



BRITISH LEARNING CENTRE

PEARSON EDEXCEL INTERNATIONAL GCSE · 4MA1

Mathematics

Trigonometry, Vectors & Transformations

Covers trigonometry and Pythagoras' theorem (4.8, with trigonometric graphs from 3.3) and vectors and transformation geometry (5.1–5.2) of Pearson Edexcel International GCSE Mathematics A (4MA1), Foundation and Higher tiers. In mark schemes, a column vector is written as (top over bottom).

Name:

Key knowledge · Practice · Exam-style questions · Mark schemes

How to use this workbook

1

Key knowledge

Read the notes, worked examples and common mistakes.

2

Practice

Build the skill on Sheet A.

3

Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [].

4

Check & reflect

Mark your work with the mark scheme, then colour the tracker.

My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

#	Spec ref	Topic · I can...	R	A	G
1	4.8A (F) Foundation & Higher	Pythagoras' theorem I can use Pythagoras' theorem in two dimensions, including distances between points.			
2	4.8B (F) Foundation & Higher	Trigonometry in right-angled triangles I can use sine, cosine and tangent to work out lengths and angles in right-angled triangles.			
3	4.8C (F), 4.8B (H) Foundation & Higher	Elevation, depression and bearings I can solve problems with angles of elevation and depression and with bearings.			
4	4.8A (H), 3.3 (H) Higher	Obtuse angles and trigonometric graphs I can use the graphs of sin, cos and tan for any angle to solve equations and describe transformations.			
5	4.8C (H) Higher	The sine rule I can use the sine rule to work out sides and angles in any triangle, including the ambiguous case.			
6	4.8C (H) Higher	The cosine rule I can use the cosine rule to work out a side or an angle in any triangle.			
7	4.8E (H), 4.8C (H) Higher	Area of a triangle and mixed problems I can use $\frac{1}{2}ab \sin C$ and choose the right rule to solve multi-step triangle problems.			
8	4.8D (H), 4.8F (H) Higher	Trigonometry in three dimensions I can use Pythagoras and trigonometry in 3D, including the angle between a line and a plane.			
9	5.2A–I Foundation & Higher	Rotations, reflections and translations I can rotate, reflect and translate shapes and describe these transformations fully.			
10	5.2J–L Foundation & Higher	Enlargement I can enlarge a shape by a positive or fractional scale factor, with or without a centre.			
11	5.2M, 5.2I Foundation & Higher	Describing and combining transformations I can describe transformations fully and replace two transformations by a single one.			
12	5.1A–F Higher	Vectors I can use vector notation, combine vectors and work out the modulus of a vector.			
13	5.1G Higher	Vector geometry and proof I can use vector methods to prove that lines are parallel and that points are collinear.			



CHECK

Starting point check

Starting point check

Name _____ Date _____ Score _____ / 12

Answer these questions to check what you already know. You may use a calculator. Diagrams are NOT accurately drawn.

- 1 Work out $7^2 + 24^2$, then find its square root.

- 2 Round 0.046819 to 3 significant figures.

- 3 Use your calculator to work out $\tan 45^\circ$.

- 4 The angles in a triangle are $2x$, $3x$ and $4x$. Find x .

- 5 Write down the three-figure bearing of south-east.

- 6 Make h the subject of $\sin 40^\circ = h/12$.

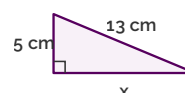
- 7 Reflect the point $(-3, 2)$ in the x -axis.

- 8 Rotate the point $(4, 0)$ through 90° anticlockwise about $(0, 0)$.

- 9 A rectangle 4 cm by 6 cm is enlarged by scale factor $1/2$. Write down its new dimensions.

Work out the length x .

10



Answer $x = \dots\dots\dots$ cm

- 11 Work out the area of a triangle with base 7 cm and height 10 cm.

- 12 Solve $x^2 = 49$, where $x > 0$.

1

UNIT 1 · KEY KNOWLEDGE

Pythagoras' theorem

I can... use Pythagoras' theorem in two dimensions, including distances between points.

Spec ref: 4.8A (F) · Foundation & Higher

Pythagoras' theorem connects the three sides of a right-angled triangle. The **hypotenuse** is the side opposite the right angle and is always the longest side.

Key points

- $a^2 + b^2 = c^2$ (c = hypotenuse)
- Hypotenuse: square, **add**, square root. Shorter side: square, **subtract**, square root.
- Distance between (x_1, y_1) and $(x_2, y_2) = \sqrt{(\text{difference in } x)^2 + (\text{difference in } y)^2}$

Key words

hypotenuse

Pythagoras' theorem

right-angled

square root

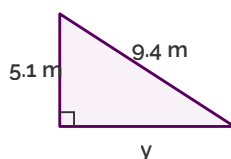
Worked example 1

A television screen is 48 cm wide and 27 cm high. Work out the length of its diagonal.

$$d^2 = 48^2 + 27^2 = 3033, \text{ so } d = \sqrt{3033} = \mathbf{55.1 \text{ cm}}$$

Worked example 2

Work out the length y .



$$y^2 = 9.4^2 - 5.1^2 = 62.35, \text{ so } y = \mathbf{7.90 \text{ m}}$$

Common mistakes

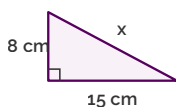
- Adding when the hypotenuse is given (the answer must be shorter than the hypotenuse).
- Writing $9.4 - 5.1$ and then squaring.

Exam tip: Write the first line of working, e.g. $9.4^2 - 5.1^2$: it usually earns the method mark.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out x .

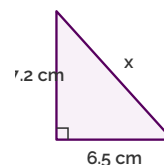
1



Answer $x = \dots\dots\dots$ cm

Work out x .

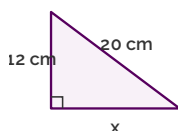
2



Answer $x = \dots\dots\dots$ cm

Work out x .

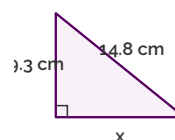
3



Answer $x = \dots\dots\dots$ cm

Work out x .

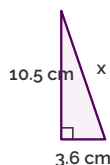
4



Answer $x = \dots\dots\dots$ cm

Work out x .

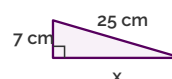
5



Answer $x = \dots\dots\dots$ cm

Work out x .

6



Answer $x = \dots\dots\dots$ cm

7

A rectangular field is 120 m by 50 m. How far is it diagonally across the field?

8

Work out the distance between the points (2, -3) and (8, 5).

9

Work out the distance between (-4, 1) and (3, 6).

10

A 4.5 m ladder reaches 4.2 m up a wall. How far is its foot from the wall?

11

Show that a triangle with sides 20 cm, 21 cm and 29 cm is right-angled.

12

A square has sides of 7 cm. Work out the length of a diagonal.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 ABCD is a trapezium. AD is parallel to BC. Angle ABC = 90° . AB = 8 cm, AD = 9 cm and BC = 15 cm.

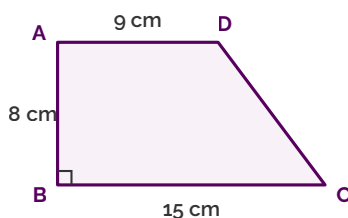


Diagram NOT accurately drawn

- (a) Work out the length of CD. [3]

Answer cm

- (b) Work out the perimeter of the trapezium. [1]

Answer cm

- (c) Work out the length of the diagonal AC. [2]

Answer cm

- 2 A rectangular park is 140 m long and 85 m wide. Ravi walks from one corner to the opposite corner along the edges. Mei walks straight across the park along the diagonal.

- (a) Work out how much further Ravi walks than Mei. [4]

Answer m

- (b) Mei walks at 1.2 m/s. How many seconds does her walk take? [1]

Answer s

- 3 P is the point $(-1, 4)$ and Q is the point $(5, -4)$.

(a) Work out the length of PQ.

[3]

Answer units

(b) Work out the coordinates of the midpoint of PQ.

[2]

Answer (..... ,)

- 4 The diagram shows a kite PQRS. The diagonals PR and QS meet at M at right angles. $PM = 5$ cm, $MR = 12$ cm and $QS = 8$ cm.

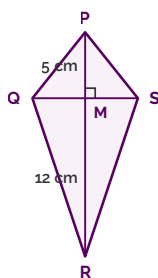


Diagram NOT
accurately drawn

(a) Work out the length PQ.

[2]

Answer cm

(b) Work out the length QR.

[2]

Answer cm

(c) Work out the perimeter of the kite.

[1]

Answer cm

- 5 A triangle has sides of length 2.8 m, 4.5 m and 5.3 m.

(a) Is the triangle right-angled? You must show your working.

[2]

(b) Work out the area of the triangle.

[2]

Answer m^2

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

2

UNIT 2 · KEY KNOWLEDGE

Trigonometry in right-angled triangles

I can... use sine, cosine and tangent to work out lengths and angles in right-angled triangles.

Spec ref: 4.8B (F) · Foundation & Higher

In a right-angled triangle, the ratios sin, cos and tan link an angle to two sides. Name the sides from the angle you are using: **opposite**, **adjacent** and **hypotenuse**.

Key points

- $\sin \theta = \text{opp} \div \text{hyp}$, $\cos \theta = \text{adj} \div \text{hyp}$, $\tan \theta = \text{opp} \div \text{adj}$
- Cover-up triangles: $O = H \times \sin \theta$, $H = O \div \sin \theta$ (and the same for cos and tan).
- To find an angle, use the inverse: $\theta = \tan^{-1}(\text{opp} \div \text{adj})$, etc.

Key words

sine

cosine

tangent

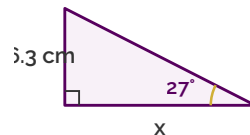
opposite

adjacent

inverse

Worked example 1

Work out the length x .



opp = 6.3, adj = x : $\tan 27^\circ = 6.3/x$, so $x = 6.3 \div \tan 27^\circ = \mathbf{12.4 \text{ cm}}$

Worked example 2

A right-angled triangle has hypotenuse 11.2 cm. The side next to angle θ is 4.7 cm. Work out θ .

$\cos \theta = 4.7 \div 11.2$, so $\theta = \cos^{-1}(4.7 \div 11.2) = \mathbf{65.2^\circ}$

Common mistakes

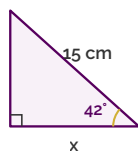
- Multiplying when the unknown side is on the bottom of the ratio (you must divide).
- Rounding the ratio before using the inverse function.

Exam tip: Check that your answer makes sense: the hypotenuse is the longest side, and angles in the triangle add to 180° .

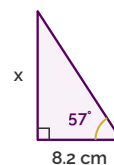
Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out x .

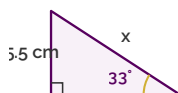
1

Answer $x = \dots\dots\dots$ cmWork out x .

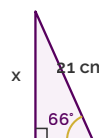
2

Answer $x = \dots\dots\dots$ cmWork out x .

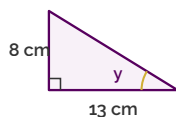
3

Answer $x = \dots\dots\dots$ cmWork out x .

4

Answer $x = \dots\dots\dots$ cmWork out the size of angle y .

5

Answer $y = \dots\dots\dots^\circ$ Work out the size of angle y .

6

Answer $y = \dots\dots\dots^\circ$ 7 Work out $9 \times \tan 52^\circ$. 8 Work out $14 \div \cos 28^\circ$. 9 Work out $\sin^{-1}(0.62)$.

10 A ramp 3.2 m long makes an angle of 6° with the ground. How high does it rise?

11 In triangle DEF, angle E = 90° , DE = 6 cm and EF = 11 cm. Work out angle EDF.

12 A rectangle is 12 cm long. Its diagonal makes an angle of 25° with the long side. Work out the width.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows the end of a symmetrical roof. The width is 9.6 m and each side of the roof makes an angle of 32° with the horizontal.

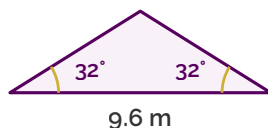


Diagram NOT accurately drawn

- (a) Work out the height of the roof. [3]

Answer m

- (b) Work out the length of one sloping side of the roof. [2]

Answer m

- (c) Work out the area of the triangular end of the roof. [2]

Answer m^2

- 2 ABC is a right-angled triangle. Angle $ABC = 90^\circ$, $AC = 17$ cm and $BC = 8$ cm.

- (a) Work out the size of angle BAC. [2]

Answer $^\circ$

- (b) D is a point on AB such that angle $BDC = 50^\circ$. Work out the length AD. [4]

Answer cm

3 An isosceles triangle has two sides of length 12 cm and a base of 9 cm.

(a) Work out the size of one of the base angles.

[3]

Answer°

(b) Work out the size of the angle at the top of the triangle.

[1]

Answer°

4 For safety, a ladder must make an angle between 70° and 78° with the ground. A ladder of length 5 m is placed with its foot 1.4 m from a vertical wall.

(a) Is the ladder safe? You must show your working.

[3]

(b) Work out the greatest height the 5 m ladder can safely reach up the wall.

[2]

Answer m

5 A straight road runs due east from a junction J. A house H is 650 m from J along a straight lane that makes an angle of 24° with the road.

(a) Work out the shortest distance from H to the road.

[2]

Answer m

(b) Work out the distance along the road from J to the point on the road nearest to H.

[2]

Answer m

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Elevation, depression and bearings

I can... solve problems with angles of elevation and depression and with bearings.

Spec ref: 4.8C (F), 4.8B (H) · Foundation & Higher

Angles of **elevation** (looking up) and **depression** (looking down) are measured from the horizontal. **Bearings** are measured clockwise from north and have three figures. Both lead to right-angled triangles.

Key points

- Elevation of B from A equals depression of A from B.
- Bearing of A from B = bearing of B from A $\pm 180^\circ$.
- Draw north lines at each point and mark the right angle before using trigonometry.

Key words

elevation

depression

bearing

clockwise

horizontal

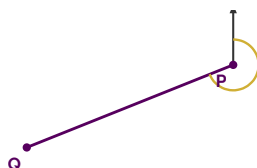
Worked example 1

Lucy's eyes are 1.5 m above the ground. She stands 28 m from a tree and looks up at the top at an angle of elevation of 39° . Work out the height of the tree.

$$28 \tan 39^\circ = 22.674\dots; \text{ add eye height: } 1.5 + 22.674 = \mathbf{24.2 \text{ m}}$$

Worked example 2

Q is 15 km west and 6 km south of P. Work out the bearing of Q from P.



$$\text{Angle from south towards west} = \tan^{-1}(15/6) = 68.2^\circ. \text{ Bearing} = 180^\circ + 68.2^\circ = \mathbf{248.2^\circ}$$

Common mistakes

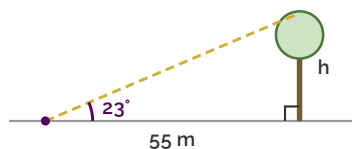
- Measuring an angle of depression from the vertical.
- Giving a bearing as 68° instead of 068° , or measuring anticlockwise.

Exam tip: A sketch is worth time: label the horizontal, the north line and the right angle.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out the height h .

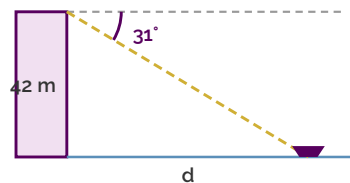
1



Answer $h = \dots\dots\dots$ m

Work out the distance d .

2



Answer $d = \dots\dots\dots$ m

3

A tower is 72 m tall. From point P on level ground, the angle of elevation of the top is 16° . How far is P from the foot of the tower?

4

From a lighthouse 48 m high, a boat is 210 m from the foot. Work out the angle of depression of the boat.

5

A drone is 120 m directly above Sam. Ana is 300 m from Sam. Work out the angle of elevation of the drone from Ana.

6

The bearing of B from A is 137° . Work out the bearing of A from B.

7

The bearing of D from C is 290° . Work out the bearing of C from D.

8

A boat sails 34 km on a bearing of 075° . How far east has it sailed?

9

A walker goes 6.8 km on a bearing of 320° . How far north is she from her start?

10

Y is 4 km north and 9 km east of X. Work out the bearing of Y from X.

11

N is 7 km south and 3 km east of M. Work out the bearing of N from M.

12

From the top of a 30 m building, the angle of depression of a car is 41° . How far is the car from the building?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 A yacht leaves harbour H and sails 22 km on a bearing of 050° to a buoy B. It then sails 16 km on a bearing of 140° to a lighthouse L.

(a) Explain why angle $HLB = 90^\circ$. [2]

(b) Work out the distance HL. [2]

Answer km

(c) Work out the bearing of L from H. [3]

Answer°

- 2 A vertical flagpole FT is 34 m tall. Points A and B are on level ground on opposite sides of the flagpole, in a straight line with its foot F. The angle of elevation of T from A is 28° and from B is 47° .

(a) Work out the distance AF. [2]

Answer m

(b) Work out the distance AB. [3]

Answer m

- 3 An aircraft is flying horizontally at a height of 1200 m towards a control tower C on the ground. At one moment the angle of depression of C from the aircraft is 9° . One minute later the angle of depression is 21° .

(a) Work out the horizontal distance the aircraft travels in that minute. [4]

Answer m

(b) Work out the speed of the aircraft in km/h. [2]

Answer km/h

4 Town T is 45 km from city C on a bearing of 235° .

(a) Work out how far west of C the town is.

[2]

Answer km

(b) Write down the bearing of C from T.

[1]

Answer $^\circ$

5 From the top of a vertical cliff 75 m high, the angle of depression of a buoy is 12° . A boat is 130 m from the foot of the cliff, between the cliff and the buoy, in a straight line.

(a) Work out the angle of depression of the boat.

[2]

Answer $^\circ$

(b) Work out the distance between the boat and the buoy.

[3]

Answer m

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Obtuse angles and trigonometric graphs

I can... use the graphs of $\sin$, $\cos$ and $\tan$ for any angle to solve equations and describe transformations.

Spec ref: 4.8A (H), 3.3 (H) · Higher

$\sin$, $\cos$ and $\tan$ are defined for angles of **any size**. $\sin x$ and $\cos x$ repeat every 360° ; $\tan x$ repeats every 180° .

Key points

- $\sin(180^\circ - x) = \sin x$; $\cos(180^\circ - x) = -\cos x$.
- Use the calculator for one solution, then symmetry and the period for the rest.
- $\sin x + a$: up a . $\sin(x + a)$: a° **left**. $a \sin x$: vertical stretch.

Key words

obtuse

period

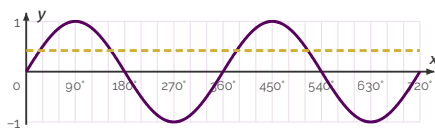
symmetry

translation

stretch

Worked example 1

Solve $\sin x = 0.42$ for $0^\circ \leq x \leq 720^\circ$.



$\sin^{-1}0.42 = 24.8^\circ$; $180 - 24.8 = 155.2^\circ$; add 360° : 384.8° , 515.2° . **$x = 24.8^\circ, 155.2^\circ, 384.8^\circ, 515.2^\circ$**

Worked example 2

Write down the maximum value of $y = 3 \cos x + 1$ and the smallest positive value of x where it occurs.

$\cos x$ has maximum 1 at 0° and 360° , so the maximum is $3 \times 1 + 1 = 4$, first at **$x = 360^\circ$** for positive x .

Common mistakes

- Stopping after the calculator answer.
- Moving $y = \sin(x + 30^\circ)$ to the right instead of the left.

Exam tip: Sketch the graph across the whole interval asked for and draw the line $y = k$: count the crossings.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

1 Write down $\sin 150^\circ$.

2 $\cos 120^\circ = -\cos$ $^\circ$

3 $\sin 100^\circ = \sin$ $^\circ$

4 Find the obtuse angle x with $\sin x = 0.9$.

5 Find the angle x between 0° and 180° with $\cos x = -0.25$.

6 Solve $\cos x = 0.6$ for $0^\circ \leq x \leq 360^\circ$.

7 Solve $\tan x = 1.5$ for $0^\circ \leq x \leq 360^\circ$.

8 Solve $\sin x = -0.7$ for $0^\circ \leq x \leq 360^\circ$.

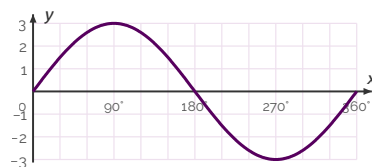
9 Solve $\cos x = 0.5$ for $-180^\circ \leq x \leq 180^\circ$.

10 Write down the maximum value of $y = 2 \sin x - 3$.

11 The graph of $y = \cos x$ is translated to give $y = \cos(x - 90^\circ)$. Describe the translation.

The graph shows $y = a \sin x$. Write down the value of a .

12

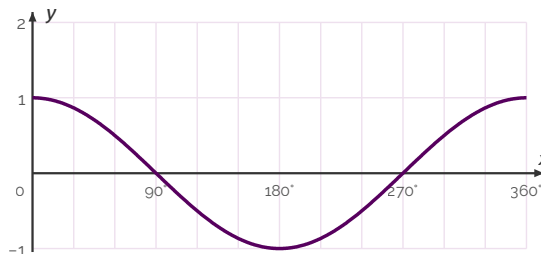


Answer $a = \dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$.



- (a) Use the graph to estimate the solutions of $\cos x = -0.4$. [2]

Answer $x = \dots\dots\dots$

- (b) On the same axes, sketch the graph of $y = \cos x + 1$. [2]

- (c) Write down the coordinates of the minimum point of $y = \cos x + 1$. [1]

Answer $(\dots\dots, \dots\dots)$

- 2 Solve these equations for $0^\circ \leq x \leq 360^\circ$. Give your answers correct to 1 decimal place.

- (a) $5 \sin x + 2 = 0$ [3]

Answer $x = \dots\dots\dots$

- (b) $2 \tan x = -5$ [3]

Answer $x = \dots\dots\dots$

- 3 In triangle ABC, angle BAC is obtuse and $\sin \text{BAC} = 0.8$.

- (a) Work out the size of angle BAC. [2]

Answer $\dots\dots^\circ$

- (b) Write down the value of $\cos \text{BAC}$. [1]

Answer $\dots\dots\dots$

4 The graph of $y = \sin x$ is transformed to the graph of $y = \sin(x + 60^\circ)$.

(a) Describe the transformation.

[2]

(b) Write down the coordinates of the maximum point of $y = \sin(x + 60^\circ)$ for $0^\circ \leq x \leq 360^\circ$. **[1]**

Answer (.....,)

(c) Solve $\sin(x + 60^\circ) = 0.5$ for $0^\circ \leq x \leq 360^\circ$.

[3]

Answer $x = \dots\dots$

5 The function $f(x) = a \cos x + b$ has a maximum value of 7 and a minimum value of -1.

(a) Work out the values of a and b , where $a > 0$.

[3]

Answer $a = \dots\dots$, $b = \dots\dots$

(b) Solve $f(x) = 5$ for $0^\circ \leq x \leq 360^\circ$.

[2]

Answer $x = \dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

The sine rule

I can... use the sine rule to work out sides and angles in any triangle, including the ambiguous case.

Spec ref: 4.8C (H) · Higher

The **sine rule** connects each side of any triangle with the angle **opposite** it. You need one complete pair (a side and its opposite angle) and one more piece of information.

Key points

- $a/\sin A = b/\sin B = c/\sin C$
- To find an angle, turn it upside down: $\sin A/a = \sin B/b$.
- An angle found from sin could also be 180° minus it (check the angle sum).

Key words

sine rule

opposite

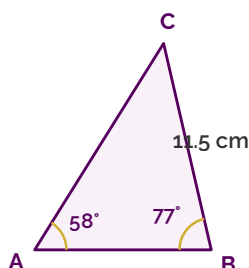
included

ambiguous

obtuse

Worked example 1

Work out the length of AB.



C = 45° , so $AB = 11.5 \times \sin 45^\circ \div \sin 58^\circ = \mathbf{9.59 \text{ cm}}$

Worked example 2

In triangle PQR, PR = 6.8 cm, PQ = 9.4 cm and angle PRQ = 118° . Work out angle PQR.

$\sin Q/6.8 = \sin 118^\circ/9.4$, so $\sin Q = 0.6387\dots$, Q = $\mathbf{39.7^\circ}$ (Q cannot be obtuse because R already is)

Common mistakes

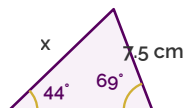
- Using a side with an angle that is not opposite it.
- Forgetting to find the third angle first when it is needed.

Exam tip: Write the rule with the letters from the question before you substitute any numbers.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out x .

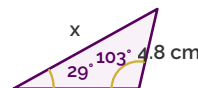
1



Answer $x = \dots\dots\dots$ cm

Work out x .

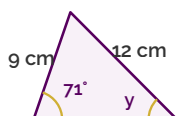
2



Answer $x = \dots\dots\dots$ cm

Work out the size of angle y .

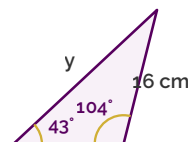
3



Answer $y = \dots\dots\dots^\circ$

Work out the length y .

4



Answer $y = \dots\dots\dots$ cm

5

In triangle ABC, $a = 13$ cm, $A = 81^\circ$ and $B = 37^\circ$. Work out b .

6

In triangle PQR, $p = 8.5$ cm, $q = 5.2$ cm and $P = 124^\circ$. Work out angle Q.

7

In triangle XYZ, $x = 14$ cm, $X = 63^\circ$ and $Z = 48^\circ$. Work out z .

8

In triangle KLM, $KL = 20$ cm, angle $M = 95^\circ$ and angle $K = 40^\circ$. Work out LM.

9

In triangle ABC, $a = 7$ cm, $b = 10$ cm and $A = 38^\circ$. Work out the two possible sizes of angle B.

10

In triangle DEF, $d = 15$ cm, $e = 11$ cm and $D = 72^\circ$. Work out angle E.

11

Write down the obtuse angle that has the same sine as 35° .

12

Two angles of a triangle are 52° and 61° . The side opposite 61° is 9 cm. Work out the side opposite 52° .

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 Two surveyors stand at points A and B on one bank of a straight river, 85 m apart. A tree C is on the other bank. Angle $CAB = 64^\circ$ and angle $CBA = 49^\circ$.

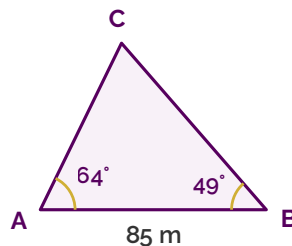


Diagram NOT accurately drawn

- (a) Work out the distance AC.

[3]

Answer m

- (b) Work out the width of the river (the shortest distance from C to AB).

[2]

Answer m

- 2 In triangle PQR, $PR = 12.6$ cm, $QR = 9$ cm and angle $QPR = 41^\circ$.

- (a) Show that there are two possible values for angle PQR, and work out both.

[4]

Answer $^\circ$ or $^\circ$

- (b) Given that angle PQR is acute, work out the length PQ.

[3]

Answer cm

- 3 Ship A is 20 km from port P on a bearing of 035° . Ship B is on a bearing of 110° from P and on a bearing of 150° from A.

(a) Show that angle $PAB = 65^\circ$.

[2]

(b) Work out the distance AB.

[3]

Answer km

(c) Work out the distance PB.

[2]

Answer km

- 4 ABCD is a quadrilateral with diagonal BD. $AD = 7.2$ cm, angle $DAB = 118^\circ$, angle $ABD = 28^\circ$, angle $DBC = 54^\circ$ and angle $BCD = 71^\circ$.

(a) Work out the length BD.

[2]

Answer BD = cm

(b) Work out the length CD.

[2]

Answer CD = cm

(c) Work out the area of triangle BCD.

[3]

Answer cm^2

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

The cosine rule

I can... use the cosine rule to work out a side or an angle in any triangle.

Spec ref: 4.8C (H) · Higher

Use the **cosine rule** when there is no complete side–angle pair: either two sides and the angle **between** them, or all three sides. It is Pythagoras' theorem with a correction for angles that are not 90° .

Key points

- $a^2 = b^2 + c^2 - 2bc \cos A$ (side)
- $\cos A = (b^2 + c^2 - a^2)/2bc$ (angle)
- If $\cos A$ is negative, angle A is obtuse — your calculator will give it correctly.

Key words

cosine rule

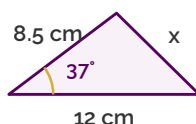
included angle

obtuse

rearrange

Worked example 1

Work out the length x .



$$x^2 = 8.5^2 + 12^2 - 2 \times 8.5 \times 12 \times \cos 37^\circ = 53.33\dots, x = \mathbf{7.30 \text{ cm}}$$

Worked example 2

A triangle has sides 5 cm, 9 cm and 12 cm. Work out its largest angle.

$$\text{Largest angle is opposite 12 cm: } \cos \theta = (25 + 81 - 144)/90 = -0.4222\dots, \theta = \mathbf{115.0^\circ}$$

Common mistakes

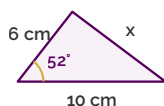
- Typing $8.5^2 + 12^2 - 2 \times 8.5 \times 12$ and then $\times \cos 37^\circ$ in two separate steps.
- Putting the wrong side as a when finding an angle: a must be opposite the angle.

Exam tip: Type the whole expression into your calculator in one go, then write down the value before you square root.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out x .

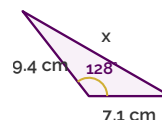
1



Answer $x = \dots\dots\dots$ cm

Work out x .

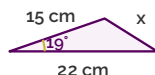
2



Answer $x = \dots\dots\dots$ cm

Work out x .

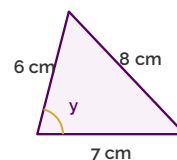
3



Answer $x = \dots\dots\dots$ cm

Work out angle y .

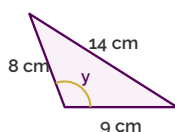
4



Answer $y = \dots\dots\dots^\circ$

Work out angle y .

5



Answer $y = \dots\dots\dots^\circ$

In triangle ABC, $b = 11$ cm, $c = 4$ cm and $A = 90^\circ$. Use the cosine rule to work out a . What do you notice?

6

In triangle PQR, $PQ = 5.5$ cm, $QR = 8$ cm and angle $PQR = 140^\circ$. Work out PR.

7

A triangle has sides 10 cm, 10 cm and 16 cm. Work out the largest angle.

8

A triangle has sides 3 cm, 7 cm and 8 cm. Work out the smallest angle.

9

Two sides of a triangle are 13 cm and 15 cm and the angle between them is 60° . Work out the third side.

10

Without calculating, is the angle opposite the 9 cm side of a 4 cm, 6 cm, 9 cm triangle acute or obtuse? Explain.

11

A triangle has sides 7 cm, 8 cm and 13 cm. Work out the largest angle exactly.

12

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 In an orienteering race, Kai runs 2.4 km from the start S to checkpoint C. He then turns and runs 3.1 km to the finish F. Angle SCF = 105° .

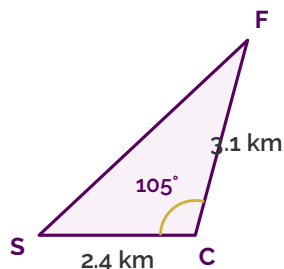


Diagram NOT accurately drawn

- (a) Work out the direct distance SF. [3]

Answer km

- (b) Work out angle CSF. [3]

Answer $^\circ$

- 2 A triangular plot of land has sides of length 17 m, 11 m and 9 m.

- (a) Work out the size of the largest angle. [3]

Answer $^\circ$

- (b) Work out the size of the smallest angle. [2]

Answer $^\circ$

- (c) Work out the area of the plot. [2]

Answer m^2

- 3 A plane flies 60 km from airport A on a bearing of 070° to point P. It then flies 45 km on a bearing of 170° to point Q.

(a) Show that angle $APQ = 80^\circ$.

[2]

(b) Work out the distance AQ.

[3]

Answer km

(c) Work out the bearing of Q from A.

[3]

Answer $^\circ$

- 4 In triangle ABC, $AB = x$ cm, $AC = (x + 3)$ cm, $BC = 13$ cm and angle $BAC = 60^\circ$.

(a) Show that $x^2 + 3x - 160 = 0$.

[3]

(b) Work out the value of x , correct to 3 significant figures.

[2]

Answer $x =$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Area of a triangle and mixed problems

I can... use $\frac{1}{2}ab \sin C$ and choose the right rule to solve multi-step triangle problems.

Spec ref: 4.8E (H), 4.8C (H) · Higher

For any triangle, area = $\frac{1}{2}ab \sin C$, where a and b are two sides and C is the angle **between** them. In mixed problems, decide at each step which rule fits the information you have.

Key points

- Area = $\frac{1}{2}ab \sin C$ (two sides and the included angle).
- Choosing a rule: a side–opposite angle pair → sine rule; otherwise → cosine rule; right angle → SOH CAH TOA or Pythagoras.
- A given area can be used backwards to find a side or an angle.

Key words

area

included angle

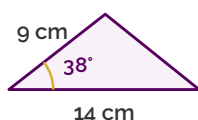
segment

sector

strategy

Worked example 1

Work out the area of the triangle.



$$\text{Area} = \frac{1}{2} \times 9 \times 14 \times \sin 38^\circ = 38.8 \text{ cm}^2$$

Worked example 2

A triangle has area 50 cm^2 . Two sides are 12 cm and 10 cm. Work out the acute angle between them.

$$\frac{1}{2} \times 12 \times 10 \times \sin \theta = 50, \text{ so } \sin \theta = 50/60 \text{ and } \theta = 56.4^\circ$$

Common mistakes

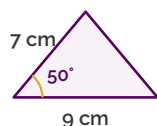
- Using an angle that is not between the two sides.
- Using $\frac{1}{2} \times \text{base} \times \text{slant side}$ — the height must be perpendicular.

Exam tip: Label the triangle with a, b, c and A, B, C before you start a multi-step problem.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Work out the area.

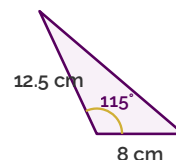
1



Answer cm^2

Work out the area.

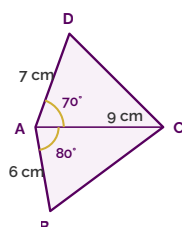
2



Answer cm^2

Work out the area of the quadrilateral.

3



Answer cm^2

Work out the area of a triangle with sides 20 cm and 16 cm and an included angle of 27° .

4

Work out the area of an equilateral triangle with sides of 12 cm.

5

A parallelogram has sides 8 cm and 11 cm with an angle of 65° . Work out its area.

6

A triangle has area 24 cm^2 . Two sides are 8 cm and 9 cm. Work out the acute angle between them.

7

A triangle has area 35 cm^2 , one side 10 cm and an included angle of 44° . Work out the other side.

8

A regular hexagon has sides of 5 cm. Use $\frac{1}{2}ab \sin C$ to work out its area.

9

A triangle has sides 6 cm, 8 cm and 10 cm. Which rule would you use first to find its area, and why?

10

In triangle ABC, $a = 9$, $b = 7$ and $c = 6$. Work out angle C, then the area.

11

A triangle has sides 5 cm and 8 cm and area 10 cm^2 . Work out the **obtuse** angle between these sides.

12

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 A triangular garden PQR has $PQ = 14$ m, $PR = 18$ m and angle $QPR = 47^\circ$.

(a) Work out the area of the garden. [2]

Answer m^2

(b) Work out the length QR. [3]

Answer m

(c) Work out angle PQR. [3]

Answer $^\circ$

- 2 The diagram shows a sector OAB of a circle, centre O, radius 8 cm. Angle $AOB = 70^\circ$. The shaded segment lies between the chord AB and the arc.

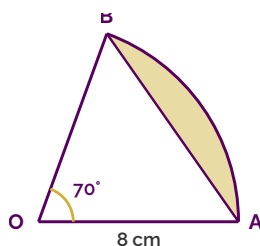


Diagram NOT accurately drawn

(a) Work out the area of triangle OAB. [2]

Answer cm^2

(b) Work out the area of the shaded segment. [3]

Answer cm^2

(c) Work out the perimeter of the shaded segment. [2]

Answer cm

3 Triangle XYZ has $XY = 11$ cm, $XZ = 15$ cm and area 60 cm^2 . Angle YXZ is acute.

(a) Work out angle YXZ.

[3]

Answer°

(b) Work out YZ.

[3]

Answer cm

4 A kite ABCD has $AB = AD = 9$ cm and $CB = CD = 15$ cm. Angle BAD = 110° .

(a) Work out the length of the diagonal BD.

[2]

Answer cm

(b) Work out the area of the kite.

[3]

Answer cm^2

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Trigonometry in three dimensions

I can... use Pythagoras and trigonometry in 3D, including the angle between a line and a plane.

Spec ref: 4.8D (H), 4.8F (H) · Higher

In 3D, find a right-angled triangle containing what you need and draw it flat.

Key points

- Space diagonal of a cuboid: $\sqrt{l^2 + w^2 + h^2}$.
- For a pyramid with its vertex above the centre of the base, use half a base diagonal.
- The angle between a line and a plane: drop a perpendicular from the top of the line to the plane, then use trig.

Key words

three dimensions

space diagonal

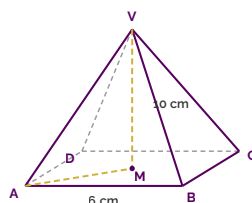
plane

projection

pyramid

Worked example 1

A square-based pyramid has base 6 cm by 6 cm and slant edges of 10 cm. Work out its vertical height.



Half diagonal of base = $\frac{1}{2}\sqrt{6^2 + 6^2} = 4.24$ cm. Height = $\sqrt{10^2 - 4.24^2} = \sqrt{82} = 9.06$ cm

Worked example 2

For the same pyramid, work out the angle between the edge VA and the base.

$\cos \theta = 4.24/10$, so $\theta = 64.9^\circ$

Common mistakes

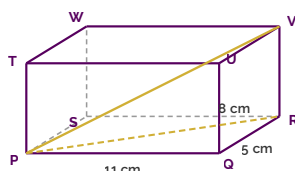
- Using a full base diagonal instead of half for a pyramid.
- Choosing the wrong angle: it must be between the line and a line **in** the plane.

Exam tip: Draw each 2D triangle you use separately, labelled with the letters from the 3D diagram.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

PQRSTUVW is a cuboid. Work out the length PR.

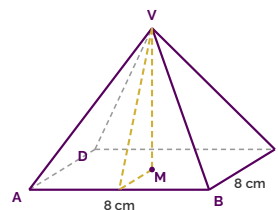
1



Answer PR = cm

VABCD is a pyramid with a square base of side 8 cm. V is 10 cm vertically above the centre M. N is the midpoint of AB. Work out VN.

2



Answer VN = cm

3

For the cuboid in question 1, work out PV.

4

Work out the angle between PV and the base PQRS.

5

Work out angle UPQ.

6

A cube has edges of 9 cm. Work out the length of a space diagonal.

7

Work out the angle between a space diagonal of a cube and its base.

8

For the pyramid in question 2, work out the angle between the face VAB and the base.

9

For the same pyramid, work out the length VA.

10

A room is 5 m long, 4 m wide and 2.5 m high. Work out the distance from one bottom corner to the opposite top corner.

11

A pole 6 m tall stands at the centre of a square field of side 30 m. Work out the angle of elevation of the top of the pole from a corner.

12

A cuboid has length 12 cm, width 9 cm and space diagonal 17 cm. Work out its height.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows a triangular prism. The cross-section ABC is a right-angled triangle with angle $ABC = 90^\circ$, $AB = 8$ cm and $BC = 6$ cm. The prism is 20 cm long: $AD = BE = CF = 20$ cm. The rectangle ABED is horizontal.

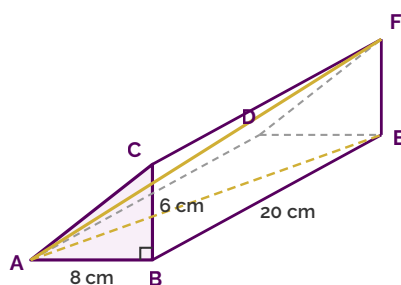


Diagram NOT accurately drawn

- (a) Work out the length AF. [3]

Answer AF = cm

- (b) Work out the angle between AF and the base ABED. [3]

Answer°

- 2 VABCD is a pyramid with a rectangular base ABCD. $AB = 12$ cm and $BC = 9$ cm. The vertex V is vertically above the centre M of the base and $VM = 14$ cm.

- (a) Work out the length VA. [3]

Answer VA = cm

- (b) Work out the angle between VA and the base. [2]

Answer°

- (c) Work out the angle between the face VBC and the base. [3]

Answer°

3 A cuboid box measures 50 cm by 30 cm by 25 cm.

(a) Work out the length of the longest straight rod that will fit in the box.

[3]

Answer cm

(b) Work out the angle the rod makes with the base of the box.

[2]

Answer°

4 A solid cone has vertex V, vertical height 12 cm and slant height 15 cm. A and B are points at opposite ends of a diameter of the base.

(a) Work out the radius of the base.

[2]

Answer cm

(b) Work out the angle between VA and the base.

[2]

Answer°

(c) Work out angle AVB.

[2]

Answer°

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Rotations, reflections and translations

I can... rotate, reflect and translate shapes and describe these transformations fully.

Spec ref: 5.2A-I · Foundation & Higher

A rotation needs a **centre** and an **angle**; a reflection a **mirror line**; a translation a **column vector**. The image is congruent to the object.

Key points

- Anticlockwise rotations are positive ($+90^\circ$); clockwise rotations are negative (-90°).
- Mirror lines you must know: $x = a$ (vertical), $y = b$ (horizontal), $y = x$, $y = -x$.
- Translation: top number = units right (+) or left (-); bottom number = units up (+) or down (-).

Key words

rotation

reflection

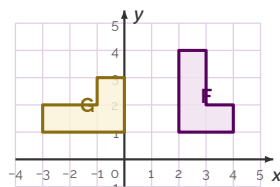
translation

column vector

congruent

Worked example 1

Shape F is rotated to shape G. Describe the rotation.



Use tracing paper to test centres: **rotation of $+90^\circ$ (anticlockwise) about (1, 0)**

Worked example 2

Reflect the point (5, 2) in the line $y = -x$.

In $y = -x$, $(x, y) \rightarrow (-y, -x)$, so the image is **(-2, -5)**.

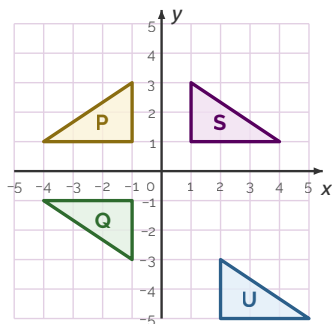
Common mistakes

- Mixing up the lines $x = 2$ (vertical) and $y = 2$ (horizontal).
- Rotating the wrong way: -90° means clockwise.

Exam tip: Check a rotation by measuring: each vertex must stay the same distance from the centre.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Describe fully the single transformation that maps S onto P.



1

2

Using the diagram in question 1, describe fully the transformation that maps S onto Q.

3

Describe fully the transformation that maps S onto U.

4

Describe fully the transformation that maps P onto Q.

5

Reflect $(-4, 1)$ in the line $y = x$.

6

Reflect $(-4, 1)$ in the line $y = 3$.

7

Rotate $(2, 5)$ through -90° about $(0, 0)$.

8

Rotate $(2, 5)$ through 180° about $(0, 2)$.

9

Rotate $(2, 5)$ through $+90^\circ$ about $(1, 1)$.

10

Translate $(-3, 4)$ by 6 right and 7 down.

11

The point $(5, -1)$ is translated to $(2, 3)$. Describe the translation in words.

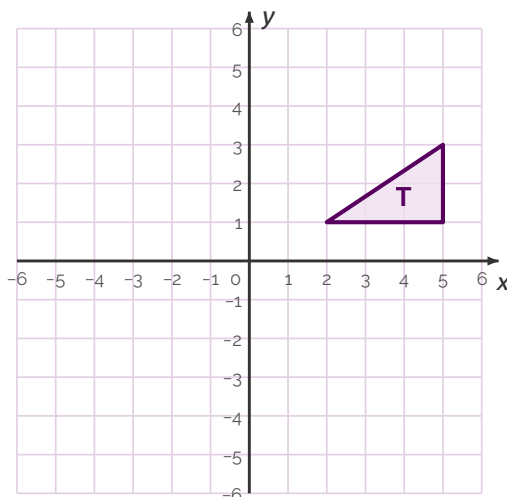
12

Which of these does not keep a shape congruent: rotation, enlargement, reflection?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows triangle T.

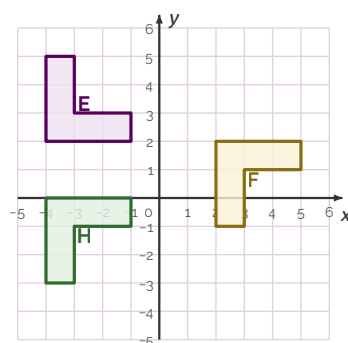


- (a) Rotate triangle T through 180° about (1, 0). Label the image A. [2]
- (b) Reflect triangle T in the line $y = x$. Label the image B. [2]
- (c) Translate triangle T by the vector shown. Label the image C. [2]

$$\mathbf{v} = \begin{pmatrix} -7 \\ 3 \end{pmatrix}$$

- (d) Rotate triangle T through -90° about (0, -1). Label the image D. [2]

- 2 The diagram shows shapes E, F and H.



- (a) Describe fully the single transformation that maps E onto F. [3]
-
- (b) Describe fully the single transformation that maps E onto H. [2]
-

- 3 The point $K(3, -2)$ is mapped to K' by a translation. K' is then reflected in the x -axis to give $K''(-1, 6)$.

(a) Find the coordinates of K' .

[2]

Answer (.....,)

(b) Write down the column vector of the translation.

[2]

- 4 Explain why a triangle and its image after a rotation are congruent.

(a) Explain.

[2]

(b) A triangle has an area of 14 cm^2 . It is reflected, then translated. Write down the area of the final image.

[1]

Answer cm^2

- 5 A rotation of 90° anticlockwise maps the point $(6, 1)$ onto the point $(2, 5)$.

(a) Find the centre of the rotation.

[3]

Answer (.....,)

(b) Find the image of $(2, 5)$ under the same rotation.

[2]

Answer (.....,)

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Enlargement

I can... enlarge a shape by a positive or fractional scale factor, with or without a centre.

Spec ref: 5.2J–L · Foundation & Higher

An **enlargement** needs a **centre** and a **scale factor**. Lengths are multiplied by the scale factor and angles stay the same: the image is **similar**.

Key points

- Distance from centre $\times$ scale factor = new distance from centre.
- Scale factor = image length $\div$ object length.
- Centre: draw lines through corresponding vertices; they cross at the centre.

Key words

enlargement

scale factor

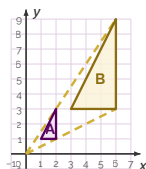
centre

similar

fractional

Worked example 1

Enlarge triangle A by scale factor 3, centre (0, 0).



Centre (0, 0), so multiply each coordinate by 3: **(3, 3), (6, 3), (6, 9)**

Worked example 2

A rectangle 10 cm by 6 cm is enlarged by scale factor $\frac{1}{2}$. Find the new size and say what happens to the angles.

Lengths $\times \frac{1}{2}$: **5 cm by 3 cm**. Angles are unchanged — still 90° .

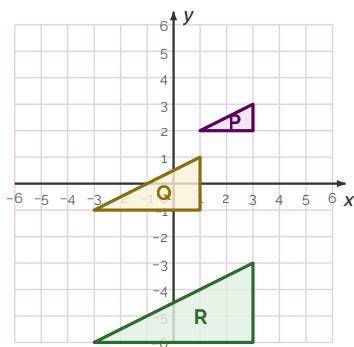
Common mistakes

- Adding the scale factor instead of multiplying.
- Counting from the origin when the centre is a different point.

Exam tip: After drawing an enlargement, check one length: image length $\div$ object length must equal the scale factor.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Describe fully the single transformation that maps P onto Q.



1

2

Using the diagram in question 1, describe fully the transformation that maps P onto R.

3

Describe fully the transformation that maps R onto P.

4

Enlarge $(3, -2)$ by scale factor 4, centre $(0, 0)$.

5

Enlarge $(8, 6)$ by scale factor $1/2$, centre $(0, 0)$.

6

Enlarge $(5, 2)$ by scale factor 3, centre $(2, 1)$.

7

Enlarge $(7, 9)$ by scale factor $1/3$, centre $(1, 3)$.

8

A triangle with sides 4 cm, 5 cm and 7 cm is enlarged by scale factor 2.5. Find the new perimeter.

9

A shape has an angle of 37° . It is enlarged by scale factor 4. What is the size of the angle in the image?

10

A line of length 12 cm becomes 3 cm after an enlargement. Find the scale factor.

11

The point $(4, 6)$ is enlarged with centre $(0, 0)$ to $(6, 9)$. Find the scale factor.

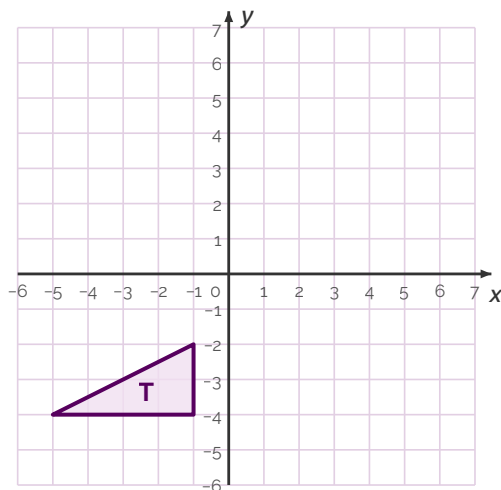
12

Is an enlargement with scale factor $1/2$ bigger or smaller than the original?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

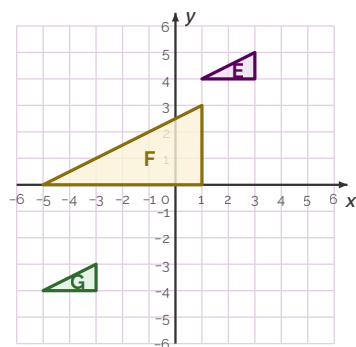
Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows triangle T.



- (a) Enlarge triangle T by scale factor 2, centre $(-6, -5)$. Label the image A. [3]
- (b) Enlarge triangle T by scale factor $\frac{1}{2}$, centre $(-5, -6)$. Label the image B. [2]

- 2 The diagram shows triangles E, F and G.



- (a) Describe fully the single transformation that maps E onto F. [3]

- (b) Describe fully the single transformation that maps F onto G. [3]

- 3 Photograph A is 15 cm wide and 10 cm high. It is enlarged to make photograph B, which is 24 cm wide.

(a) Work out the scale factor of the enlargement.

[1]

Answer

(b) Work out the height of photograph B.

[2]

Answer cm

(c) Photograph B is reduced to make photograph C, 6 cm wide. Work out the scale factor from B to C, as a fraction.

[2]

Answer

- 4 Triangle J has vertices (2, 2), (6, 2) and (6, 8).

(a) J is enlarged by scale factor $\frac{1}{2}$, centre (0, 0). Write down the coordinates of the image.

[2]

(b) Explain why the image is similar but not congruent to J.

[2]

- 5 An enlargement has scale factor 3. It maps the point (2, 1) onto the point (8, 7).

(a) Find the coordinates of the centre of enlargement.

[3]

Answer (.....,

(b) Find the image of (0, 3) under the same enlargement.

[2]

Answer (.....,

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Describing and combining transformations

I can... describe transformations fully and replace two transformations by a single one.

Spec ref: 5.2M, 5.2I · Foundation & Higher

To **describe fully**, name the transformation and give every detail. Two transformations in a row can often be replaced by a **single** one.

Key points

- Rotation: centre, angle, direction. Reflection: mirror line. Translation: vector. Enlargement: centre, scale factor.
- Reflections in two perpendicular lines = rotation of 180° about their crossing point. Two translations = one translation.

Key words

describe fully

combined

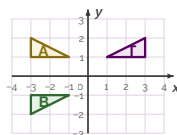
single transformation

congruent

equivalent

Worked example 1

T is reflected in the y-axis to give A. A is reflected in the x-axis to give B. Describe the single transformation that maps T onto B.



$(1, 1) \rightarrow (-1, 1) \rightarrow (-1, -1)$, and the same for every vertex: **rotation of 180° about $(0, 0)$**

Worked example 2

A translation 2 right and 5 down is followed by a translation 4 left and 1 up. Describe the single translation.

Add the parts: across $2 + (-4) = -2$, up $-5 + 1 = -4$. **2 left and 4 down**, column vector with top -2 and bottom -4 .

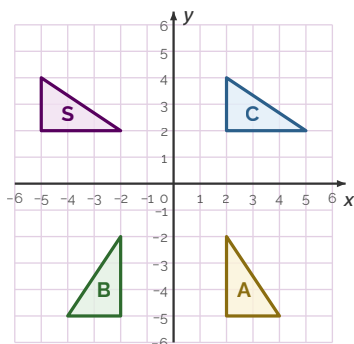
Common mistakes

- Writing two transformations when the question asks for a single one.
- Leaving out the centre of a rotation or enlargement.

Exam tip: Count the marks: 3 marks for a rotation (type, angle, centre), 2 for a reflection (type, line).

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

Describe fully the single transformation that maps S onto A.



1

Using the diagram in question 1, describe fully the transformation that maps S onto B.

2

Describe fully the transformation that maps S onto C.

3

Describe fully the transformation that maps A onto B.

4

A shape is reflected in $x = 2$ and then in $x = 5$. What single transformation is this?

5

A shape is rotated 90° clockwise about $(0, 0)$ twice. What single transformation is this?

6

Reflection in the x -axis followed by reflection in the y -axis is the same as which single transformation?

7

Translate by 3 right 1 up, then by 5 left 4 up. Give the single translation in words.

8

Enlargement scale factor 2 then scale factor 3, both centre $(0, 0)$. Single transformation?

9

Which transformations preserve both length and angle?

10

A rotation maps $(1, 0)$ to $(0, 1)$ and $(2, 0)$ to $(0, 2)$. Describe it fully.

11

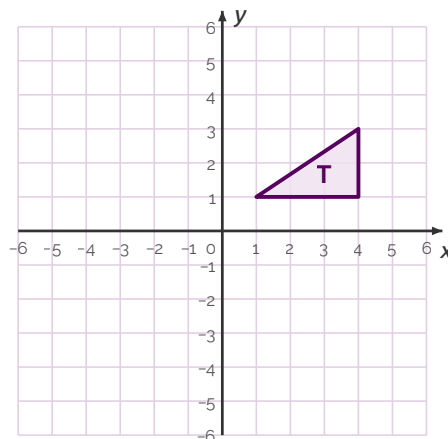
Rotation of 360° about any point is the same as which transformation?

12

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 The diagram shows triangle T.



- (a) Reflect triangle T in the y-axis. Label the image P. [2]
- (b) Rotate triangle P through 90° anticlockwise about (0, 0). Label the image Q. [2]
- (c) Describe fully the single transformation that maps T onto Q. [2]

- 2 Shape M has vertices (2, -1), (5, -1) and (5, -3).

- (a) M is mapped onto N with vertices (0, 1), (-3, 1), (-3, 3). Describe fully the single transformation. [3]

- (b) M is translated 4 left and 2 up, then that image is translated 1 left and 6 up. Describe fully the single transformation. [2]

- (c) Describe fully the single transformation that maps N onto M. [2]

- 3 A transformation R is a reflection in the line $y = 2$. A transformation S is a reflection in the line $y = 5$.

(a) The point (3, 1) is transformed by R and then by S. Find its final position. [2]

Answer (.....,)

(b) Describe fully the single transformation equivalent to R followed by S. [2]

- 4 Triangle V has vertices (2, 2), (4, 2) and (4, 6). Triangle W has vertices (3, 3), (4, 3) and (4, 5).

(a) Describe fully the single transformation that maps V onto W. [3]

(b) Is W congruent to V? Give a reason. [1]

- 5 A shape is rotated through 90° clockwise about (0, 0), and then the image is reflected in the x-axis.

(a) Find the final image of the point (2, 5). [2]

Answer (.....,)

(b) Describe fully the single transformation equivalent to these two transformations. [2]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Vectors

I can... use vector notation, combine vectors and work out the modulus of a vector.

Spec ref: 5.1A–F · Higher

A **vector** has **magnitude** (size) and **direction**. It can be written in bold (**a**), as AB with an arrow, or as a column vector. The **resultant** of two vectors is the single vector that has the same effect as one followed by the other.

Key points

- Add, subtract and multiply by a scalar row by row (top with top, bottom with bottom).
- $k\mathbf{a}$ is parallel to $\mathbf{a}$; if k is negative it points the opposite way.
- Modulus $|\mathbf{a}| = \sqrt{(\text{top})^2 + (\text{bottom})^2}$ (Pythagoras).

Key words

vector

magnitude

modulus

resultant

scalar

Worked example 1

Work out $3\mathbf{u} + 2\mathbf{v}$ and $|\mathbf{u} + \mathbf{v}|$.

$$\mathbf{u} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$3\mathbf{u} + 2\mathbf{v}$: top $15 + (-2) = \mathbf{13}$, bottom $-6 + 6 = \mathbf{0}$.

$\mathbf{u} + \mathbf{v}$: top 4, bottom 1; $|\mathbf{u} + \mathbf{v}| = \sqrt{16 + 1} = \mathbf{4.12}$

Worked example 2

OA has top 2, bottom 7 and OB has top 6, bottom -1. Work out AB.

AB = AO + OB = -OA + OB: top $-2 + 6 = \mathbf{4}$, bottom $-7 + (-1) = \mathbf{-8}$.

Common mistakes

- Squaring a negative number wrongly when finding the modulus: $(-3)^2 = 9$.
- Writing AB as OA - OB instead of OB - OA.

Exam tip: For AB, think "end minus start": OB - OA.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

a, **b** and **c** are shown below. Work out **a** + **c**.

1 $\mathbf{a} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Using question 1, work out **c** - **b**.

2

3 Work out **4b**.

4 Work out **2a** - **3b**.

5 Work out $\frac{1}{2}\mathbf{c}$ + **a**.

6 Work out **|c|** to 3 s.f.

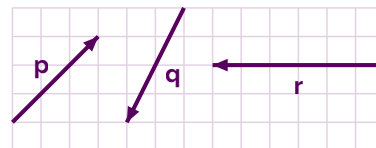
7 Work out **|a + b|** to 3 s.f.

8 Find **d** such that **2d** + **b** = **c**.

Write **p**, **q** and **r** as column vectors.

9 Is **3b** parallel to **b**? Is it the same length?

10



11 P is (-1, 3) and Q is (4, -9). Work out **|PQ|**.

12

Work out the resultant of 3 right 2 up, then 7 left 4 up, then 1 right 3 down.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 **p** and **q** are shown below.

$$\mathbf{p} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

- (a) Work out $3\mathbf{p} + \mathbf{q}$ as a column vector. [2]

- (b) Work out $|\mathbf{p} - \mathbf{q}|$. [3]

Answer

- (c) $\mathbf{r} = m\mathbf{p} + n\mathbf{q}$ is equal to the vector shown. Work out m and n . [3]

$$\mathbf{r} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

Answer $m = \dots\dots\dots$, $n = \dots\dots\dots$

- 2 O is the origin. $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, as shown below.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

- (a) Write down the coordinates of A. [1]

Answer (.....,

- (b) Work out $\mathbf{AB}$ as a column vector. [2]

- (c) Work out $|\mathbf{AB}|$, leaving your answer as a surd in its simplest form. [2]

Answer

- (d) M is the midpoint of AB. Work out $\mathbf{OM}$ as a column vector. [2]

- (e) Work out $|\mathbf{OM}|$. [1]

Answer

- 3 A boat's velocity in still water is shown by the vector $\mathbf{v}$ (km/h, east and north). A current adds the vector $\mathbf{c}$.

$$\mathbf{v} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

(a) Work out the resultant velocity. [1]

(b) Work out the speed of the boat. [2]

Answer km/h

(c) Work out the bearing on which the boat travels. [2]

Answer°

- 4 The vector $\mathbf{w}$ has top number t and bottom number $2t$, where $t > 0$. $|\mathbf{w}| = 10$.

(a) Work out the exact value of t . [3]

Answer $t =$

(b) Write down the bottom number of $\mathbf{w}$ as an exact value. [1]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Vector geometry and proof

I can... use vector methods to prove that lines are parallel and that points are collinear.

Spec ref: 5.1G · Higher

Write each vector as a route through known vectors and simplify. A multiple means **parallel**; parallel with a shared point means **collinear**.

Key points

- Find a route: $XY = XO + OY$. Reverse a vector by changing its sign.
- Ratio: if X divides PQ in the ratio $m : n$, then $PX = m/(m + n)$ PQ.
- For a proof, finish with a sentence: "... is a multiple of ..., so ... are parallel".

Key words

proof

parallel

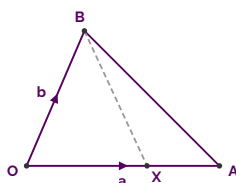
collinear

ratio

position vector

Worked example 1

$OA = a$, $OB = b$. X is on OA with $OX : XA = 5 : 3$. Find BX.



$$OX = \frac{5}{8}a. \quad BX = BO + OX = -b + \frac{5}{8}a$$

Worked example 2

$PQ = 3a - 6b$ and $RS = a - 2b$. What can you say about PQ and RS?

$$PQ = 3(a - 2b) = 3RS, \text{ so } PQ \text{ is parallel to } RS \text{ and three times as long.}$$

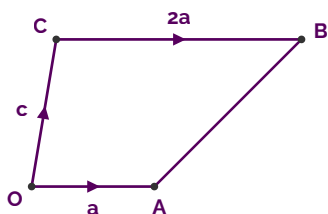
Common mistakes

- Writing the ratio $5 : 3$ as $5/3$ instead of $5/8$.
- Saying points are collinear without stating the common point.

Exam tip: In a "show that" question, every step of the route should be written down — the final line alone scores nothing.

Show your working. Give answers correct to 3 significant figures (angles to 1 d.p.) unless told otherwise. Diagrams are NOT accurately drawn.

OABC is a trapezium. $OA = \mathbf{a}$, $OC = \mathbf{c}$ and $CB = 2\mathbf{a}$. Find OB in terms of $\mathbf{a}$ and $\mathbf{c}$.



1

2

Using the trapezium in question 1, find AB .

3

Find BO .

4

Find AC .

5

M is the midpoint of CB . Find OM .

6

Find AM . What do you notice about AM and OC ?

7

$OA = \mathbf{a}$, $OB = \mathbf{b}$. X divides AB in the ratio $2 : 1$. Find AX .

8

For the same points, find OX .

9

Simplify $4(\mathbf{a} + 2\mathbf{b}) - 3(\mathbf{a} + \mathbf{b})$.

10

Is $6\mathbf{a} - 9\mathbf{b}$ parallel to $2\mathbf{a} - 3\mathbf{b}$?

11

Is $\mathbf{a} + 2\mathbf{b}$ parallel to $2\mathbf{a} + \mathbf{b}$?

12

$PQ = 2\mathbf{a} + \mathbf{b}$ and $QR = 6\mathbf{a} + 3\mathbf{b}$. Show that P , Q and R are collinear.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show clear working. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place) unless the question says otherwise.

- 1 OPQR is a parallelogram. $OP = \mathbf{p}$ and $OR = \mathbf{r}$. S is the midpoint of PQ. X is the point on OQ such that $OX : XQ = 2 : 1$.

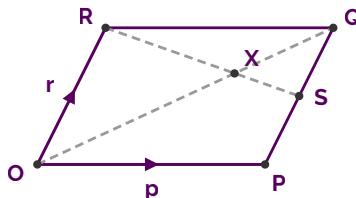


Diagram NOT accurately drawn

- (a) Find OS in terms of $\mathbf{p}$ and $\mathbf{r}$. [1]

Answer OS =

- (b) Find RX in terms of $\mathbf{p}$ and $\mathbf{r}$. [2]

Answer RX =

- (c) Prove that R, X and S lie on a straight line. [3]

- 2 In triangle OAB, $OA = 3\mathbf{a}$ and $OB = 3\mathbf{b}$. P is on OA with $OP : PA = 2 : 1$, and Q is on OB with $OQ : QB = 2 : 1$.

- (a) Find PQ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

Answer PQ =

- (b) Show that PQ is parallel to AB, and find the ratio $PQ : AB$. [3]

- 3 OABC is a trapezium with $OA = 6\mathbf{a}$, $OC = 6\mathbf{c}$ and $CB = 3\mathbf{a}$. M is the midpoint of AB.

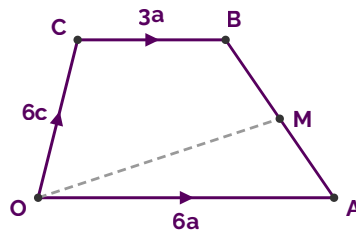


Diagram NOT accurately drawn

- (a) Find AB in terms of $\mathbf{a}$ and $\mathbf{c}$.

[2]

Answer AB =

- (b) Find OM in terms of $\mathbf{a}$ and $\mathbf{c}$. Simplify your answer.

[2]

Answer OM =

- (c) N is the point on OC with $ON = 2\mathbf{c}$. Find NM and explain whether NM is parallel to OA. [2]

- 4 Relative to an origin O, the position vectors of P, Q and R are $OP = 2\mathbf{a} + \mathbf{b}$, $OQ = 4\mathbf{a} + 3\mathbf{b}$ and $OR = 7\mathbf{a} + k\mathbf{b}$.

- (a) Find PQ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[1]

Answer PQ =

- (b) P, Q and R lie on a straight line. Find the value of k .

[3]

Answer $k =$

- 5 ABCD is a parallelogram. $AB = \mathbf{p}$ and $AD = \mathbf{q}$. E is the midpoint of BC. X is the point on BD such that $BX : XD = 1 : 2$.

- (a) Find BD in terms of $\mathbf{p}$ and $\mathbf{q}$.

[1]

Answer BD =

- (b) Find AX in terms of $\mathbf{p}$ and $\mathbf{q}$. Simplify your answer.

[2]

Answer AX =

- (c) Prove that A, X and E are collinear, and find the ratio AX : XE.

[2]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

End-of-book check

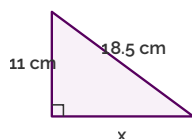
End-of-book check

Name _____ Date _____ Score _____ / 12

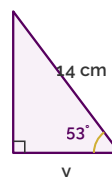
Answer every question. Give non-exact answers correct to 3 significant figures (angles to 1 decimal place). Diagrams are NOT accurately drawn.

Work out x .

1

Answer $x = \dots\dots\dots$ cmWork out y .

2

Answer $y = \dots\dots\dots$ cm

3

From the top of a 60 m cliff, the angle of depression of a boat is 18° . How far is the boat from the foot of the cliff?

4

C is 5 km east and 12 km north of D. Work out the bearing of C from D.

5

Solve $\tan x = -1$ for $0^\circ \leq x \leq 360^\circ$.

6

In triangle ABC, $b = 9$ cm, $B = 64^\circ$ and $C = 41^\circ$. Work out c .

7

A triangle has sides 6 cm, 7 cm and 11 cm. Work out the largest angle.

8

Work out the area of a triangle with sides 8 cm and 13 cm and included angle 150° .

9

A cuboid is 2 cm by 6 cm by 9 cm. Work out its space diagonal.

10

Describe fully the transformation that maps (1, 2) to (2, 4) and (3, 1) to (6, 2), centre (0, 0).

11

Vector $\mathbf{v}$ has top number 8 and bottom number -15 . Work out $|\mathbf{v}|$.

12

$OA = \mathbf{a}$, $OB = \mathbf{b}$. Find AB.



Practice paper

Name _____ Date _____ Marks _____ / 60

Time: 1 hour

Calculator allowed

Total: 60 marks

Give non-exact answers correct to 3 significant figures (angles to 1 decimal place).

Answer all questions. Show your working: an answer with no working may not score full marks.

- 1 The diagram shows a right-angled triangle ABC. $AB = 9.2$ cm and angle $BAC = 34^\circ$.

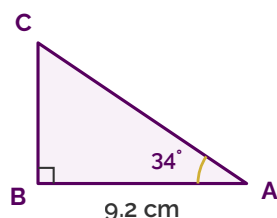


Diagram NOT accurately drawn

- (a) Work out the length of BC.

[2]

Answer cm

- (b) Work out the length of AC.

[2]

Answer cm

- 2 A ship sails 26 km from port P on a bearing of 118° .

- (a) Work out how far south of P the ship is.

[2]

Answer km

- (b) Work out how far east of P the ship is.

[2]

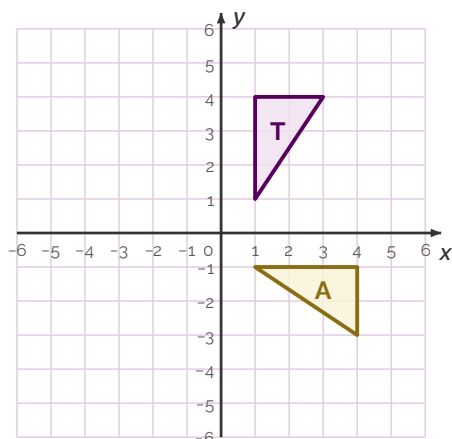
Answer km

- (c) Write down the bearing of P from the ship.

[1]

Answer $^\circ$

- 3 The diagram shows triangle T and triangle A.



- (a) Describe fully the single transformation that maps T onto A. [3]

- (b) Enlarge triangle T by scale factor 2, centre (6, 6). [2]

- 4 Solve these equations for $0^\circ \leq x \leq 360^\circ$.

(a) $10 \cos x + 3 = 0$ [3]

Answer $x = \dots\dots$

(b) $\sin(x - 30^\circ) = 0.5$ [3]

Answer $x = \dots\dots$

- 5 In triangle ABC, AB = 16 cm, AC = 21 cm and angle BAC = 54° .

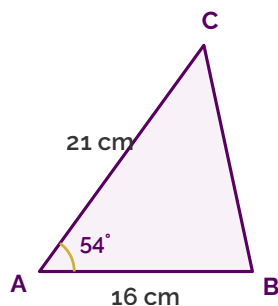


Diagram NOT accurately drawn

- (a) Work out the length of BC.

[3]

Answer BC = cm

- (b) Work out the size of angle ABC.

[3]

Answer $^\circ$

- (c) Work out the area of triangle ABC.

[2]

Answer cm^2

- 6 ABCDEFGH is a cuboid. AB = 14 cm, BC = 6 cm and CG = 5 cm.

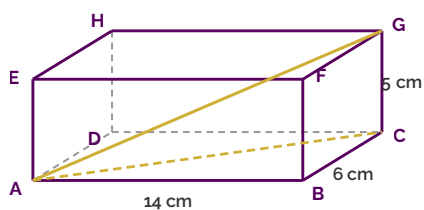


Diagram NOT accurately drawn

- (a) Work out the length AG.

[3]

Answer AG = cm

- (b) Work out the angle between AG and the plane ABCD.

[2]

Answer $^\circ$

- 7 **a** and **b** are shown below.

$$\mathbf{a} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

- (a) Work out $2\mathbf{a} - \mathbf{b}$.

[2]

- (b) Work out $|2\mathbf{a} - \mathbf{b}|$.

[2]

Answer

- 8 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. X is the point on AB such that $AX : XB = 1 : 2$. Y is the point such that $OY = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$.

- (a) Find OX in terms of **a** and **b**. Simplify your answer.

[2]

Answer OX =

- (b) Prove that O, X and Y lie on a straight line.

[2]

- (c) Find the ratio OX : XY.

[1]

Answer :

- 9 Triangle PQR has $PQ = 10$ cm, $QR = 12$ cm and area 42 cm^2 . Angle PQR is obtuse.

- (a) Work out the size of angle PQR.

[3]

Answer°

- (b) Work out the length of PR.

[2]

Answer PR = cm

- 10 A tower stands on level ground. From a point 40 m from the foot of the tower, the angle of elevation of the top is 35° .

- (a) Work out the height of the tower.

[2]

Answer m

- (b) Ali walks towards the tower until the angle of elevation is 60° . How far did he walk?

[3]

Answer m

- 11** In triangle XYZ, $XY = 8.6$ cm, $YZ = 6.4$ cm and angle $YXZ = 42^\circ$. Angle XZY is obtuse.

(a) Work out the size of angle XZY.

[4]

Answer°

- 12** Triangle J has vertices $(2, 2)$, $(6, 2)$ and $(2, 4)$.

(a) J is enlarged by scale factor $\frac{1}{2}$, centre $(0, 0)$, to give K. Write down the vertices of K.

[2]

(b) K is reflected in the line $y = x$ to give L. Write down the vertices of L.

[2]



FOR TEACHERS & PARENTS

Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

Starting point check

- 1 625, 25
- 2 0.0468
- 3 1
- 4 20
- 5 135°
- 6 $h = 12 \sin 40^\circ$
- 7 $(-3, -2)$
- 8 $(0, 4)$
- 9 2 cm by 3 cm
- 10 12.0 cm
- 11 35 cm^2
- 12 7

Unit 1 · Sheet A — Pythagoras' theorem

- 1 17 cm
- 2 9.70 cm
- 3 16 cm
- 4 11.5 cm
- 5 11.1 cm
- 6 24 cm
- 7 130 m
- 8 10 units
- 9 8.60 units
- 10 1.62 m
- 11 $20^2 + 21^2 = 400 + 441 = 841 = 29^2$
- 12 9.90 cm

Unit 1 · Sheet B — Pythagoras' theorem

- 1
(a) horizontal difference $15 - 9 = 6$ B1; $\sqrt{6^2 + 8^2}$ M1; 10 cm A1 [3]
(b) $8 + 9 + 15 + 10 = 42$ cm B1ft [1] (c) $\sqrt{8^2 + 15^2}$ M1; 17 cm A1 [2]
- 2 (a) $\sqrt{140^2 + 85^2}$ M1; = 164 A1; $225 - 164$ M1; 61.2 m A1 [4] (b) $164 \div 1.2 = 136$ s B1ft [1]
- 3 (a) differences 6 and 8 B1; $\sqrt{6^2 + 8^2}$ M1; 10 A1 [3] (b) $((-1 + 5)/2, (4 - 4)/2)$ M1; $(2, 0)$ A1 [2]
- 4 (a) $\sqrt{5^2 + 4^2}$ M1; 6.40 cm A1 [2] (b) $\sqrt{12^2 + 4^2}$ M1; 12.6 cm A1 [2] (c) $2(6.40 + 12.6) = 38.1$ cm B1ft [1]
- 5 (a) $2.8^2 + 4.5^2 = 28.09$ M1; $5.3^2 = 28.09$, equal so yes, right-angled A1 [2] (b) $\frac{1}{2} \times 2.8 \times 4.5$ M1; 6.3 m^2 A1 [2]

Unit 2 · Sheet A — Trigonometry in right-angled triangles

- 1 11.1 cm
- 2 12.6 cm
- 3 10.1 cm
- 4 19.2 cm
- 5 31.6°
- 6 31.8°
- 7 11.5
- 8 15.9
- 9 38.3°
- 10 0.334 m
- 11 61.4°
- 12 5.60 cm

Unit 2 · Sheet B — Trigonometry in right-angled triangles

- 1 (a) half width 4.8 B1; $\tan 32^\circ = h/4.8$ M1; $h = 3.00$ m A1 [3] (b) $4.8 \div \cos 32^\circ$ M1; 5.66 m A1 [2] (c) $\frac{1}{2} \times 9.6 \times 3.00$ M1; 14.4 m^2 A1ft [2]
- 2 (a) $\sin A = 8/17$ M1; 28.1° A1 [2] (b) $AB = \sqrt{17^2 - 8^2} = 15$ B1; $DB = 8 \div \tan 50^\circ$ M1; = 6.71 A1; $AD = 8.29$ cm A1 [4]
- 3 (a) $\cos \theta = 4.5/12$ M1A1; 68.0° A1 [3] (b) $180 - 2 \times 68.0 = 44.0^\circ$ B1ft [1]
- 4 (a) $\cos \theta = 1.4/5$ M1; $\theta = 73.7^\circ$ A1; between 70 and 78, so safe A1ft [3] (b) $5 \sin 78^\circ$ M1; 4.89 m A1 [2]
- 5 (a) $650 \sin 24^\circ$ M1; 264 m A1 [2] (b) $650 \cos 24^\circ$ M1; 594 m A1 [2]

Unit 3 · Sheet A — Elevation, depression and bearings

- 1 23.3 m
- 2 69.9 m
- 3 251 m
- 4 12.9°
- 5 21.8°
- 6 317°
- 7 110°
- 8 32.8 km
- 9 5.21 km
- 10 066.0°
- 11 156.8°
- 12 34.5 m

Unit 3 · Sheet B — Elevation, depression and bearings

1 (a) angle between bearing of H from B (230°) and of L from B (140°) is 90° M1A1 (or $180 - 50 = 130$, $130 - 40 = 90$) [2] (b) $\sqrt{(22^2 + 16^2)}$ M1; 27.2 km A1 [2] (c) $\tan^{-1}(16/22) = 36.0$ M1A1; $050^\circ + 36.0 = 086.0^\circ$ A1 [3]

2 (a) $34 \div \tan 28^\circ$ M1; 63.9 m A1 [2] (b) $BF = 34 \div \tan 47^\circ = 31.7$ M1A1; $AB = 95.7$ m A1 [3]

3 (a) $1200/\tan 9^\circ = 7580$ M1A1; $1200/\tan 21^\circ = 3130$ M1; difference 4450 m A1 [4] (b) $4450 \text{ m per minute} \times 60 \div 1000$ M1; 267 km/h A1ft [2]

4 (a) $45 \sin 55^\circ$ M1; 36.9 km A1 [2] (b) 055° B1 [1]

5 (a) $\tan^{-1}(75/130)$ M1; 30.0° A1 [2] (b) $75 \div \tan 12^\circ$ M1; $= 353$ A1; $-130 = 223$ m A1 [3]

Unit 4 · Sheet A — Obtuse angles and trigonometric graphs

1 0.5

2 60

3 80

4 115.8°

5 104.5°

6 53.1° , 306.9°

7 56.3° , 236.3°

8 224.4° , 315.6°

9 -60° , 60°

10 -1

11 90° to the right (vector (90 over 0))

12 3

Unit 4 · Sheet B — Obtuse angles and trigonometric graphs

1 (a) 113 to 115 B1; 245 to 247 B1 [2] (b) same shape moved up 1: max 2 at 0 and 360, min 0 at 180 B2 [2] (c) (180, 0) B1 [1]

2 (a) $\sin x = -0.4$ M1; 203.6° , 336.4° A1A1 [3] (b) $\tan x = -2.5$ M1; 111.8° , 291.8° A1A1 [3]

3 (a) $\sin^{-1}0.8 = 53.1$ M1; $180 - 53.1 = 126.9^\circ$ A1 [2] (b) -0.6 B1 [1]

4 (a) translation B1; 60° to the left (vector (-60 over 0)) B1 [2] (b) (30, 1) B1 [1]

(c) $x + 60 = 30$, 150, 390 M1; $x = 90$ A1; $x = 330$ A1 [3]

5 (a) $a + b = 7$, $-a + b = -1$ M1; $a = 4$ A1; $b = 3$ A1 [3] (b) $4 \cos x + 3 = 5$, $\cos x = 0.5$ M1; 60° , 300° A1 [2]

Unit 5 · Sheet A — The sine rule

1 10.1 cm

2 9.65 cm

3 45.2°

4 22.8 cm

5 7.92 cm

6 30.5°

7 11.7 cm

8 12.9 cm

9 61.6° or 118.4°

10 44.2°

11 145°

12 8.11 cm

Unit 5 · Sheet B — The sine rule

1 (a) angle C = 67° B1; $AC = 85 \sin 49^\circ \div \sin 67^\circ$ M1; 69.7 m A1 [3]

(b) $69.7 \times \sin 64^\circ$ M1; 62.6 m A1ft [2]

2 (a) $\sin Q = 12.6 \sin 41^\circ \div 9$ M1; $= 0.9185$ A1; 66.7° A1; 113.3° A1 [4]

(b) angle R = $180 - 41 - 66.7 = 72.3$ B1; $PQ = 9 \sin R \div \sin 41^\circ$ M1; 13.1 cm A1 [3]

3 (a) bearing of P from A is 215° B1; $215 - 150 = 65$ B1 [2] (b) angle APB = 75° , angle PBA = 40° B1; $AB = 20 \sin 75^\circ \div \sin 40^\circ$ M1; 30.1 km A1 [3] (c) $20 \sin 65^\circ \div \sin 40^\circ$ M1; 28.2 km A1 [2]

4 (a) $BD = 7.2 \sin 118^\circ \div \sin 28^\circ$ M1; 13.5 cm A1 [2] (b) $CD = 13.5 \sin 54^\circ \div \sin 71^\circ$ M1; 11.6 cm A1ft [2] (c) angle BDC = 55° B1; $\frac{1}{2} \times 13.5 \times 11.6 \times \sin 55^\circ$ M1; 64.3 cm² A1 [3]

Unit 6 · Sheet A — The cosine rule

1 7.88 cm

2 14.9 cm

3 9.22 cm

4 75.5°

5 110.7°

6 11.7 cm — same as Pythagoras, as $\cos 90^\circ = 0$

7 12.7 cm

8 106.3°

9 21.8°

10 14.1 cm

11 Obtuse: $4^2 + 6^2 = 52 < 81$

12 120° ($\cos = -1/2$)

Unit 6 · Sheet B — The cosine rule

1 (a) $2.4^2 + 3.1^2 - 2 \times 2.4 \times 3.1 \cos 105^\circ$ M1A1; 4.38 km A1 [3] (b) $\sin S/3.1 = \sin 105^\circ/4.38$ (or cosine rule) M1; $= 0.6830$ A1; 43.1° A1 [3]

2 (a) $\cos \theta = (11^2 + 9^2 - 17^2) \div (2 \times 11 \times 9)$ M1; $= -0.4394$ A1; 116.1° A1 [3] (b) opposite 9 m: $\cos = (17^2 + 11^2 - 9^2) \div (2 \times 17 \times 11)$ M1; 28.4° A1 [2] (c) $\frac{1}{2} \times 11 \times 9 \times \sin 116.1^\circ$ M1; 44.5 m² A1ft [2]

3 (a) bearing of A from P is 250° B1; $250 - 170 = 80$ B1 [2] (b) $60^2 + 45^2 - 2 \times 60 \times 45 \cos 80^\circ$ M1A1; 68.5 km A1 [3] (c) $\sin PAQ = 45 \sin 80^\circ \div 68.5$ M1; $PAQ = 40.3^\circ$ A1; bearing 110.3° A1 [3]

4 (a) $13^2 = x^2 + (x + 3)^2 - 2x(x + 3) \cos 60^\circ$ M1; $\cos 60^\circ = \frac{1}{2}$ used M1; $169 = x^2 + 3x + 9 \rightarrow x^2 + 3x - 160 = 0$ A1 [3] (b) formula with $a = 1$, $b = 3$, $c = -160$ M1; $x = 11.2$ A1 [2]

Unit 7 · Sheet A — Area of a triangle and mixed problems

1 24.1 cm²

2 45.3 cm²

3 56.2 cm²

4 72.6 cm²

5 62.4 cm²

6 79.8 cm²

7 41.8°

8 10.1 cm

9 65.0 cm²

10 It is right-angled ($6^2 + 8^2 = 10^2$), so $\frac{1}{2} \times 6 \times 8 = 24$ cm²

11 $C = 41.8^\circ$, area = 21.0 cm²

12 150°

Unit 7 · Sheet B — Area of a triangle and mixed problems

- 1** (a) $\frac{1}{2} \times 14 \times 18 \times \sin 47^\circ$ M1; 92.2 m² A1 [2] (b) $14^2 + 18^2 - 2 \times 14 \times 18 \cos 47^\circ$ M1A1; 13.3 m A1 [3] (c) $\cos Q = (14^2 + 13.3^2 - 18^2) \div (2 \times 14 \times 13.3)$ M1; = 0.1299 A1; Q = 82.5° A1 [3]
- 2** (a) $\frac{1}{2} \times 8 \times 8 \times \sin 70^\circ$ M1; 30.1 cm² A1 [2] (b) sector = $70/360 \times \pi \times 8^2 = 39.1$ M1A1; segment = 9.03 cm² A1 [3] (c) arc $70/360 \times 2\pi \times 8 = 9.77$ and chord $2 \times 8 \sin 35^\circ = 9.18$ M1; 19.0 cm A1 [2]
- 3** (a) $\frac{1}{2} \times 11 \times 15 \times \sin X = 60$ M1; $\sin X = 0.7273$ A1; 46.7° A1 [3] (b) $11^2 + 15^2 - 2 \times 11 \times 15 \cos 46.7^\circ$ M1A1; 10.9 cm A1 [3]
- 4** (a) $9^2 + 9^2 - 2 \times 9 \times 9 \cos 110^\circ$ M1; 14.7 cm A1 [2] (b) triangle ABD = $\frac{1}{2} \times 81 \times \sin 110^\circ = 38.1$ B1; angle BCD from cosine rule = 58.9° M1; total 134 cm² A1 [3]

Unit 8 · Sheet A — Trigonometry in three dimensions

- 1** 12.1 cm
2 10.8 cm
3 14.5 cm
4 33.5°
5 36.0°
6 15.6 cm
7 35.3°
8 68.2°
9 11.5 cm
10 6.87 m
11 15.8°
12 8.00 cm

Unit 8 · Sheet B — Trigonometry in three dimensions

- 1** (a) $AC = \sqrt{8^2 + 6^2} = 10$ B1; $AF = \sqrt{10^2 + 20^2}$ M1; 22.4 cm A1 [3] (b) F is above E; $AE = \sqrt{8^2 + 20^2} = 21.5$ M1; $\tan \theta = 6/21.5$ M1; 15.6° A1 [3]
- 2** (a) $AC = 15$, $AM = 7.5$ B1; $\sqrt{7.5^2 + 14^2}$ M1; 15.9 cm A1 [3] (b) $\tan^{-1}(14/7.5)$ M1; 61.8° A1 [2] (c) midpoint of BC is 6 cm from M B1; $\tan^{-1}(14/6)$ M1; 66.8° A1 [3]
- 3** (a) $\sqrt{50^2 + 30^2 + 25^2}$ M1A1; 63.4 cm A1 [3] (b) $\tan^{-1}(25/\sqrt{50^2 + 30^2})$ M1; 23.2° A1 [2]
- 4** (a) $\sqrt{15^2 - 12^2}$ M1; 9 cm A1 [2] (b) $\sin^{-1}(12/15)$ or $\tan^{-1}(12/9)$ M1; 53.1° A1 [2] (c) $2 \times \sin^{-1}(9/15)$ M1; 73.7° A1 [2]

Unit 9 · Sheet A — Rotations, reflections and translations

- 1** reflection in the y-axis ($x = 0$)
2 rotation 180° about (0, 0)
3 translation by (1 over -6)
4 reflection in the x-axis ($y = 0$)
5 (1, -4)
6 (-4, 5)
7 (5, -2)
8 (-2, -1)
9 (-3, 2)
10 (3, -3)
11 3 left and 4 up (vector -3 over 4)
12 enlargement (scale factor $\neq \pm 1$)

Unit 9 · Sheet B — Rotations, reflections and translations

- 1** (a) vertices (0, -1), (-3, -1), (-3, -3) B2 (B1 for 180° rotation about another point) [2] (b) vertices (1, 2), (1, 5), (3, 5) B2 (B1 for 2 correct vertices) [2] (c) vertices (-5, 4), (-2, 4), (-2, 6) B2 (B1 for one component correct) [2] (d) vertices (2, -3), (2, -6), (4, -6) B2 (B1 for correct orientation) [2]
- 2** (a) rotation B1; -90° (90° clockwise) B1; centre (-1, -1) B1 [3] (b) reflection B1; in the line $y = 1$ B1 [2]
- 3** (a) reflect back: (-1, -6) M1A1 [2] (b) (-4 over -4) B2 (B1 for one component) [2]
- 4** (a) rotations preserve lengths B1 and angles B1, so all sides and angles match (SSS) — congruent [2] (b) 14 cm² B1 [1]
- 5** (a) centre (2, 1) B3 (M1 for perpendicular bisector of the two points; M1 for checking 90°) [3] (b) (-2, 1) B2 [2]

Unit 10 · Sheet A — Enlargement

- 1** enlargement, scale factor 2, centre (5, 5)
2 enlargement, scale factor 3, centre (3, 6)
3 enlargement, scale factor 1/3, centre (3, 6)
4 (12, -8)
5 (4, 3)
6 (11, 4)
7 (3, 5)
8 40 cm
9 37°
10 1/4
11 1.5 (3/2)
12 smaller

Unit 10 · Sheet B — Enlargement

- 1** (a) vertices (-4, -3), (4, -3), (4, 1) B3 (B2 for correct size in wrong place; B1 for 2 correct vertices) [3] (b) vertices (-5, -5), (-3, -5), (-3, -4) B2 (B1 for correct size and orientation) [2]
- 2** (a) enlargement B1; scale factor 3 B1; centre (4, 6) B1 [3] (b) enlargement B1; scale factor 1/3 B1; centre (-5, -6) B1 [3]
- 3** (a) $24 \div 15 = 1.6$ B1 [1] (b) 10×1.6 M1; 16 cm A1 [2] (c) $6 \div 24$ M1; 1/4 A1 [2]
- 4** (a) (1, 1), (3, 1), (3, 4) B2 (B1 for two correct) [2] (b) angles unchanged so similar B1; lengths halved so not congruent B1 [2]
- 5** (a) $c + 3((2, 1) - c) = (8, 7)$ or draw a line through both points and use distances M1; $2c = (6, 3) - (8, 7)$ M1; centre (-1, -2) A1 [3] (b) (2, 13) B2 [2]

Unit 11 · Sheet A — Describing and combining transformations

- 1** reflection in the line $y = x$
2 rotation +90° (anticlockwise) about (0, 0)
3 translation by (7 over 0)
4 reflection in $x = 0$
5 translation 6 right (vector 6 over 0)
6 rotation 180° about (0, 0)
7 rotation 180° about (0, 0)
8 2 left and 5 up
9 enlargement scale factor 6, centre (0, 0)
10 rotation, reflection, translation
11 rotation +90° (anticlockwise) about (0, 0)
12 no change (identity) / translation by zero vector

Unit 11 · Sheet B — Describing and combining transformations

- 1** (a) vertices $(-1, 1)$, $(-4, 1)$, $(-4, 3)$ B2 [2] (b) vertices $(-1, -1)$, $(-1, -4)$, $(-3, -4)$ B2ft [2] (c) reflection B1; in the line $y = -x$ B1ft [2]
2 (a) rotation B1; 180° B1; centre $(1, 0)$ B1 [3] (b) translation B1; by $(-5 \text{ over } 8)$ B1 [2] (c) rotation 180° B1; about $(1, 0)$ B1 [2]
3 (a) $(3, 3)$ B1; $(3, 7)$ B1 [2] (b) translation B1; by $(0 \text{ over } 6)$ B1 [2]
4 (a) enlargement B1; scale factor $1/2$ B1; centre $(4, 4)$ B1 [3] (b) No — the lengths are halved B1 [1]
5 (a) $(5, -2)$ B1; $(5, 2)$ B1ft [2] (b) reflection B1; in the line $y = x$ B1 [2]

Unit 12 · Sheet A — Vectors

- 1** $(8 \text{ over } -1)$
2 $(9 \text{ over } 3)$
3 $(-12 \text{ over } 4)$
4 $(13 \text{ over } -13)$
5 $(5 \text{ over } -3)$
6 7.21
7 4.12
8 $(4.5 \text{ over } 1.5)$
9 parallel, three times as long
10 $p = (3 \text{ over } 3)$, $q = (-2 \text{ over } -4)$, $r = (-6 \text{ over } 0)$
11 13
12 3 left 3 up $(-3 \text{ over } 3)$

Unit 12 · Sheet B — Vectors

- 1** (a) $(10 \text{ over } 0)$ B2 (B1 for one component) [2] (b) $p - q = (6 \text{ over } -4)$ B1; $\sqrt{36 + 16}$ M1; 7.21 A1 [3] (c) $4m - 2n = 2$ and $-m + 3n = 7$ M1; elimination M1; $m = 2$, $n = 3$ A1 [3]
2 (a) $(3, 5)$ B1 [1] (b) $b - a$ M1; $(4 \text{ over } -8)$ A1 [2] (c) $\sqrt{16 + 64} = \sqrt{80}$ M1; $4\sqrt{5}$ A1 [2] (d) $a + \frac{1}{2}AB$ M1; $(5 \text{ over } 1)$ A1 [2] (e) $\sqrt{26} = 5.10$ B1ft [1]
3 (a) $(9 \text{ over } 4)$ B1 [1] (b) $\sqrt{81 + 16}$ M1; 9.85 km/h A1 [2] (c) $\tan^{-1}(9/4)$ M1; 066.0° A1 [2]
4 (a) $t^2 + 4t^2 = 100$ M1; $5t^2 = 100$, $t^2 = 20$ M1; $t = 2\sqrt{5}$ A1 [3] (b) $4\sqrt{5}$ B1ft [1]

Unit 13 · Sheet A — Vector geometry and proof

- 1** $2a + c$
2 $a + c$
3 $-2a - c$
4 $-a + c$
5 $a + c$
6 c — $AM = OC$, so AM is parallel to OC and equal in length
7 $-2/3a + 2/3b$
8 $1/3a + 2/3b$
9 $a + 5b$
10 Yes: $6a - 9b = 3(2a - 3b)$
11 No: neither is a multiple of the other
12 $QR = 3PQ$, parallel with common point Q , so collinear

Unit 13 · Sheet B — Vector geometry and proof

- 1** (a) $p + \frac{1}{2}r$ B1 [1] (b) $OX = 2/3(p + r)$ M1; $RX = 2/3 p - 1/3 r$ A1 [2] (c) $XS = OS - OX = 1/3 p - 1/6 r$ M1A1; $RX = 2XS$, parallel with common point X , so collinear A1 [3]
2 (a) $OP = 2a$, $OQ = 2b$ M1; $PQ = 2b - 2a$ A1 [2] (b) $AB = 3b - 3a$ B1; $PQ = 2/3 AB$ M1; parallel, ratio $2 : 3$ A1 [3]
3 (a) $AO + OC + CB$ M1; $-3a + 6c$ A1 [2] (b) $OA + \frac{1}{2}AB$ M1; $9/2a + 3c$ A1 [2] (c) $NM = 9/2a + c$ B1; not a multiple of a alone, so not parallel to OA B1 [2]
4 (a) $2a + 2b$ B1 [1] (b) $PR = 5a + (k - 1)b$ M1; must be a multiple of $a + b$, so $k - 1 = 5$ M1; $k = 6$ A1 [3]
5 (a) $q - p$ B1 [1] (b) $p + 1/3(q - p)$ M1; $2/3 p + 1/3 q$ A1 [2] (c) $AE = p + \frac{1}{2}q$ and $AX = 2/3 AE$, common point A , so collinear B1; $AX : XE = 2 : 1$ B1 [2]

End-of-book check

- 1** 14.9 cm
2 8.43 cm
3 185 m
4 022.6°
5 135° , 315°
6 6.57 cm
7 115.4°
8 26 cm^2
9 11 cm
10 enlargement, scale factor 2, centre $(0, 0)$
11 17
12 $b - a$

Practice paper

- 1** (a) $9.2 \tan 34^\circ$ M1; 6.21 cm A1 [2] (b) $9.2 \div \cos 34^\circ$ or Pythagoras M1; 11.1 cm A1 [2]
2 (a) $26 \cos 62^\circ$ M1; 12.2 km A1 [2] (b) $26 \sin 62^\circ$ M1; 23.0 km A1 [2] (c) 298° B1 [1]
3 (a) rotation B1; -90° (90° clockwise) B1; about $(0, 0)$ B1 [3] (b) vertices $(-4, -4)$, $(-4, 2)$, $(0, 2)$ B2 (B1 for correct size, wrong position) [2]
4 (a) $\cos x = -0.3$ M1; 107.5° A1; 252.5° A1 [3] (b) $x - 30 = 30$ or 150 M1; $x = 60^\circ$ A1; $x = 180^\circ$ A1 [3]
5 (a) $16^2 + 21^2 - 2 \times 16 \times 21 \cos 54^\circ$ M1A1; 17.4 cm A1 [3] (b) $\cos B = (16^2 + BC^2 - 21^2)/(2 \times 16 \times BC)$ M1; $= 0.2104$ A1; 77.9° A1 [3] (c) $\frac{1}{2} \times 16 \times 21 \times \sin 54^\circ$ M1; 136 cm^2 A1 [2]
6 (a) $AC^2 = 14^2 + 6^2 = 232$ M1; $AG = \sqrt{232 + 25}$ M1; 16.0 cm A1 [3] (b) $\tan^{-1}(5/\sqrt{232})$ M1; 18.2° A1 [2]
7 (a) $(9 \text{ over } -12)$ B2 (B1 for one component) [2] (b) $\sqrt{81 + 144}$ M1; 15 A1 [2]
8 (a) $a + 1/3(b - a)$ M1; $2/3a + 1/3b$ A1 [2] (b) $OY = 9/4 OX$ M1; parallel with common point O , so collinear A1 [2] (c) $4 : 5$ B1 [1]
9 (a) $\frac{1}{2} \times 10 \times 12 \sin Q = 42$ M1; $\sin Q = 0.7$ A1; $Q = 180 - 44.4 = 135.6^\circ$ A1 [3] (b) $10^2 + 12^2 - 240 \cos 135.6^\circ$ M1; 20.4 cm A1 [2]
10 (a) $40 \tan 35^\circ$ M1; 28.0 m A1 [2] (b) $28.0 \div \tan 60^\circ = 16.2$ M1A1; $40 - 16.2 = 23.8$ m A1 [3]
11 (a) $\sin Z/8.6 = \sin 42^\circ/6.4$ M1; $\sin Z = 0.8991$ A1; acute value 64.0° B1; obtuse $Z = 116.0^\circ$ A1 [4]
12 (a) $(1, 1)$, $(3, 1)$, $(1, 2)$ B2 (B1 for two correct) [2] (b) $(1, 1)$, $(1, 3)$, $(2, 1)$ B2ft [2]