse.

15 Opposite 7 cm, hypotenuse 10 cm. Find the angle.

16 Adjacent 4 cm, hypotenuse 9 cm. Find the angle.

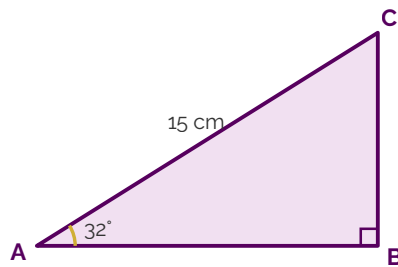
17 Opposite 11 cm, adjacent 5 cm. Find the angle.

18 Which ratio uses the opposite and adjacent sides?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 ABC is a right-angled triangle. Angle B = 90° , angle A = 32° and AC = 15 cm.



Not to scale

- (a) Work out the length of BC.

[2]

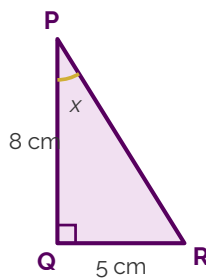
Answer cm

- (b) Work out the length of AB.

[2]

Answer cm

- 2 PQR is a right-angled triangle. PQ = 8 cm and QR = 5 cm.



Not to scale

- (a) Work out the size of angle x.

[3]

Answer $x = \dots\dots\dots^\circ$

- (b) Work out the length of PR.

[2]

Answer cm

3 A ramp rises 1.2 m over a horizontal distance of 9 m.

(a) Work out the angle the ramp makes with the horizontal.

[2]

Answer °

(b) Safety rules say the angle must be less than 8°. Does the ramp meet the rules?

[1]

4 A ladder 5 m long makes an angle of 72° with horizontal ground. It leans against a vertical wall.

(a) How high up the wall does the ladder reach?

[2]

Answer m

(b) How far is the foot of the ladder from the wall?

[2]

Answer m

5 An isosceles triangle has sides 10 cm, 10 cm and 12 cm.

(a) Work out the size of one of the base angles.

[3]

Answer °

(b) Work out the area of the triangle.

[2]

Answer cm²

6 A boat sails 20 km on a bearing of 040°.

(a) How far North has the boat travelled?

[2]

Answer km

(b) How far East has the boat travelled?

[2]

Answer km

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Similarity and congruence

I can... use congruence conditions and scale factors in similar shapes.

Curriculum ref: Geometry and measures

Congruent shapes are exactly the same shape and size. Two triangles are congruent if they satisfy SSS, SAS, ASA (or AAS) or RHS.

Similar shapes have equal angles and their sides are in the same ratio. One is an enlargement of the other by a **scale factor**.

Key words

congruent

similar

scale factor

enlargement

corresponding

Key points

- Scale factor = new length $\div$ matching original length
- Angles do not change in an enlargement.
- If lengths are multiplied by k , areas are multiplied by k^2 .
- Match the sides carefully: the shortest with the shortest, and so on.

Worked example

Triangles PQR and XYZ are similar. PQ = 4 cm, XY = 10 cm and QR = 6 cm. Find YZ.

$$\text{Scale factor} = 10 \div 4 = 2.5$$

$$YZ = 6 \times 2.5 = \mathbf{15 \text{ cm}}$$

Worked example 2

Explain why two triangles with sides 5, 7, 9 cm and 9, 7, 5 cm are congruent.

All three pairs of sides are equal, so they are congruent by **SSS**.

Common mistakes

- Adding instead of multiplying: an enlargement from 4 cm to 10 cm is $\times 2.5$, not $+6$.
- Thinking AAA proves congruence. Equal angles only prove triangles are similar.
- Matching the wrong sides when the second shape is turned or flipped.

Exam tip: Redraw similar triangles side by side, the same way round, before finding the scale factor.

Answer each question in the box. Show working on spare paper.

1 All three sides equal. Which congruence condition?

2 Two sides and the angle between them equal. Which condition?

3 Two angles and a corresponding side equal. Which condition?

4 Right angle, hypotenuse and one side equal. Which condition?

5 Are two triangles with three equal angles always congruent?

6 A length of 3 cm becomes 12 cm. Find the scale factor.

7 A side of 5 cm is enlarged by scale factor 1.5. New length?

8 Similar rectangles: 4 cm × 6 cm and 10 cm × x cm. Find x.

9 $ABC \sim DEF$. $AB = 5$, $DE = 15$, $BC = 7$. Find EF .

10 $ABC \sim DEF$. $AB = 5$, $DE = 15$, $DF = 18$. Find AC .

11 Are rectangles 3 cm × 5 cm and 6 cm × 9 cm similar?

12 Are all squares similar?

13 Are all rectangles similar?

14 An angle of 40° in a shape is enlarged by scale factor 3. New angle?

15 Lengths are multiplied by 3. What are areas multiplied by?

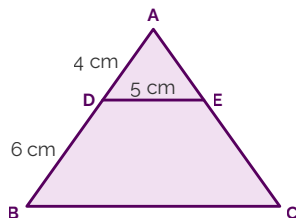
16 A model is built to a scale of 1 : 50. A real wall is 8 m long. Model length?

17 A 2 m pole casts a 3 m shadow. A tree casts a 15 m shadow at the same time. Tree height?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 In the diagram, DE is parallel to BC. AD = 4 cm, DB = 6 cm, DE = 5 cm and AC = 9 cm.



Not to scale

- (a) Explain why triangle ADE is similar to triangle ABC. [2]

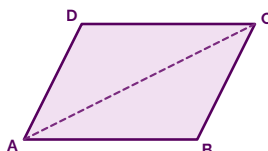
- (b) Work out the length of BC. [2]

Answer cm

- (c) Work out the length of AE. [2]

Answer cm

- 2 ABCD is a parallelogram. AC is a diagonal.



- (a) Prove that triangle ABC is congruent to triangle CDA. [3]

- (b) Hence explain why angle ABC = angle CDA. [1]

- 3 Mia is 1.5 m tall. Her shadow is 2.4 m long. At the same time, the shadow of a building is 36 m long.

Work out the height of the building.

[3]

Answer m

- 4 A photo measures 15 cm by 10 cm. It is enlarged so that the shorter side becomes 25 cm.

(a) Work out the longer side of the enlarged photo.

[2]

Answer cm

(b) How many times larger is the perimeter of the enlarged photo?

[1]

Answer

(c) How many times larger is the area of the enlarged photo?

[1]

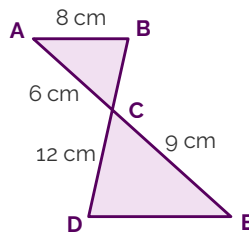
Answer

(d) A smaller copy of the original photo has a longer side of 6 cm. Work out its shorter side.

[2]

Answer cm

- 5 In the diagram, AB is parallel to DE. ACE and BCD are straight lines.



Not to scale

(a) Explain why triangles ABC and EDC are similar.

[2]

(b) Work out the length of DE.

[2]

Answer cm

(c) Work out the length of BC.

[2]

Answer cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Circles, sectors and volume

I can... find arc lengths, sector areas and volumes of cylinders, cones and spheres.

Curriculum ref: Geometry and measures

A **sector** is a slice of a circle between two radii. Its **arc** is a fraction of the circumference: the fraction is $\text{angle} \div 360$.

The volume of a **prism** or **cylinder** is $\text{area of cross-section} \times \text{length}$. A cone is one third of the matching cylinder.

Key points

- Arc length = $\frac{\theta}{360} \times 2\pi r$; sector area = $\frac{\theta}{360} \times \pi r^2$
- Cylinder: $V = \pi r^2 h$
- Cone: $V = \frac{1}{3}\pi r^2 h$; sphere: $V = \frac{4}{3}\pi r^3$
- Perimeter of a sector = arc + 2 radii.

Key words

arc

sector

radius

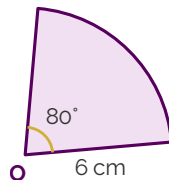
cylinder

cone

sphere

Worked example 1

Find the arc length and area of a sector with radius 6 cm and angle 80° .



$$\text{Arc} = \frac{80}{360} \times 2 \times \pi \times 6 = \mathbf{8.38 \text{ cm}}$$

$$\text{Area} = \frac{80}{360} \times \pi \times 6^2 = \mathbf{25.1 \text{ cm}^2}$$

Worked example 2

Find the volume of a cone with radius 3 cm and height 4 cm.

$$\frac{1}{3} \times \pi \times 3^2 \times 4 = 12\pi = \mathbf{37.7 \text{ cm}^3}$$

Common mistakes

- Using the diameter instead of the radius in πr^2 .
- Forgetting to add the two radii when finding the perimeter of a sector.

Exam tip: If a question says "in terms of π ", leave π in your answer, e.g. $12\pi \text{ cm}^3$.

Answer each question in the box. Show working on spare paper.

1 Circumference of a circle, radius 5 cm

2 Area of a circle, diameter 10 cm

3 Arc length: radius 10 cm, angle 90°

4 Arc length: radius 8 cm, angle 45°

5 Sector area: radius 6 cm, angle 60°

6 Sector area: radius 12 cm, angle 150°

7 Sector area: radius 4 cm, angle 270°

8 Perimeter of a semicircle, diameter 10 cm

9 Arc length in terms of π : radius 9 cm, angle 40°

10 Sector area in terms of π : radius 6 cm, angle 120°

11 Radius of a circle with circumference 40 cm

12 Volume of a cylinder: radius 3 cm, height 10 cm

13 Volume in terms of π : cylinder radius 2 cm, height 5 cm

14 Volume of a cone: radius 6 cm, height 10 cm

15 Volume in terms of π : cone radius 3 cm, height 7 cm

16 Volume of a sphere: radius 3 cm

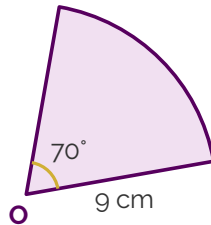
17 Volume in terms of π : sphere radius 6 cm

18 Volume of a hemisphere: radius 5 cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 The diagram shows a sector of a circle, centre O. The radius is 9 cm and the angle is 70° .



Not to scale

- (a) Work out the arc length. [2]

Answer cm

- (b) Work out the perimeter of the sector. [1]

Answer cm

- (c) Work out the area of the sector. [2]

Answer cm^2

- 2 A tin is a cylinder with radius 4 cm and height 11 cm.

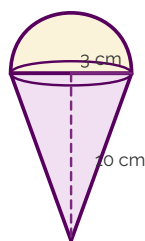
- (a) Work out the volume of the tin. [2]

Answer cm^3

- (b) A different cylinder has radius 4 cm and volume 1000 cm^3 . Work out its height. [2]

Answer cm

- 3 An ice cream is a cone of radius 3 cm and height 10 cm, with a hemisphere of radius 3 cm on top.



Not to scale

Work out the total volume. Give your answer to 1 decimal place.

[4]

Answer cm^3

- 4 A sector has radius 12 cm and arc length 15 cm.

Work out the angle of the sector. Give your answer to 1 decimal place.

[3]

Answer $^\circ$

- 5 A metal sphere of radius 6 cm is melted down and made into a cylinder of radius 4 cm.

Work out the height of the cylinder.

[3]

Answer cm

- 6 A garden is a quarter of a circle with radius 8 m.

(a) Work out the length of fencing needed to go all the way round the garden.

[3]

Answer m

(b) Turf costs £4.50 per m^2 . Work out the cost of covering the garden with turf.

[3]

Answer £

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Probability and tree diagrams

I can... use tree diagrams and relative frequency to find probabilities.

Curriculum ref: Probability

Events are **independent** if one does not affect the other. A **tree diagram** shows all the outcomes of two or more events.

Relative frequency = number of successes ÷ number of trials.
It estimates a probability, and more trials give a more reliable estimate.

Key points

- Multiply **along** the branches; add **between** different routes.
- The branches from one point add up to 1.
- Without replacement, the second set of probabilities changes.
- Expected frequency = probability × number of trials

Key words

independent

tree diagram

relative frequency

expected frequency

mutually exclusive

Worked example

The probability that it rains on any day is 0.3. Find the probability that it rains on exactly one of two days.

Rain then dry: $0.3 \times 0.7 = 0.21$. Dry then rain: $0.7 \times 0.3 = 0.21$

Add the two routes: $0.21 + 0.21 = \mathbf{0.42}$

Worked example 2

A bag has 3 red and 2 blue balls. Two are taken without replacement. Find P(both red).

$\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \mathbf{\frac{3}{10}}$

Common mistakes

- Adding along the branches instead of multiplying.
- Forgetting there are two routes for "one of each".
- Not reducing the total after an item is not replaced.

Exam tip: For "at least one", it is usually quicker to work out $1 - P(\text{none})$.

Answer each question in the box. Show working on spare paper.

1 $P(A) = 0.35$. Find $P(\text{not } A)$.

2 $P(\text{red}) = 2/7$. Find $P(\text{not red})$.

3 A bag has 3 red, 5 blue and 2 green counters. $P(\text{blue})$?

4 Same bag. $P(\text{not green})$?

5 A spinner lands on A, B or C. $P(A) = 0.2$, $P(B) = 0.35$. $P(C)$?

6 A coin is thrown and a dice is rolled. How many outcomes?

7 Same coin and dice. $P(\text{head and } 6)$?

8 Two coins are thrown. $P(\text{two heads})$?

9 A and B are independent. $P(A) = 0.4$, $P(B) = 0.5$. $P(A \text{ and } B)$?

10 A and B are mutually exclusive. $P(A) = 0.3$, $P(B) = 0.25$. $P(A \text{ or } B)$?

11 A drawing pin lands point up 37 times in 50 throws. Relative frequency?

12 $P(\text{event}) = 0.15$. Expected number of times in 200 trials?

13 A fair dice is rolled 300 times. Expected number of sixes?

14 Relative frequency 0.12 after 400 trials. How many successes?

15 $P(\text{pass}) = 0.6$ for each of two independent tests. $P(\text{pass both})$?

16 Same tests. $P(\text{pass neither})$?

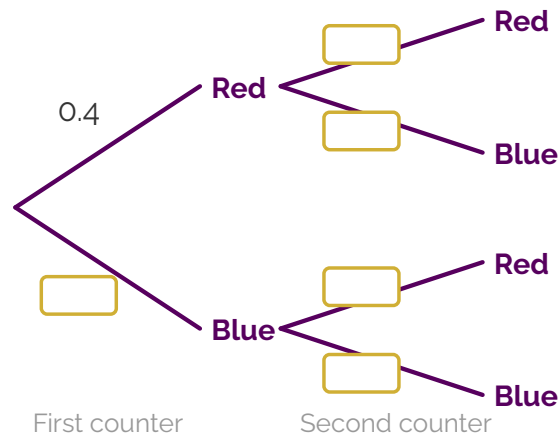
17 5 red and 3 blue sweets. Two taken without replacement. $P(\text{both red})$?

18 Which gives a more reliable estimate: 20 trials or 500 trials?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 A bag contains 4 red and 6 blue counters. Sam takes a counter at random, records its colour and puts it back. He then takes a second counter.



(a) Complete the tree diagram. [2]

(b) Work out the probability that both counters are red. [2]

Answer

(c) Work out the probability that the counters are different colours. [3]

Answer

- 2 A box contains 5 dark and 3 milk chocolates. Kim eats two chocolates, chosen at random.

(a) Work out the probability that both chocolates are milk. [2]

Answer

(b) Work out the probability that at least one chocolate is dark. [2]

Answer

- 3 A four-sided spinner is spun 200 times. The results are shown in the table.

Letter	A	B	C	D
Frequency	46	58	64	32

- (a) Work out the relative frequency of C.

[1]

Answer

- (b) Do you think the spinner is fair? Give a reason.

[2]

- (c) The spinner is spun 1000 times. Estimate how many times it lands on B.

[2]

Answer

- (d) Explain how the estimate in (c) could be made more reliable.

[1]

- 4 The probability that Leo is late for school on any day is 0.1. Each day is independent.

- (a) Work out the probability that Leo is late on exactly one of two days.

[3]

Answer

- (b) Work out the probability that Leo is late on at least one day in a school week of 5 days.

[2]

Answer

- 5 Two fair spinners are each numbered 1, 2, 3, 4. Both are spun and the numbers are added.

- (a) Work out the probability that the total is 5.

[2]

Answer

- (b) Work out the probability that the total is greater than 6.

[1]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Cumulative frequency and comparing data

I can... draw cumulative frequency graphs and compare two distributions.

Curriculum ref: Statistics

A **cumulative frequency** is a running total. Plot each running total at the **upper end** of its class and join the points with a smooth curve.

To **compare distributions**, compare an average (median or mean) **and** a measure of spread (range or IQR), and say what they mean in context.

Key words

cumulative frequency

median

quartile

interquartile range

distribution

Key points

- Median: read across from $n/2$; lower quartile from $n/4$; upper quartile from $3n/4$.
- Interquartile range (IQR) = upper quartile – lower quartile
- A smaller IQR means the data are more consistent.
- Estimated mean from a grouped table: $\Sigma(\text{frequency} \times \text{midpoint}) \div \text{total frequency}$

Worked example

Frequencies are 4, 10, 16 and 10 for 0–10, 10–20, 20–30 and 30–40 minutes. Write the cumulative frequencies and the median position.

Running totals: 4, 14, 30, 40

Plot at 10, 20, 30, 40. The median is the $40 \div 2 = 20\text{th}$ value (in the 20–30 class).

Worked example 2

Team A: median 15 s, IQR 3 s. Team B: median 17 s, IQR 6 s (times to run 100 m).

Team A was faster on average because its median time is lower.

Team A's times were more consistent because its IQR is smaller.

Common mistakes

- Plotting cumulative frequency at the midpoint of each class.
- Saying "higher is better" without thinking about the context, e.g. times.

Exam tip: Draw lines on the graph to show where you read your values. They can earn method marks.

Answer each question in the box. Show working on spare paper.

1 Median of 3, 7, 8, 12, 15, 15, 20

2 Mode of the same data

3 Range of the same data

4 Mean of the same data (2 d.p.)

5 Lower quartile of the same data

6 Upper quartile of the same data

7 Interquartile range of the same data

8 Values 1, 2, 3, 4 have frequencies 3, 5, 8, 4. Find the mean.

9 Groups 0–10, 10–20, 20–30 have frequencies 5, 8, 7. Estimate the mean.

10 Frequencies 4, 9, 12, 5. Write the cumulative frequencies.

11 A cumulative frequency total is 80. Which value gives the median?

12 Same total of 80. Which value gives the lower quartile?

13 On a cumulative frequency graph, where do you plot each point?

14 Box plot: min 5, LQ 12, median 18, UQ 25, max 40. IQR?

15 Same box plot. Range?

16 Class A IQR = 12, class B IQR = 30. Which is more consistent?

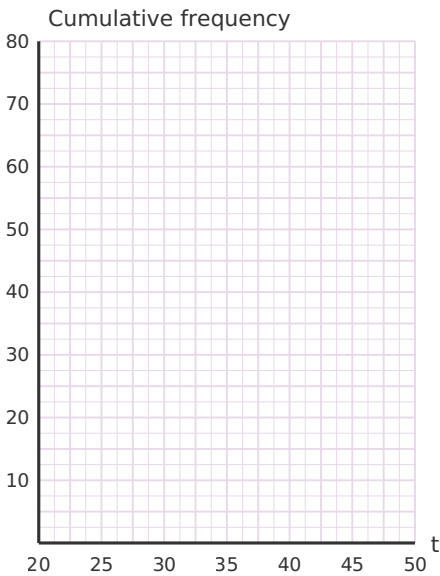
Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 The table shows the times taken by 80 runners to finish a race.

Time, t (min)	Frequency	Cumulative frequency
$20 < t \leq 25$	6	
$25 < t \leq 30$	14	
$30 < t \leq 35$	24	
$35 < t \leq 40$	20	
$40 < t \leq 45$	12	
$45 < t \leq 50$	4	

- (a) Complete the cumulative frequency column. [1]
- (b) Draw a cumulative frequency graph on the grid. [2]



- 2 The mean of five numbers is 12. A sixth number is added and the mean becomes 13.

Work out the sixth number. [3]

Answer

3 Use your cumulative frequency graph from question 1.

(a) Estimate the median time.

[1]

Answer min

(b) Estimate the interquartile range.

[2]

Answer min

(c) Estimate how many runners took more than 42 minutes.

[2]

Answer

4 Two classes sat the same test. Class P: median 64, IQR 12. Class Q: median 58, IQR 25.

Compare the results of the two classes. Give two comparisons in context.

[4]

5 The table shows the masses of 40 apples.

Mass, m (g)	$100 < m \leq 120$	$120 < m \leq 140$	$140 < m \leq 160$	$160 < m \leq 180$
Frequency	5	12	15	8

(a) Work out an estimate of the mean mass.

[4]

Answer g

(b) Explain why your answer is an estimate.

[1]

(c) Write down the modal class.

[1]

Answer

6 Here are 11 test scores: 12, 15, 18, 21, 22, 25, 27, 30, 34, 36, 41

Find the median, the range and the interquartile range.

[4]

Answer median range IQR

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Checkpoint-style mixed revision

I can... choose and combine methods from the whole year to solve problems.

Curriculum ref: All strands

This unit mixes topics from the whole year, in the style of an end-of-stage Checkpoint test. Real tests have **two papers**: one **without a calculator** and one **with a calculator**.

Questions often combine two or more topics, for example Pythagoras' theorem with bearings, or percentages with standard form.

Key words

Checkpoint

non-calculator

calculator

method mark

accuracy

Key points

- Read the question twice. Underline the key numbers and what you must find.
- Show all working: method marks are given even if the final answer is wrong.
- Give answers to the accuracy asked for (e.g. 3 s.f., 1 d.p., in terms of π).
- Without a calculator: estimate first, then check your answer is sensible.
- Check units: cm, cm², cm³, degrees, £.

Worked example

A cone has radius 5 cm and volume 100π cm³. Find its height.

$$\frac{1}{3} \times \pi \times 5^2 \times h = 100\pi$$

$$25h/3 = 100, \text{ so } h = 300 \div 25 = \mathbf{12 \text{ cm}}$$

Common mistakes

- Losing marks by rounding too early. Keep full calculator values until the last step.
- Leaving a question blank. Write down any relevant formula or first step.
- Forgetting to answer the question asked, e.g. giving x when the question asks for a length.

Exam tip: Spend about one minute per mark, and come back to hard questions at the end.

Mixed practice from all units. Answer each question in the box.

1 Work out 3^{-2}

2 $(2.5 \times 10^3) \times (4 \times 10^{-5})$ in standard form

3 Round 0.004 987 1 to 3 significant figures

4 Work out $\sqrt{8} \times \sqrt{2}$

5 Decrease £650 by 18%

6 $y \propto x$; $y = 12$ when $x = 4$. Find y when $x = 7$.

7 Expand $(x - 5)^2$

8 Factorise $x^2 + 3x - 18$

9 Make h the subject of $V = \pi r^2 h$

10 Find the n th term: 1, 7, 17, 31, ...

11 Gradient of the line through $(-2, 5)$ and $(2, -3)$

12 Shorter sides 1.2 m and 0.5 m. Find the hypotenuse.

13 Angle 35° , adjacent 10 cm. Find the opposite side (3 s.f.).

14 A triangle with sides 4, 5, 6 cm is enlarged so the shortest side is 10 cm. Longest side?

15 Sector area: radius 10 cm, angle 36° (in terms of π)

16 Volume of a sphere of radius 1.5 cm (1 d.p.)

17 The bearing of A from B is 310° . Bearing of B from A?

18 $P(A) = 0.7$ on each of two independent trials. $P(A \text{ both times})$?

19 Median of 4, 9, 2, 7, 10, 3

20 Estimate $5.87 \times 0.312 \div 0.0198$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in [].

- 1 A school minibus travels 156 km from Pattaya to Bangkok. The journey takes 2 hours 15 minutes.

(a) Work out the average speed. Give your answer to 1 decimal place. [2]

Answer km/h

(b) Fuel costs 35 baht per litre. The price rises by 8% each year. Work out the price after 3 years. [2]

Answer baht

- 2 (a) Solve $x^2 - 2x - 24 = 0$ by factorising. [3]

Answer $x =$ or

(b) Solve the simultaneous equations $3a + 2b = 21$ and $a + b = 8$. [3]

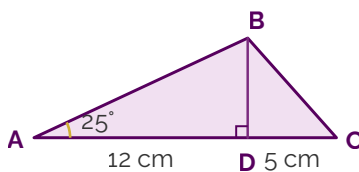
Answer $a =$ $b =$

- 3 The n th term of a sequence is $n^2 + 3n$.

Is 70 a term of the sequence? If so, which term? [2]

Answer

- 4 In the diagram, BD is perpendicular to AC. AD = 12 cm, DC = 5 cm and angle BAD = 25° .



Not to scale

- (a) Work out the length of BD. [2]

Answer cm

- (b) Work out the length of BC. [2]

Answer cm

- (c) Work out the size of angle BCD. [2]

Answer $^\circ$

- 5 A water tank is a cylinder with radius 0.8 m and height 1.5 m.

- (a) Work out the volume of the tank in m^3 . [2]

Answer m^3

- (b) $1 \text{ m}^3 = 1000$ litres. How many litres does the tank hold? [1]

Answer litres

- (c) Water flows into the empty tank at 12 litres per minute. How long does it take to fill? Give your answer in hours and minutes. [2]

Answer h min

- 6 The probability that a day is sunny is 0.7. If it is sunny, the probability that Nina goes to the beach is 0.8. If it is not sunny, the probability is 0.2.

- (a) Work out the probability that Nina goes to the beach on a given day. [3]

Answer

- (b) Estimate the number of days Nina goes to the beach in 50 days. [1]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

End-of-book check

End-of-book check

Name _____ Date _____ Score _____ / 12

Answer all the questions. Compare your score with your starting point check.

1 Simplify $(x^3)^2 \times x^{-4}$

2 $(3 \times 10^5) \times (4 \times 10^{-2})$ in standard form

3 Simplify $\sqrt{45}$

4 £2000 at 5% compound interest for 3 years. Final value?

5 Factorise $x^2 - x - 20$

6 Make r the subject of $A = 4\pi r^2$

7 Solve $x + 2y = 8$ and $x - y = 2$

8 Find the n th term: 3, 9, 19, 33, ...

9 Equation of the line through (0, 4) and (2, 10)

10 Angle 30° , hypotenuse 14 cm. Find the opposite side.

11 Arc length: radius 10 cm, angle 72° (1 d.p.)

12 3 red and 2 blue balls. Two taken without replacement. $P(\text{both red})$?



Checkpoint-style practice test Name _____ Date _____ Marks _____ / 50

Paper 1 (Questions 1–6): **no calculator** · about 25 minutes
Paper 2 (Questions 7–13): **calculator allowed** · about 25 minutes
Total: 50 marks · You need a pen, pencil, ruler and protractor.

Answer all questions. Show your working. Marks are shown in [].

1 Paper 1 — no calculator

(a) Write 0.000 47 in standard form. [1]

Answer

(b) Work out $(6 \times 10^5) \div (2 \times 10^{-3})$. Give your answer in standard form. [2]

Answer

2 (a) Work out $\sqrt{12} \times \sqrt{3}$ [1]

Answer

(b) Write $2^5 \times 2^{-2} \div 2^0$ as a single power of 2. [1]

Answer

(c) Simplify $\sqrt{50} - \sqrt{8}$ [2]

Answer

3 (a) Expand and simplify $(x - 4)(x + 7)$ [2]

Answer

(b) Factorise $x^2 + 9x + 20$ [2]

Answer

- 4 Solve the simultaneous equations.

$$2x + 3y = 13$$

$$5x - 3y = 1$$

[3]

Answer $x = \dots\dots\dots y = \dots\dots\dots$

- 5 Here is a sequence: 6, 15, 28, 45, ...

(a) Find the n th term.

[3]

Answer $\dots\dots\dots$

(b) Work out the 10th term.

[1]

Answer $\dots\dots\dots$

- 6 (a) The bearing of B from A is 125° . Work out the bearing of A from B.

[1]

Answer $\dots\dots\dots^\circ$

(b) Find the point where the lines $y = 3x - 2$ and $y = -x + 6$ meet.

[2]

Answer ($\dots\dots\dots$, $\dots\dots\dots$)

- 7 Paper 2 — calculator allowed

A car costs £18 500. Its value falls by 12% each year.

(a) Work out the value of the car after 4 years. Give your answer to the nearest pound.

[3]

Answer £ $\dots\dots\dots$

(b) A price including 20% tax is £84. Work out the price before tax.

[2]

Answer £ $\dots\dots\dots$

- 8 A rectangle has length 4.6 m and width 3.2 m, both correct to 1 decimal place.

(a) Work out the upper bound of the area.

[2]

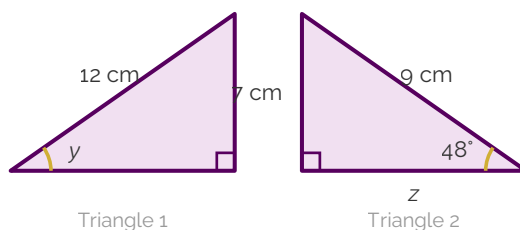
Answer m^2

(b) Work out the lower bound of the perimeter.

[2]

Answer m

9



Not to scale

(a) Work out the angle y in Triangle 1.

[2]

Answer $y = \dots\dots\dots^\circ$

(b) Work out the length z in Triangle 2.

[2]

Answer $z = \dots\dots\dots \text{cm}$

- 10 A rectangular field measures 85 m by 40 m. A straight path goes from one corner to the opposite corner.

Work out the length of the path. Give your answer to 1 decimal place.

[2]

Answer m

11

(a) Work out the volume of a cone with radius 5 cm and height 12 cm.

[2]

Answer cm^3

(b) Work out the area of a sector with radius 7 cm and angle 135° .

[2]

Answer cm^2

- 12** The table shows the distances, d km, that 30 students travel to school.

Distance, d (km)	$0 < d \leq 10$	$10 < d \leq 20$	$20 < d \leq 30$	$30 < d \leq 40$
Frequency	4	9	12	5

- (a)** Work out an estimate of the mean distance.

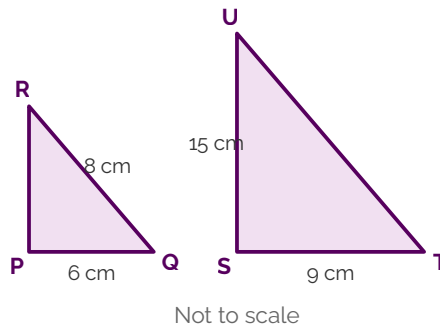
[3]

Answer km

- (b)** For a second group, the median is 12 km and the interquartile range is 15 km. The first group has median 21 km and IQR 10 km. Make one comparison of the distances.

[2]

- 13** Triangles PQR and STU are similar. Angle P = angle S, angle Q = angle T.



- (a)** Work out the length of TU.

[2]

Answer cm

- (b)** Work out the length of PR.

[2]

Answer cm

- (c)** A model of a bridge is made to a scale of 1 : 200. The model is 45 cm long. Work out the length of the real bridge in metres.

[1]

Answer m



FOR TEACHERS & PARENTS

Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

Starting point check

- 1 2^7
- 2 5.2×10^4
- 3 3.5
- 4 £92
- 5 20 and 25
- 6 $x^2 + 7x + 12$
- 7 $x = 6$
- 8 $3n + 1$
- 9 5
- 10 10 cm
- 11 28.3 cm^2
- 12 0.65

Unit 1 · Sheet A — Indices and standard form

- 1 5^7
- 2 x^5
- 3 y^{12}
- 4 1
- 5 $1/8$
- 6 0.01
- 7 $1/4$
- 8 $9/4$
- 9 $12a^7$
- 10 $4b^6$
- 11 $8x^9$
- 12 x^3
- 13 4.5×10^7
- 14 6.2×10^{-4}
- 15 308 000
- 16 0.0071
- 17 8×10^8
- 18 3×10^5
- 19 3×10^8
- 20 5.1×10^6
- 21 3.2×10^{-4}

Unit 1 · Sheet B — Indices and standard form

- 1 (a) p^7 B1 [1] (b) $27x^6y^3$ B2 (B1 for two of the three parts correct) [2] (c) $3a^6$ B2 (B1 for $3a^k$ or ka^6) [2] (d) $n = -5$ B1 [1]
- 2 (a) 0.000065 B1 [1] (b) $6.5 \times 10^{-5} \times 4 \times 10^6$ M1; = 260 g, ÷ 1000 M1; 2.6×10^{-1} kg A1 [3]
- 3 (a) $1.5 \times 10^8 \div 3 \times 10^5$ M1; 500 s A1 [2] (b) 8 minutes 20 seconds B1 ft [1]
- 4 (a) $1 + 1/9$ M1; $10/9$ (or $1\frac{1}{9}$) A1 [2] (b) 8 B1 [1] (c) $2^6 \times 2^6$ M1; 2^{12} A1 [2]
- 5 (a) $2.4 \times 10^7 \div 8 \times 10^5$ M1; 30 A1 [2] (b) 24 000 000 + 800 000 M1; 2.48×10^7 A1 [2]
- 6 (a) 25×10^{-6} M1; = 2.5×10^{-5} shown A1 [2] (b) 0.00003 (= 3×10^{-5}) B1 [1]

Unit 2 · Sheet A — Surds, rounding and bounds

- 1 5000
- 2 0.04
- 3 63 000
- 4 0.0070
- 5 2.04
- 6 200 000
- 7 $40 \times 5 \div 0.5 = 400$
- 8 $10 \times 4 = 40$
- 9 $7.5 \leq l < 8.5$
- 10 $3.55 \leq m < 3.65$
- 11 $235 \leq n < 245$
- 12 72.5 s
- 13 $3\sqrt{2}$
- 14 $4\sqrt{3}$
- 15 $5\sqrt{3}$
- 16 6
- 17 42
- 18 $4\sqrt{2}$
- 19 $5\sqrt{5}$
- 20 $2/3$

Unit 2 · Sheet B — Surds, rounding and bounds

- 1 (a) 6.522114554... B2 (B1 for $\sqrt{41.6} = 6.4498...$ or $2.3^2 = 5.29$ seen) [2] (b) 6.52 B1 ft [1]
- 2 (a) $11.5 \leq l < 12.5$ B2 (B1 for one bound correct) [2] (b) 12.5×5.5 M1; 68.75 cm^2 A1 [2] (c) $2 \times (11.5 + 4.5)$ M1; 32 cm A1 [2]
- 3 (a) $\sqrt{36} \times \sqrt{2}$ M1; $6\sqrt{2}$ A1 [2] (b) $2\sqrt{2} + 5\sqrt{2}$ M1 (either surd simplified); $= 7\sqrt{2}$ A1 [2] (c) $\sqrt{3} \times \sqrt{3} = 3$ M1; $3 + 4\sqrt{3}$ A1 [2]
- 4 (a) 5, 30 and 0.2 seen M1; $150 \div 0.2$ M1; 750 A1 [3] (b) 788 B1 [1]
- 5 upper distance 125 and lower time 1.45 B1; $125 \div 1.45$ M1; 86.2 km/h A1 [3]
- 6 (a) $\sqrt{50}$ M1; $5\sqrt{2}$ cm A1 [2] (b) $20\sqrt{2}$ cm B1 ft [1]

Unit 3 · Sheet A — Percentages, ratio and proportion

- 1 115
- 2 0.92
- 3 1.025
- 4 324
- 5 73.5
- 6 £540.80
- 7 £7369.50
- 8 £80
- 9 £60
- 10 26%
- 11 3 : 4
- 12 £36 and £48
- 13 45
- 14 1 : 4.5
- 15 £5.44
- 16 15 days
- 17 70
- 18 20

Unit 3 · Sheet B — Percentages, ratio and proportion

- 1 (a) 1.035 seen M1; 6000×1.035^4 M1; \$6885.14 A1 [3] (b) trials of 6000×1.035^n M1; 9 years A1 (value \$8177.38) [2]
- 2 (a) $540 \div 0.75$ M1; £720 A1 [2] (b) $540 \times 1.25 = £675$ M1; 25% of 540 is less than 25% of 720 (different starting amount) A1 [2] (c) $675 \div 720 = 0.9375$ M1; decrease of 6.25% A1 [2]
- 3 (a) 1 part = 12 M1; 9 parts M1; 108 A1 [3] (b) $48 + 6 = 54 : 60$ M1; 9 : 10 A1 [2]
- 4 (a) $8 \times 9 = 72$ pump-hours M1; 12 hours A1 [2] (b) $72 \div 4$ M1; 18 pumps A1 [2]
- 5 (a) $45 = k \times 9$ M1; $y = 5x^2$ A1 [2] (b) 80 B1 ft [1] (c) 7 B1 ft [1]
- 6 2000×1.12^6 M1; 3948 A1 [2]

Unit 4 · Sheet A — Expanding and factorising

- 1 $6x - 15$
- 2 $x^2 + 7x$
- 3 $x^2 + 8x + 12$
- 4 $x^2 - 5x - 36$
- 5 $x^2 - 11x + 24$
- 6 $x^2 + 10x + 25$
- 7 $x^2 - 12x + 36$
- 8 $2x^2 + 7x + 3$
- 9 $x^2 - 49$
- 10 $6x^2 + 11x - 10$
- 11 $4(2x + 3)$
- 12 $x(x - 9)$
- 13 $3y(2y + 5)$
- 14 $(x + 2)(x + 5)$
- 15 $(x + 4)(x - 3)$
- 16 $(x - 3)(x - 7)$
- 17 $(x + 5)(x - 5)$
- 18 $(x - 5)(x + 3)$
- 19 $(x + 6)^2$
- 20 $(x + 10)(x - 10)$

Unit 4 · Sheet B — Expanding and factorising

- 1 (a) $x^2 - 2x + 6x - 12$ M1 (3 of 4 terms correct); $x^2 + 4x - 12$ A1 [2] (b) $(2x - 3)(2x - 3)$ with 3 of 4 terms correct M1; $4x^2 - 12x + 9$ A1 [2] (c) $3x + 6$ B1; $x^2 - 3x - 4$ B1; $-x^2 + 6x + 10$ A1 [3]
- 2 (a) $6ab(2a - 3b)$ B2 (B1 for a partial factorisation, e.g. $6a(2ab - 3b^2)$) [2] (b) $(x - 7)(x + 4)$ B2 (B1 for brackets giving two correct terms) [2] (c) $(x + 8)(x - 8)$ B1 [1]
- 3 (a) $(x + 5)(x - 2)$ M1; $x^2 - 2x + 5x - 10 = x^2 + 3x - 10$ A1 [2] (b) $x^2 + 3x - 10 = 44$ rearranged B1 [1] (c) $(x + 9)(x - 6) = 0$ M1; $x = 6$ (reject -9) A1; 11 cm by 4 cm A1 [3]
- 4 $(x - 7)(x + 2) = 0$ M1 A1; $x = 7$ or $x = -2$ A1 [3]
- 5 (a) $n^2 + 6n + 9$ B1; $n^2 - 6n + 9$ B1; subtraction with brackets handled correctly to give $12n$ A1 [3] (b) $n = 100$; $12 \times 100 = 1200$ B1 [1]

Unit 5 · Sheet A — Formulae and simultaneous equations

- 1 $x = y - 7$
- 2 $x = y/4$
- 3 $x = (y + 5)/3$
- 4 $x = 2(y - 1)$
- 5 $x = 3y + 4$
- 6 $x = p/5 - q$
- 7 $a = (v - u)/t$
- 8 $h = 2A/b$
- 9 $x = \pm\sqrt{y + 3}$
- 10 $r = C/(2\pi)$
- 11 $t = 2s/(u + v)$
- 12 19
- 13 $x = 7, y = 3$
- 14 $x = 4, y = 3$
- 15 $x = 2, y = 3$
- 16 $x = 4, y = 3$
- 17 $x = -1, y = 3$
- 18 $x = 2, y = -1$

Unit 5 · Sheet B — Formulae and simultaneous equations

- 1 (a) 95 B1 [1] (b) $F - 32 = 9C/5$ M1; $C = 5(F - 32)/9$ A1 oe [2] (c) -10 B1 ft [1]
- 2 (a) $4k = 3t - a$ M1; $t = (4k + a)/3$ A1 [2] (b) $r^2 = 3V/(\pi h)$ M1; $r = \sqrt{3V/(\pi h)}$ A1 [2] (c) $ax - cx = d - b$ M1; $x(a - c) = d - b$ M1; $x = (d - b)/(a - c)$ A1 oe [3]
- 3 multiply second equation by 3 (or 2) M1; $10x = 30, x = 3$ A1; $y = -2$ A1 [3]
- 4 (a) $2a + 3c = 41$ B1; $a + 4c = 33$ B1 [2] (b) correct elimination or substitution M1; $c = 5$ A1; $a = 13$ A1 [3]
- 5 $x + 2x + 1 = 10$ M1; $x = 3$ A1; $P(3, 7)$ A1 [3]
- 6 multiply to match coefficients, e.g. $\times 2$ and $\times 3$ M1; correct subtraction, e.g. $17y = -17$ M1; $y = -1$ A1; $x = 3$ A1 [4]

Unit 6 · Sheet A — Linear and quadratic sequences

- 1 19, 23
- 2 $5n + 1$
- 3 $23 - 3n$
- 4 113
- 5 No ($3n = 98$ has no whole-number solution)
- 6 21st term
- 7 4, 7, 12, 19
- 8 1, 7, 17, 31
- 9 15
- 10 2
- 11 $n^2 + 1$
- 12 $n^2 - 1$
- 13 $3n^2$
- 14 $n^2 + n$
- 15 $2n^2 + n + 2$
- 16 13
- 17 48
- 18 $\times 3$

Unit 6 · Sheet B — Linear and quadratic sequences

- 1 (a) $3n$ B1; $3n + 2$ B1 [2] (b) $3n + 2 = 80$ M1; pattern 26 A1 [2] (c) $3n = 98$, but 98 is not a multiple of 3 B1 [1]
- 2 (a) first differences 9, 13, 17, 21; second differences all 4 B1 [1] (b) $2n^2$ seen M1; subtract to get 5, 8, 11, 14, 17 M1; $2n^2 + 3n + 2$ A1 [3] (c) 232 B1 ft [1]
- 3 (a) 5, 12, 21 B1 [1] (b) try $n = 10$ M1; $100 + 40 = 140$ A1 [2] (c) $n = 12$ gives 192 and $n = 13$ gives 221 M1; No, 200 is between them A1 [2]
- 4 compares terms, e.g. $n = 5$: $A = 23, B = 23$ M1; $n = 6$: $A = 27, B = 34$ M1; $n = 6$ A1 [3]
- 5 (a) 15, 21 B1 [1] (b) second difference 1 so $\frac{1}{2}n^2$ M1; remainder $\frac{1}{2}, 1, 1\frac{1}{2}, 2 = \frac{1}{2}n$ M1; $\frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}n(n + 1)$ A1 [3] (c) 1275 B1 [1]
- 6 3, -3, -13 B2 (B1 for two correct) [2]

Unit 7 · Sheet A — Linear and quadratic graphs

- 1 $m = 3, c = -4$
- 2 $m = -2, c = 7$
- 3 $m = 3, c = 4$
- 4 -1
- 5 2
- 6 -2
- 7 $1/2$
- 8 $y = 5x - 2$
- 9 $y = 3x + 1$
- 10 $y = 3x - 1$
- 11 $y = 4x - 3$
- 12 Yes ($3 \times 3 + 1 = 10$)
- 13 (3, 0)
- 14 5
- 15 $x = 3$ and $x = -3$
- 16 (0, -8)
- 17 $x = 2$ and $x = -4$
- 18 $x = 3$
- 19 (3, -4)
- 20 $y = x^2 + 1$

Unit 7 · Sheet B — Linear and quadratic graphs

- 1 (a) 0, -4, 0 B2 (B1 for two correct) [2] (b) all 7 points plotted correctly B1 ft; smooth U-shaped curve through them B1 [2] (c) $x = -1$ and $x = 3$ B1 [1] (d) (1, -4) B1 [1] (e) line $y = 2$ drawn or used M1; $x \approx -1.4$ and $x \approx 3.4$ A1 (accept -1.5 to -1.4 and 3.4 to 3.5) [2]
- 2 (a) (4, 0) B1; (0, 6) B1 [2] (b) rearrange to $y = -1.5x + 6$ M1; $-3/2$ A1 oe [2] (c) $3(2) + 2(3) = 12$, so yes B1 [1]
- 3 (a) $(7 - (-1)) \div (3 - (-1))$ M1; 2 A1 [2] (b) $y = 2x + c$ with a point substituted M1; $y = 2x + 1$ A1 [2] (c) $y = 2x + 5$ B1 ft [1] (d) (1, 3) B1 [1] (e) $4^2 + 8^2 = 80$ M1; 8.94 A1 [2]
- 4 (a) the cost per hour (£30 per hour) B1 [1] (b) a fixed call-out charge of £45 B1 [1] (c) £150 B1 [1] (d) $30h + 45 = 40h + 20$ M1; 2.5 hours A1 [2]

Unit 8 · Sheet A — Pythagoras' theorem and bearings

- 1 15 cm
- 2 13 cm
- 3 12.2 cm
- 4 6.5 m
- 5 15 cm
- 6 7.1 cm
- 7 7 cm
- 8 No ($36 + 64 = 100 \neq 121$)
- 9 Yes ($400 + 441 = 841 = 29^2$)
- 10 17 cm
- 11 7.07 cm
- 12 10
- 13 13
- 14 8 cm
- 15 090°
- 16 225°
- 17 250°
- 18 120°
- 19 10 km

Unit 8 · Sheet B — Pythagoras' theorem and bearings

- 1 (a) $6.5^2 - 2.5^2$ M1; $\sqrt{36}$ M1; 6 m A1 [3] (b) $\sqrt{(6.5^2 - 3^2)} = 5.7663\dots$ M1; 0.23 m A1 [2]
- 2 $8^2 + 6^2$ M1; $AC = 10$ A1; $10^2 + 24^2$ M1; $x = 26$ cm A1 [4]
- 3 (a) sketch with a right angle or $12^2 + 5^2$ M1; $\sqrt{169}$ M1; 13 km A1 [3] (b) less than 135° B1; the ship went further East (12 km) than South (5 km), so the angle below East is less than 45° B1 [2]
- 4 (a) differences 6 and 8 M1; $6^2 + 8^2 = 100$ M1; 10 A1 [3] (b) (5, 3) B1 [1]
- 5 (a) $065 + 180$ M1; 245° A1 [2] (b) bearing of P from R = 295° or angle with North line $180 - 115$ M1; $360 - 295 = 65^\circ$ A1 [2] (c) angle $PQR = 245 - 180 = 65^\circ$ M1; two equal angles (65°), so isosceles A1 [2]
- 6 $55^2 - 48^2 = 721$ M1; 26.9 cm A1 [2]

Unit 9 · Sheet A — Trigonometry with SOH CAH TOA

- 1 0.5
- 2 0.5
- 3 1
- 4 0.788
- 5 0.951
- 6 2.747
- 7 36.9°
- 8 75.5°
- 9 63.4°
- 10 7.71 cm
- 11 7.25 cm
- 12 8.66 cm
- 13 15.7 cm
- 14 9.33 cm
- 15 44.4°
- 16 63.6°
- 17 65.6°
- 18 tan

Unit 9 · Sheet B — Trigonometry with SOH CAH TOA

- 1 (a) $15 \times \sin 32^\circ$ M1; 7.95 cm A1 [2] (b) $15 \times \cos 32^\circ$ M1; 12.7 cm A1 [2]
- 2 (a) $\tan x = 5/8$ M1; $\tan^{-1}(5/8)$ M1; 32.0° A1 [3] (b) $8^2 + 5^2 = 89$ M1; 9.43 cm A1 [2]
- 3 (a) $\tan^{-1}(1.2/9)$ M1; 7.6° A1 [2] (b) Yes, $7.6^\circ < 8^\circ$ B1 ft [1]
- 4 (a) $5 \times \sin 72^\circ$ M1; 4.76 m A1 [2] (b) $5 \times \cos 72^\circ$ M1; 1.55 m A1 [2]
- 5 (a) split into right-angled triangles with base 6 M1; $\cos^{-1}(6/10)$ M1; 53.1° A1 [3] (b) height = 8 cm M1; 48 cm^2 A1 [2]
- 6 (a) $20 \times \cos 40^\circ$ M1; 15.3 km A1 [2] (b) $20 \times \sin 40^\circ$ M1; 12.9 km A1 [2]

Unit 10 · Sheet A — Similarity and congruence

- 1 SSS
- 2 SAS
- 3 ASA (or AAS)
- 4 RHS
- 5 No (only similar)
- 6 4
- 7 7.5 cm
- 8 15
- 9 21
- 10 6
- 11 No ($6/3 = 2$ but $9/5 = 1.8$)
- 12 Yes
- 13 No
- 14 40°
- 15 9
- 16 16 cm
- 17 10 m

Unit 10 · Sheet B — Similarity and congruence

- 1 (a) angle A is common B1; angle ADE = angle ABC (corresponding angles, DE parallel to BC) B1 [2] (b) scale factor $10/4 = 2.5$ M1; 12.5 cm A1 [2] (c) $9 \div 2.5$ M1; 3.6 cm A1 [2]
- 2 (a) $AB = CD$ and $BC = DA$ (opposite sides of a parallelogram) B1; AC is common B1; so congruent by SSS B1 [3] (b) corresponding angles in congruent triangles are equal B1 [1]
- 3 scale factor $36 \div 2.4 = 15$ M1; 1.5×15 M1; 22.5 m A1 [3]
- 4 (a) scale factor 2.5 M1; 37.5 cm A1 [2] (b) 2.5 B1 [1] (c) 6.25 B1 [1] (d) scale factor $6/15 = 0.4$ M1; 4 cm A1 [2]
- 5 (a) vertically opposite angles at C are equal B1; alternate angles equal (e.g. angle BAC = angle DEC) B1 [2] (b) scale factor $9/6 = 1.5$ M1; 12 cm A1 [2] (c) $12 \div 1.5$ M1; 8 cm A1 [2]

Unit 11 · Sheet A — Circles, sectors and volume

- 1 31.4 cm
- 2 78.5 cm^2
- 3 15.7 cm
- 4 6.28 cm
- 5 18.8 cm^2
- 6 188.5 cm^2
- 7 37.7 cm^2
- 8 25.7 cm
- 9 $2\pi \text{ cm}$
- 10 $12\pi \text{ cm}^2$
- 11 6.37 cm
- 12 282.7 cm^3
- 13 $20\pi \text{ cm}^3$
- 14 377.0 cm^3
- 15 $21\pi \text{ cm}^3$
- 16 113.1 cm^3
- 17 $288\pi \text{ cm}^3$
- 18 261.8 cm^3

Unit 11 · Sheet B — Circles, sectors and volume

- 1 (a) $70/360 \times 2 \times \pi \times 9 \text{ M1}$; 11.0 cm A1 [2] (b) 29.0 cm B1 ft [1] (c) $70/360 \times \pi \times 9^2 \text{ M1}$; $49.5 \text{ cm}^2 \text{ A1}$ [2]
- 2 (a) $\pi \times 4^2 \times 11 \text{ M1}$; $552.9 \text{ cm}^3 \text{ A1}$ [2] (b) $1000 \div (\pi \times 16) \text{ M1}$; 19.9 cm A1 [2]
- 3 cone: $1/3 \times \pi \times 3^2 \times 10 = 30\pi \text{ M1}$; hemisphere: $2/3 \times \pi \times 3^3 = 18\pi \text{ M1}$; add M1; $150.8 \text{ cm}^3 \text{ A1}$ [4]
- 4 $\theta/360 \times 2\pi \times 12 = 15 \text{ M1}$; $\theta = 15 \times 360 \div (24\pi) \text{ M1}$; 71.6° A1 [3]
- 5 $4/3 \times \pi \times 6^3 = 288\pi \text{ M1}$; $\pi \times 4^2 \times h = 288\pi \text{ M1}$; $h = 18 \text{ cm A1}$ [3]
- 6 (a) $2\pi \times 8 \div 4 \text{ M1}$; + 16 M1; 28.6 m A1 [3] (b) $\pi \times 8^2 \div 4 = 50.27 \text{ M1}$; $\times 4.50 \text{ M1}$; £226.19 A1 (accept £226.08 to £226.20) [3]

Unit 12 · Sheet A — Probability and tree diagrams

- 1 0.65
- 2 $5/7$
- 3 $1/2$
- 4 $4/5$
- 5 0.45
- 6 12
- 7 $1/12$
- 8 $1/4$
- 9 0.2
- 10 0.55
- 11 0.74
- 12 30
- 13 50
- 14 48
- 15 0.36
- 16 0.16
- 17 $5/14$
- 18 500 trials

Unit 12 · Sheet B — Probability and tree diagrams

- 1 (a) 0.6 for first Blue B1; 0.4 and 0.6 correctly placed on all four second branches B1 [2] (b) $0.4 \times 0.4 \text{ M1}$; $4/25 (0.16) \text{ A1}$ [2] (c) $0.4 \times 0.6 \text{ M1}$; $\times 2$ or $0.24 + 0.24 \text{ M1}$; $12/25 (0.48) \text{ A1}$ [3]
- 2 (a) $3/8 \times 2/7 \text{ M1}$; $3/28 \text{ A1}$ [2] (b) $1 - 3/28 \text{ M1}$; $25/28 \text{ A1}$ [2]
- 3 (a) $64/200 = 0.32 \text{ B1}$ [1] (b) No B1; for a fair spinner expect about 50 each, but C = 64 and D = 32 are far from 50 B1 [2] (c) $58/200 \times 1000 \text{ M1}$; 290 A1 [2] (d) spin the spinner more times (more trials) B1 [1]
- 4 (a) $0.1 \times 0.9 \text{ M1}$; $\times 2 \text{ M1}$; 0.18 A1 [3] (b) $1 - 0.9^5 \text{ M1}$; 0.410 A1 (accept 0.40951) [2]
- 5 (a) 16 outcomes listed or a grid drawn M1; $1/4 \text{ A1}$ [2] (b) $3/16 \text{ B1}$ [1]

Unit 13 · Sheet A — Cumulative frequency and comparing data

- 1 12
- 2 15
- 3 17
- 4 11.43
- 5 7
- 6 15
- 7 8
- 8 2.65
- 9 16
- 10 4, 13, 25, 30
- 11 40th
- 12 20th
- 13 at the upper class boundary
- 14 13
- 15 35
- 16 Class A

Unit 13 · Sheet B — Cumulative frequency and comparing data

- 1 (a) 6, 20, 44, 64, 76, 80 B1 [1] (b) points plotted at upper class boundaries (25, 6), (30, 20), ..., (50, 80) B1 ft; smooth curve or line segments joining them B1 [2]
- 2 $6 \times 13 = 78 \text{ M1}$; $5 \times 12 = 60 \text{ M1}$; 18 A1 [3]
- 3 (a) about 34.2 min B1 (accept 33.5 to 34.5) [1] (b) $LQ \approx 30$ and $UQ \approx 39 \text{ M1}$; $IQR \approx 9 \text{ min A1}$ (accept 8 to 10) [2] (c) reads cumulative frequency at 42 (≈ 69) M1; $\approx 11 \text{ A1}$ (accept 10 to 13) [2]
- 4 Class P did better on average B1 because its median is higher ($64 > 58$) B1; Class P's marks were more consistent B1 because its IQR is smaller ($12 < 25$) B1 [4]
- 5 (a) midpoints 110, 130, 150, 170 M1; $\Sigma fx = 5720 \text{ M1}$; $\div 40 \text{ M1}$; 143 g A1 [4] (b) the exact masses are not known; midpoints were used B1 [1] (c) $140 < m \leq 160 \text{ B1}$ [1]
- 6 median 25 B1; range 29 B1; LQ 18 and UQ 34 M1; IQR 16 A1 [4]

Unit 14 · Sheet A — Checkpoint-style mixed revision

- 1 $1/9$
- 2 1×10^{-1}
- 3 0.00499
- 4 4
- 5 £533
- 6 21
- 7 $x^2 - 10x + 25$
- 8 $(x + 6)(x - 3)$
- 9 $h = V/(\pi r^2)$
- 10 $2n^2 - 1$
- 11 -2
- 12 1.3 m
- 13 7.00 cm
- 14 15 cm
- 15 $10\pi \text{ cm}^2$
- 16 14.1 cm^3
- 17 130°
- 18 0.49
- 19 5.5
- 20 $6 \times 0.3 \div 0.02 = 90$

Unit 14 · Sheet B — Checkpoint-style mixed revision

- 1 (a) $156 \div 2.25$ M1; 69.3 km/h A1 [2] (b) 35×1.08^3 M1; 44.09 baht A1 [2]
- 2 (a) $(x - 6)(x + 4)$ M1 A1; $x = 6$ or $x = -4$ A1 [3] (b) multiply second by 2 and subtract M1; $a = 5$ A1; $b = 3$ A1 [3]
- 3 tries $n^2 + 3n = 70$ or substitutes values M1; yes, the 7th term ($49 + 21 = 70$) A1 [2]
- 4 (a) $12 \times \tan 25^\circ$ M1; 5.60 cm A1 [2] (b) $\sqrt{(BD^2 + 5^2)}$ M1; 7.50 cm A1 ft [2] (c) $\tan^{-1}(BD \div 5)$ M1; 48.2° A1 ft [2]
- 5 (a) $\pi \times 0.8^2 \times 1.5$ M1; 3.016 m^3 A1 [2] (b) 3016 litres B1 ft [1] (c) $3016 \div 12 = 251.3$ min M1; 4 hours 11 minutes A1 (accept 4 h 12 min) [2]
- 6 (a) $0.7 \times 0.8 = 0.56$ M1; $0.3 \times 0.2 = 0.06$ M1; 0.62 A1 [3] (b) 31 B1 ft [1]

End-of-book check

- 1 x^2
- 2 1.2×10^4
- 3 $3\sqrt{5}$
- 4 £2315.25
- 5 $(x - 5)(x + 4)$
- 6 $r = \sqrt{(A/(4\pi))}$
- 7 $x = 4, y = 2$
- 8 $2n^2 + 1$
- 9 $y = 3x + 4$
- 10 7 cm
- 11 12.6 cm
- 12 $3/10$

Checkpoint-style practice test

- 1 (a) 4.7×10^{-4} B1 [1] (b) $6 \div 2 = 3$ and $10^{5-(-3)}$ M1; 3×10^8 A1 [2]
- 2 (a) 6 B1 [1] (b) 2^3 B1 [1] (c) $5\sqrt{2} - 2\sqrt{2}$ M1; $3\sqrt{2}$ A1 [2]
- 3 (a) 3 of 4 terms correct M1; $x^2 + 3x - 28$ A1 [2] (b) $(x + 4)(x + 5)$ B2 (B1 for one correct bracket) [2]
- 4 add the equations M1; $x = 2$ A1; $y = 3$ A1 [3]
- 5 (a) second difference 4, so $2n^2$ M1; remainder 4, 7, 10, 13 M1; $2n^2 + 3n + 1$ A1 [3] (b) 231 B1 ft [1]
- 6 (a) 305° B1 [1] (b) $3x - 2 = -x + 6$ M1; (2, 4) A1 [2]
- 7 (a) 0.88 seen M1; $18\,500 \times 0.88^4$ M1; £11094 A1 [3] (b) $84 \div 1.2$ M1; £70 A1 [2]
- 8 (a) 4.65×3.25 M1; 15.1125 m^2 A1 [2] (b) $2 \times (4.55 + 3.15)$ M1; 15.4 m A1 [2]
- 9 (a) $\sin^{-1}(7/12)$ M1; 35.7° A1 [2] (b) $9 \times \cos 48^\circ$ M1; 6.02 cm A1 [2]
- 10 $85^2 + 40^2$ M1; 93.9 m A1 [2]
- 11 (a) $\frac{1}{3} \times \pi \times 5^2 \times 12$ M1; 314.2 cm^3 A1 [2] (b) $135/360 \times \pi \times 7^2$ M1; 57.7 cm^2 A1 [2]
- 12 (a) midpoints $\times$ frequencies M1; $\Sigma fx = 630$ M1; 21.0 km A1 [3] (b) e.g. the second group travel less far on average (median $12 < 21$) B1; with a reason in context B1 (or: second group less consistent, IQR $15 > 10$) [2]
- 13 (a) scale factor $9/6 = 1.5$ M1; 12 cm A1 [2] (b) $15 \div 1.5$ M1; 10 cm A1 [2] (c) 90 m B1 [1]