



BRITISH LEARNING CENTRE

PEARSON EDEXCEL INTERNATIONAL GCSE · 4MA1

# Mathematics

## Statistics & Probability

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Area 6, Statistics and probability, of the Pearson Edexcel International GCSE Mathematics A (4MA1) specification, Foundation and Higher tiers. Each unit has key knowledge, worked examples, a practice sheet and exam-style questions with full mark schemes.

Name: .....

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Key knowledge · Practice · Exam-style questions · Mark schemes

# How to use this workbook

1

## Key knowledge

Read the notes, worked examples and common mistakes.

2

## Practice

Build the skill on Sheet A.

3

## Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [ ].

4

## Check & reflect

Mark your work with the mark scheme, then colour the tracker.

## My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

| #  | Spec ref                   | Topic · I can...                                                                                                                                      | R | A | G |
|----|----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|
| 1  | 6.3<br>Foundation & Higher | <b>Probability and the probability scale</b><br>I can use the language of probability and calculate the probability of an event and its complement.   |   |   |   |
| 2  | 6.3<br>Foundation & Higher | <b>Relative frequency and expected frequency</b><br>I can estimate probabilities from data and use probability to work out expected frequency.        |   |   |   |
| 3  | 6.3<br>Foundation & Higher | <b>Sample spaces and mutually exclusive events</b><br>I can list outcomes systematically and add probabilities of mutually exclusive events.          |   |   |   |
| 4  | 6.3<br>Foundation & Higher | <b>Probability from Venn diagrams</b><br>I can find probabilities from Venn diagrams using set notation.                                              |   |   |   |
| 5  | 6.3<br>Higher              | <b>Tree diagrams and independent events</b><br>I can draw and use tree diagrams and find the probability of two or more independent events.           |   |   |   |
| 6  | 6.3<br>Higher              | <b>Conditional probability</b><br>I can find conditional probabilities when combining events, including selections without replacement.               |   |   |   |
| 7  | 6.1<br>Foundation          | <b>Collecting data and two-way tables</b><br>I can organise data in tally charts, frequency tables, grouped tables and two-way tables.                |   |   |   |
| 8  | 6.1<br>Foundation          | <b>Pictograms and bar charts</b><br>I can draw and interpret pictograms, bar charts, comparative bar charts and composite bar charts.                 |   |   |   |
| 9  | 6.1<br>Foundation          | <b>Pie charts</b><br>I can draw pie charts and use them to find frequencies, fractions and percentages.                                               |   |   |   |
| 10 | 6.2<br>Foundation          | <b>Mean, median, mode and range</b><br>I can calculate and use the mean, median, mode and range for lists and frequency tables.                       |   |   |   |
| 11 | 6.2<br>Foundation & Higher | <b>Grouped data: estimating the mean</b><br>I can estimate the mean of grouped data and identify the modal class and the class containing the median. |   |   |   |
| 12 | 6.1, 6.2<br>Higher         | <b>Cumulative frequency, median and IQR</b><br>I can draw cumulative frequency graphs and estimate the median and interquartile range.                |   |   |   |
| 13 | 6.1<br>Higher              | <b>Histograms</b><br>I can construct and interpret histograms for continuous data with unequal class intervals.                                       |   |   |   |



CHECK

## Starting point check

### Starting point check

Name \_\_\_\_\_ Date \_\_\_\_\_ Marks \_\_\_\_\_ / 14

Answer as many questions as you can. This check shows what you already know — some topics may be new to you.

- 1 A fair spinner has 10 equal sections numbered 1 to 10. Find the probability that it lands on a square number. [1]

Answer .....

- 2 The probability that a team wins its next match is 0.62. What is the probability that it does not win? [1]

Answer .....

- 3 A dice is rolled 200 times and lands on 4 thirty-eight times. Find the relative frequency of a 4. [1]

Answer .....

- 4 The probability that a light bulb is faulty is 0.04. How many faulty bulbs would you expect in a box of 250? [1]

Answer .....

- 5 A coin is thrown and a spinner numbered 1 to 5 is spun. How many possible outcomes are there? [1]

Answer .....

- 6 A and B are mutually exclusive.  $P(A) = 0.3$  and  $P(B) = 0.45$ . Find  $P(A \text{ or } B)$ . [1]

Answer .....

- 7  $n(A) = 14$ ,  $n(B) = 11$  and  $n(A \cap B) = 5$ . Find  $n(A \cup B)$ .

[2]

Answer .....

- 8 Find the mean of 12, 7, 9, 15, 7.

[1]

Answer .....

- 9 Find the mode and the range of 3, 8, 8, 5, 11, 2.

[2]

Answer mode ..... range .....

- 10 72 people were asked to choose a colour. 24 chose blue. Work out the angle for blue in a pie chart.

[1]

Answer .....

- 11 Write down the mid-point of the class  $20 < t \leq 30$ .

[1]

Answer .....

- 12 **(Higher)** The class  $5 < x \leq 15$  has frequency 32. Work out its frequency density.

[1]

Answer .....

# Probability and the probability scale

**I can...** use the language of probability and calculate the probability of an event and its complement.

Spec ref: 6.3 · Foundation & Higher

An **event** is something that may happen, such as 'rolling a 6'. Its possible results are **outcomes**; they are **equally likely** if each has the same chance, e.g. a fair dice.

For equally likely outcomes:  $P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$ .  $P(\text{certain}) = 1$  and  $P(\text{impossible}) = 0$ .

$P(\text{event does not happen}) = 1 - P(\text{event happens})$ .

## Key words

outcome

event

equally likely

random

probability scale

## Key points

- Probabilities lie on a scale from 0 to 1
- $P(\text{not } A) = 1 - P(A)$
- The probabilities of all possible outcomes add up to 1

## Worked example 1

A spinner has 10 equal sections: 4 blue, 3 red, 2 green and 1 yellow. Find  $P(\text{red})$  and  $P(\text{not blue})$ .

$$P(\text{red}) = \frac{3}{10}$$

$$P(\text{not blue}) = \frac{6}{10} = \frac{3}{5}$$

## Worked example 2

$P(A) = 0.25$ ,  $P(B) = 0.15$ ,  $P(C) = x$  and  $P(D) = x$  are the only outcomes. Find  $x$ .

$$0.25 + 0.15 + 2x = 1$$

$$2x = 0.6, \text{ so } x = 0.3$$

## Common mistakes

- Writing probabilities as ratios or words ('3 out of 10').
- Assuming outcomes are equally likely when they are not.

**Exam tip:** Write probabilities as fractions, decimals or percentages; fractions do not have to be simplified unless the question asks.

Answer each question. Give probabilities as fractions, decimals or percentages.

1

A bag has 6 red and 9 yellow sweets. One is taken at random. Find  $P(\text{red})$ .

2

For the same bag, find  $P(\text{yellow})$ .

3

For the same bag, find  $P(\text{green})$ .

4

Which word describes a probability of 1: impossible, unlikely, even, likely or certain?

5

Which word describes getting a head when a fair coin is thrown?

6

A card is taken at random from cards numbered 1 to 12. Find  $P(\text{factor of 12})$ .

7

A card is taken at random from cards numbered 1 to 12. Find  $P(\text{number greater than 8})$ .

8

A card is taken at random from cards numbered 1 to 12. Find  $P(\text{odd prime number})$ .

9

A letter is chosen at random from the word PROBABILITY. Find  $P(B)$ .

10

A letter is chosen at random from PROBABILITY. Find  $P(\text{not a vowel})$ .

11

The probability that it will rain tomorrow is 0.35. Find the probability that it will not rain.

12

A team can win, draw or lose.  $P(\text{lose}) = 0.48$  and  $P(\text{draw}) = 0.17$ . Find  $P(\text{win})$ .

13

A bag has 40 counters.  $P(\text{blue}) = 0.35$ . How many blue counters are there?

14

Jamal says  $P(\text{win}) = 1/3$  because there are three outcomes: win, draw, lose. Why is he wrong?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 A biased spinner has four sections numbered 1, 2, 3 and 4. The table shows the probability that the spinner lands on each number.

| Number      | 1   | 2    | 3   | 4    |
|-------------|-----|------|-----|------|
| Probability | 0.3 | 0.25 | $x$ | $2x$ |

- (a) Work out the probability that the spinner lands on 4. [3]

Answer .....

- (b) Work out the probability that the spinner lands on an odd number. [2]

Answer .....

- (c) Ella says the spinner is fair. Explain why she is wrong. [1]

- 2 A box contains 5 black pens, 8 blue pens and 7 red pens. A pen is taken at random.



- (a) Write down the probability that the pen is blue. [1]

Answer .....

- (b) Work out the probability that the pen is not red. [1]

Answer .....

- (c) On the probability scale, mark with a cross (×) the probability that the pen is red. [1]

- (d) How many black pens must be added to the box so that the probability of taking a black pen is  $1/2$ ? [3]

Answer .....

- 3** A bag contains only red, white and blue counters. The probability of taking a red counter is 0.2. The probability of taking a white counter is 0.45.

**(a)** Work out the probability of taking a blue counter. [1]

Answer .....

**(b)** There are 12 red counters in the bag. Work out the total number of counters. [2]

Answer .....

**(c)** Work out the number of blue counters. [1]

Answer .....

- 4** A fair 8-sided dice has faces numbered 1 to 8. It is rolled once.

**(a)** Find the probability that the dice shows a multiple of 3. [1]

Answer .....

**(b)** Find the probability that the dice does not show 5. [1]

Answer .....

**(c)** Kemi says, 'The probability of rolling a prime number is  $\frac{1}{2}$ .' Is she correct? Show how you decide. [2]

**(d)** Explain why 'rolling an even number' and 'rolling a number less than 9' are not equally likely. [1]

- 5** 400 tickets are sold in a raffle. One ticket is chosen at random to win.

**(a)** The probability that Sara wins is 0.035. How many tickets did Sara buy? [2]

Answer .....

**(b)** Leo bought three times as many tickets as Sara. Work out the probability that Leo wins. Give your answer as a fraction in its simplest form. [2]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

## Relative frequency and expected frequency

**I can...** estimate probabilities from data and use probability to work out expected frequency.

Spec ref: 6.3 · Foundation & Higher

When we cannot work out a probability from equally likely outcomes, we **estimate** it from collected data. Relative frequency = number of successes ÷ number of trials.

More trials give a more reliable estimate. A spinner or dice is **biased** if its outcomes are not equally likely.

**Expected frequency** = probability × number of trials. It is what you expect on average, not what must happen.

### Key words

relative frequency

trial

estimate

expected frequency

biased

### Key points

- Relative frequency = successes ÷ trials
- Expected frequency = probability × number of trials
- Combine results to get a better estimate

### Worked example

**In a survey, 42 of 120 cars passing a school were red. Estimate P(red car) and the number of red cars in the next 500.**

Relative frequency =  $42 \div 120 = 0.35$

Expected number =  $0.35 \times 500 = 175$  red cars

### Common mistakes

- Dividing by the number of successes instead of the number of trials.
- Thinking an unusual result proves bias after only a few trials.

**Exam tip:** If two people's results are given, add the successes and add the trials to get the best overall estimate.

Answer each question. Give probabilities as fractions, decimals or percentages.

- 1 A spinner lands on blue 36 times in 90 spins. Estimate  $P(\text{blue})$ .

- 2 A coin is thrown 50 times and shows heads 23 times. Find the relative frequency of heads.

- 3  $P(\text{scoring a goal}) = 0.3$ . A player takes 70 shots. Work out the expected number of goals.

- 4 A fair dice is rolled 300 times. How many times would you expect to get 5?

- 5  $P(\text{rain on a day in April}) = 0.24$ . How many rainy days would you expect in April (30 days)?

- 6 45 out of 60 seeds grew. Estimate the probability that a seed grows.

- 7 Using question 6, how many of 200 seeds would you expect to grow?

- 8 After 250 trials the relative frequency of success is 0.62. How many successes were there?

- 9 Which gives the best estimate of a probability: 20, 100 or 1000 trials?

- 10 A coin is thrown 10 times and shows 7 heads. Does this prove the coin is biased?

- 11  $P(\text{bus is late}) = 1/8$ . On how many of 64 days would you expect the bus to be late?

- 12 3 in every 200 phones made are faulty. How many faulty phones would you expect in 12 000?

- 13 An event has an expected frequency of 45 in 180 trials. Find its probability.

- 14 A bag has 10 counters. In 50 draws (with replacement), a red counter was taken 20 times. Estimate the number of red counters.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 Rosa rolls a six-sided dice 120 times. The table shows her results.

| Score     | 1  | 2  | 3  | 4  | 5  | 6  |
|-----------|----|----|----|----|----|----|
| Frequency | 18 | 22 | 19 | 20 | 17 | 24 |

- (a) Work out the relative frequency of rolling a 6. [1]

Answer .....

- (b) Estimate the probability of rolling an even number. [2]

Answer .....

- (c) Do you think the dice is fair? Give a reason. [1]

- (d) Rosa rolls the dice 900 more times. Use her results to estimate the number of times it shows 6. [2]

Answer .....

- 2 Chen recorded the types of 250 vehicles passing his school.

| Vehicle   | Car | Van | Lorry | Motorbike |
|-----------|-----|-----|-------|-----------|
| Frequency | 160 | 45  | 20    | 25        |

- (a) Work out the relative frequency of a van. [1]

Answer .....

- (b) Estimate the number of lorries in the next 1000 vehicles. [2]

Answer .....

- (c) Give one reason why your answer to (b) might not be reliable. [1]

- 3 At a fair, it costs \$2 to play a game. The probability of winning is 0.15. Each winner gets \$10. 300 people play the game.

(a) Work out the expected number of winners. [2]

Answer .....

(b) Work out the expected profit for the fair. [3]

Answer \$ .....

- 4 Amir and Bella each throw the same coin. Amir throws it 40 times and gets 14 heads. Bella throws it 160 times and gets 70 heads.

(a) Work out the relative frequency of heads for each person. [2]

Answer Amir ..... Bella .....

(b) Combine the results to give the best estimate of the probability of heads. [2]

Answer .....

(c) Whose result is more reliable? Give a reason. [1]

- 5 The probability that a spinner lands on red is 0.32.

(a) The spinner is spun 250 times. Work out the expected number of reds. [2]

Answer .....

(b) In fact, red came up 95 times. Does this mean the probability 0.32 is wrong? Explain. [1]

(c) The spinner is spun  $n$  times. The expected number of reds is 48. Find  $n$ . [2]

Answer  $n$  = .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Sample spaces and mutually exclusive events

**I can...** list outcomes systematically and add probabilities of mutually exclusive events.

Spec ref: 6.3 · Foundation & Higher

The **sample space** is the set of all possible outcomes. List outcomes in a **systematic** order, or use a sample space diagram (a grid) for two events.

Events are **mutually exclusive** if they cannot happen at the same time, e.g. 'roll a 2' and 'roll a 5'.

For mutually exclusive events:  $P(A \text{ or } B) = P(A) + P(B)$ .

## Key words

sample space

systematic

mutually exclusive

outcome

addition rule

## Key points

- Number of outcomes for two events = (first) × (second)
- Mutually exclusive:  $P(A \text{ or } B) = P(A) + P(B)$
- Only add probabilities when the events cannot overlap

## Worked example 1

**A coin is thrown and a spinner (red, blue, green) is spun. List the outcomes and find  $P(\text{head and green})$ .**

H-R, H-B, H-G, T-R, T-B, T-G: 6 equally likely outcomes

$P(\text{head and green}) = \frac{1}{6}$

## Worked example 2

**$P(\text{red}) = 0.3$  and  $P(\text{blue}) = 0.25$  for a spinner. Find  $P(\text{red or blue})$ .**

Red and blue are mutually exclusive.

$P(\text{red or blue}) = 0.3 + 0.25 = 0.55$

## Common mistakes

- Missing outcomes such as (2, 1) when (1, 2) is listed.
- Adding probabilities of events that can happen together.

**Exam tip:** For two dice or spinners, draw the full grid of outcomes before you count.

Answer each question. Give probabilities as fractions, decimals or percentages.

1 A coin is thrown and a dice is rolled. How many possible outcomes are there?

2 A café has 3 starters, 4 main courses and 2 desserts. How many different three-course meals are there?

3 Two fair dice are rolled and the scores multiplied. Find  $P(\text{product} = 6)$ .

4 Two fair dice are rolled and the scores multiplied. Find  $P(\text{product is odd})$ .

5 Two fair dice are rolled and the scores multiplied. Find  $P(\text{product} > 20)$ .

6 Two fair spinners, each numbered 1–4, are spun and the scores added. Find  $P(\text{total} = 5)$ .

7 For the spinners in question 6, find  $P(\text{total is 7 or more})$ .

8 A and B are mutually exclusive.  $P(A) = 0.4$  and  $P(B) = 0.35$ . Find  $P(A \text{ or } B)$ .

9  $P(\text{lemon sweet}) = 0.2$  and  $P(\text{lime sweet}) = 0.15$ . Find  $P(\text{lemon or lime})$ .

10 Are 'rolling a 6' and 'rolling an even number' mutually exclusive?

11 A card is taken from cards numbered 1–10. Find  $P(\text{less than 3 or greater than 8})$ .

12 A coin is thrown three times. How many possible outcomes are there?

13 A coin is thrown three times. Find  $P(\text{at least one head})$ .

14 A spinner is red, blue or green.  $P(\text{red}) = 0.35$  and  $P(\text{blue}) = 0.4$ . Find  $P(\text{green})$ .

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 Spinner A is numbered 1, 3, 5. Spinner B is numbered 2, 3, 4, 5. Both spinners are fair. They are spun and the scores are **added**.

| + | 2 | 3 | 4 | 5  |
|---|---|---|---|----|
| 1 | 3 |   |   |    |
| 3 |   |   |   |    |
| 5 |   |   |   | 10 |

(a) Complete the sample space diagram. [2]

(b) Find the probability that the total is 8. [1]

Answer .....

(c) Find the probability that the total is an odd number. [2]

Answer .....

(d) Find the probability that the total is greater than 10. [1]

Answer .....

- 2 Freya chooses one ice-cream flavour (vanilla, strawberry or chocolate) and one topping (sprinkles or sauce).

(a) List all the possible combinations. [2]

(b) Freya chooses at random. Find the probability that she chooses chocolate with sauce. [1]

Answer .....

- 3 A bag contains red, blue, green and white beads. A bead is taken at random. The table shows some of the probabilities.

| Colour      | Red  | Blue | Green | White |
|-------------|------|------|-------|-------|
| Probability | 0.15 | 0.3  |       | 0.2   |

- (a) Work out the probability that the bead is green. [2]

Answer .....

- (b) Work out the probability that the bead is red or white. [1]

Answer .....

- (c) There are 80 beads in the bag. How many of them are blue? [2]

Answer .....

- 4 Two fair four-sided dice, each numbered 1 to 4, are rolled. The score is the total of the two numbers.

- (a) Show all the possible outcomes in a sample space diagram. [2]

- (b) Which score is most likely? Give its probability. [2]

Answer score ..... probability .....

- (c) Find the probability that the score is a prime number. [2]

Answer .....

- 5 A card is taken at random from 20 cards numbered 1 to 20. Event A is 'the number is even'. Event B is 'the number is a multiple of 5'.

**(a)** Are events A and B mutually exclusive? Give a reason. [1]

**(b)** Find  $P(A \text{ or } B)$ . [2]

Answer .....

**(c)** Explain why  $P(A) + P(B)$  does not give the answer to (b). [2]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Probability from Venn diagrams

**I can...** find probabilities from Venn diagrams using set notation.

Spec ref: 6.3 · Foundation & Higher

A **Venn diagram** shows how items of the universal set  $\mathcal{E}$  are split between sets. Each region shows a number of items, or a probability.

$A \cap B$  means in both A **and** B.  $A \cup B$  means in A **or** B (or both).  $A'$  means **not** in A.

$P(\text{region}) = \text{number in the region} \div \text{number in } \mathcal{E}$ . If the diagram shows probabilities, all the regions add up to 1.

## Key words

**Venn diagram**

**universal set  $\mathcal{E}$**

**intersection  $n$**

**union  $u$**

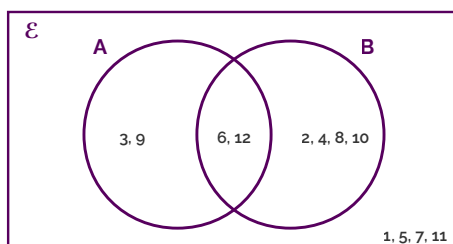
**complement  $A'$**

## Key points

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- Fill the overlap first, then the rest of each set, then the outside
- All regions together make the whole of  $\mathcal{E}$

## Worked example

$\mathcal{E} = \{1, 2, \dots, 12\}$ ,  $A = \{\text{multiples of 3}\}$ ,  $B = \{\text{even numbers}\}$ . Find  $P(A \cap B)$  and  $P(A \cup B)$ .



$A \cap B = \{6, 12\}$ ;  $A \text{ only} = \{3, 9\}$ ;  $B \text{ only} = \{2, 4, 8, 10\}$

$P(A \cap B) = 2/12 = \mathbf{1/6}$

$P(A \cup B) = 8/12 = \mathbf{2/3}$

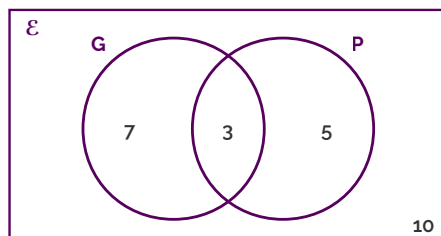
## Common mistakes

- Writing the whole of set A in the 'A only' region.
- Forgetting the items outside the circles.

**Exam tip:** Check your Venn diagram: the numbers in all the regions must add up to  $n(\mathcal{E})$ .

Answer each question. Give probabilities as fractions, decimals or percentages.

- 1 The Venn diagram shows how many of 25 students play guitar (G) and piano (P). A student is chosen at random.



(a) Find  $P(G \cap P)$ .

Answer .....

(b) Find  $P(G)$ .

Answer .....

(c) Find  $P(G \cup P)$ .

Answer .....

(d) Find  $P(P')$ .

Answer .....

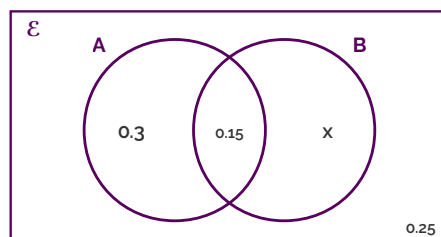
(e) Find  $P(G \cap P')$ .

Answer .....

(f) Find  $P(G' \cap P')$ .

Answer .....

- 2 The Venn diagram shows the probabilities of events A and B.



(a) Find the value of x.

Answer  $x =$  .....

(b) Find  $P(B)$ .

Answer .....

(c) Find  $P(A \cup B)$ .

Answer .....

3  $\mathcal{E} = \{1, 2, \dots, 12\}$ ,  $A = \{\text{multiples of 4}\}$ ,  $B = \{\text{factors of 12}\}$ . List the members of  $A \cap B$ .

Answer .....

4 Using the sets in question 3, find  $P(A \cup B)$  for a number chosen at random from  $\mathcal{E}$ .

Answer .....

5  $n(\mathcal{E}) = 50$ ,  $n(A) = 30$ ,  $n(B) = 25$  and  $n(A \cap B) = 12$ . Find  $n(A \cup B)$ .

Answer .....

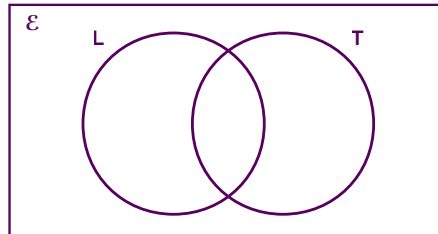
6 Using question 5, find  $P(A \cap B)$ .

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 60 students were asked if they have a laptop (L) or a tablet (T). 34 have a laptop, 28 have a tablet and 9 have neither.



- (a) Work out how many students have both a laptop and a tablet. [2]

Answer .....

- (b) Complete the Venn diagram. [2]

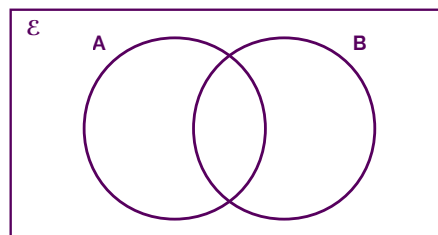
- (c) A student is chosen at random. Find the probability that the student has a laptop but not a tablet. [1]

Answer .....

- (d) Find  $P((L \cup T)')$ . [1]

Answer .....

- 2  $\mathcal{E} = \{1, 2, 3, \dots, 16\}$ ,  $A = \{\text{square numbers}\}$ ,  $B = \{\text{factors of 16}\}$ .



- (a) Complete the Venn diagram by writing each number of  $\mathcal{E}$  in the correct region. [3]

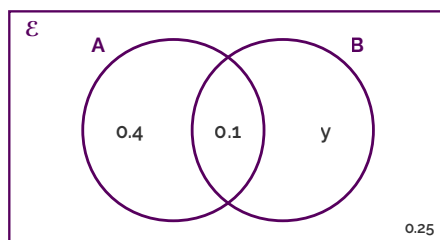
- (b) A number is chosen at random from  $\mathcal{E}$ . Find  $P(A \cap B)$ . [1]

Answer .....

- (c) Find  $P(A' \cap B)$ . [1]

Answer .....

- 3 The Venn diagram shows the probabilities of two events A and B.



- (a) Find the value of  $y$ .

[2]

**Answer**  $y = \dots\dots$

- (b) Find  $P(B)$ .

[1]

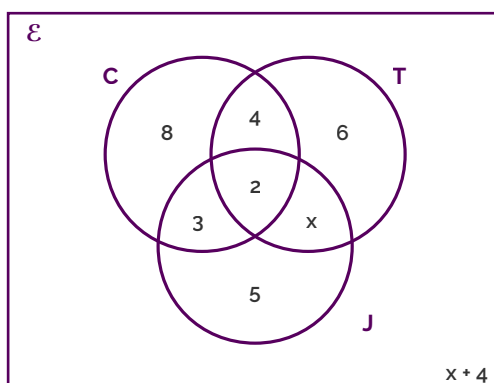
**Answer**  $\dots\dots$

- (c) Find  $P(A \cup B)$ .

[1]

**Answer**  $\dots\dots$

- 4 50 people were asked whether they drink coffee (C), tea (T) or juice (J). The Venn diagram shows the results.



- (a) Find the value of  $x$ .

[2]

**Answer**  $x = \dots\dots$

- (b) A person is chosen at random. Find the probability that they drink exactly one of the three drinks.

[2]

**Answer**  $\dots\dots$

- (c) Find the probability that the person drinks tea.

[1]

**Answer**  $\dots\dots$

**5** In a group of 30 students, 17 play football (F), 13 play hockey (H) and 6 play both.

**(a)** Find  $n(F \cup H)$ .

**[2]**

**Answer .....**

**(b)** A student is chosen at random. Find  $P(F \cap H)$ .

**[1]**

**Answer .....**

**(c)** Find  $P(F \cup H)$ .

**[2]**

**Answer .....**

---

**Reflect:**   ☐ Red — not yet   ☐ Amber — nearly   ☐ Green — secure

# Tree diagrams and independent events

**I can...** draw and use tree diagrams and find the probability of two or more independent events.

Spec ref: 6.3 · Higher

Events are **independent** if the outcome of one does not change the probability of the other. Then  $P(A \text{ and } B) = P(A) \times P(B)$ .

A **tree diagram** shows each stage on a new set of branches.

**Multiply** along a path; **add** the paths you need.

For 'at least one', use  $1 - P(\text{none})$ .

## Key words

independent

tree diagram

branch

at least one

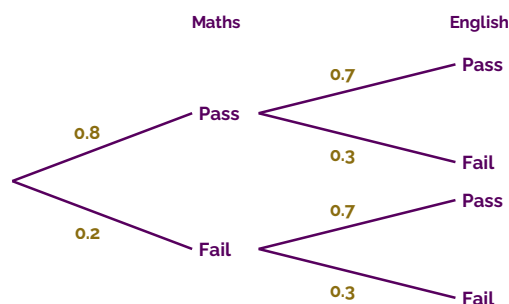
combined event

## Key points

- Independent:  $P(A \text{ and } B) = P(A) \times P(B)$
- $P(\text{at least one}) = 1 - P(\text{none})$

## Worked example

$P(\text{Ana passes Maths}) = 0.8$  and  $P(\text{passes English}) = 0.7$ , independently. Find  $P(\text{she passes exactly one})$ .



Pass M, fail E:  $0.8 \times 0.3 = 0.24$

Fail M, pass E:  $0.2 \times 0.7 = 0.14$

$P(\text{exactly one}) = 0.24 + 0.14 = \mathbf{0.38}$

## Common mistakes

- Adding along a path instead of multiplying.
- Forgetting that 'exactly one' can happen in two different orders.

**Exam tip:** Write the probability of each final outcome at the end of its path; they should add up to 1.

Answer each question. Give probabilities as fractions, decimals or percentages.

- 1 A and B are independent.  $P(A) = 0.6$  and  $P(B) = 0.5$ . Find  $P(A \text{ and } B)$ .

- 2 A fair dice is rolled twice. Find  $P(\text{two 6s})$ .

- 3 A fair dice is rolled twice. Find  $P(\text{no 6s})$ .

- 4 A fair dice is rolled twice. Find  $P(\text{at least one 6})$ .

- 5  $P(\text{red}) = 0.3$  for a spinner. It is spun twice. Find  $P(\text{red then not red})$ .

- 6 For the spinner in question 5, find  $P(\text{exactly one red})$ .

- 7  $P(\text{rain on any day}) = 0.2$ . Find  $P(\text{rain on all of three days})$ .

- 8 For question 7, find  $P(\text{no rain on any of the three days})$ .

- 9 For question 7, find  $P(\text{rain on at least one of the days})$ .

- 10 A bag has 3 red and 2 blue balls. One is taken and replaced, then another. Find  $P(\text{blue, blue})$ .

- 11 For question 10, find  $P(\text{the two balls are different colours})$ .

- 12  $P(\text{bus late}) = 0.1$  and  $P(\text{train late}) = 0.25$ , independently. Find  $P(\text{both on time})$ .

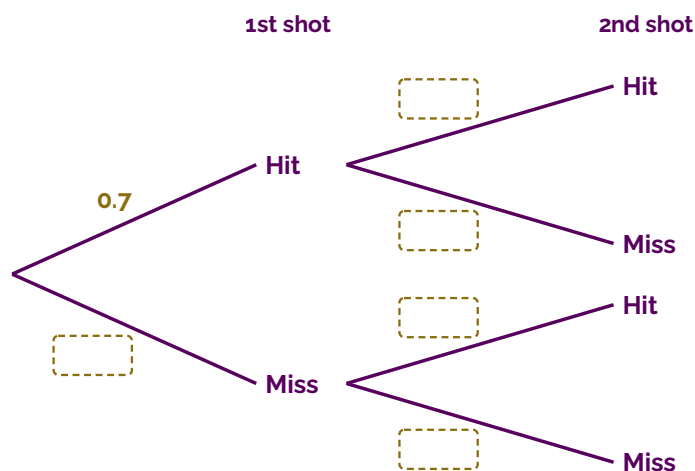
- 13 For question 12, find  $P(\text{at least one is late})$ .

- 14 A and B are independent.  $P(A) = 0.4$  and  $P(A \text{ and } B) = 0.12$ . Find  $P(B)$ .

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 Ravi is an archer. The probability that he hits the target with any shot is 0.7. He takes two shots, which are independent.



(a) Complete the probability tree diagram. [2]

(b) Work out the probability that Ravi hits the target with both shots. [2]

Answer .....

(c) Work out the probability that Ravi hits the target with exactly one shot. [3]

Answer .....

- 2** Box A contains 3 red and 5 green balls. Box B contains 4 red and 2 green balls. One ball is taken at random from each box.

**(a)** Draw a probability tree diagram to show this information.

**[3]**

**(b)** Work out the probability that both balls are red.

**[2]**

**Answer** .....

**(c)** Work out the probability that at least one ball is green.

**[2]**

**Answer** .....

- 3** The probability that Kim walks to school on any day is 0.6. The days are independent.

**(a)** Work out the probability that Kim walks to school on Monday, Tuesday and Wednesday.

**[2]**

**Answer** .....

**(b)** Work out the probability that Kim walks to school on exactly two of these three days. **[3]**

**Answer** .....

- 4** The probability that a team wins any match is  $p$ . The matches are independent. The probability that the team wins two matches in a row is 0.36.

**(a)** Find the value of  $p$ .

**[2]**

**Answer**  $p =$  .....

**(b)** Find the probability that the team wins neither of the two matches.

**[1]**

**Answer** .....

**(c)** Find the probability that the team wins at least one of the two matches.

**[1]**

**Answer** .....

- 5 Sam rolls a fair dice. He keeps rolling until he gets a 6, or until he has rolled three times.

Work out the probability that Sam gets his first 6 on the third roll.

[2]

Answer .....

---

Reflect:   ☐ Red — not yet   ☐ Amber — nearly   ☐ Green — secure

# Conditional probability

**I can...** find conditional probabilities when combining events, including selections without replacement.

Spec ref: 6.3 · Higher

When the first event changes the probabilities for the second, the events are **dependent**. Taking items **without replacement** is the most common example.

$P(B | A)$  means the probability of B **given that** A has happened. On a tree diagram, the second-stage branches are conditional probabilities.

From a table:  $P(B | A) = \text{number in A and B} \div \text{number in A}$ .

## Key words

conditional

dependent

without replacement

given that

$P(B | A)$

## Key points

- Without replacement: the total and the chosen group both go down by 1
- $P(A \text{ and } B) = P(A) \times P(B | A)$
- 'Given that' tells you the new total

## Worked example

**A bag has 5 blue and 4 white beads. Two are taken without replacement. Find  $P(\text{one of each colour})$ .**

$$P(B \text{ then } W) = 5/9 \times 4/8 = 20/72$$

$$P(W \text{ then } B) = 4/9 \times 5/8 = 20/72$$

$$P(\text{one of each}) = 40/72 = \mathbf{5/9}$$

## Common mistakes

- Keeping the same probabilities on the second branches when there is no replacement.
- Dividing by the overall total instead of the 'given' total.

**Exam tip:** In 'without replacement' questions, write the new numbers on the second branches before you multiply.

**Answer each question. Give probabilities as fractions, decimals or percentages.**

- 1** A bag contains 7 red and 3 yellow sweets. Two sweets are taken at random without replacement.

**(a)** Find  $P(\text{2nd is red} \mid \text{1st is red})$ .

**Answer** .....

**(b)** Find  $P(\text{both red})$ .

**Answer** .....

**(c)** Find  $P(\text{both yellow})$ .

**Answer** .....

**(d)** Find  $P(\text{one of each colour})$ .

**Answer** .....

- 2** The table gives information about 50 people. One person is chosen at random.

|              | Glasses | No glasses | Total |
|--------------|---------|------------|-------|
| Left-handed  | 4       | 6          | 10    |
| Right-handed | 14      | 26         | 40    |
| Total        | 18      | 32         | 50    |

**(a)** Find  $P(\text{wears glasses})$ .

**Answer** .....

**(b)** Find  $P(\text{wears glasses} \mid \text{left-handed})$ .

**Answer** .....

**(c)** Find  $P(\text{left-handed} \mid \text{wears glasses})$ .

**Answer** .....

**(d)** Find  $P(\text{right-handed} \mid \text{no glasses})$ .

**Answer** .....

- 3**  $P(A \text{ and } B) = 0.18$  and  $P(A) = 0.6$ . Find  $P(B \mid A)$ .

**Answer** .....

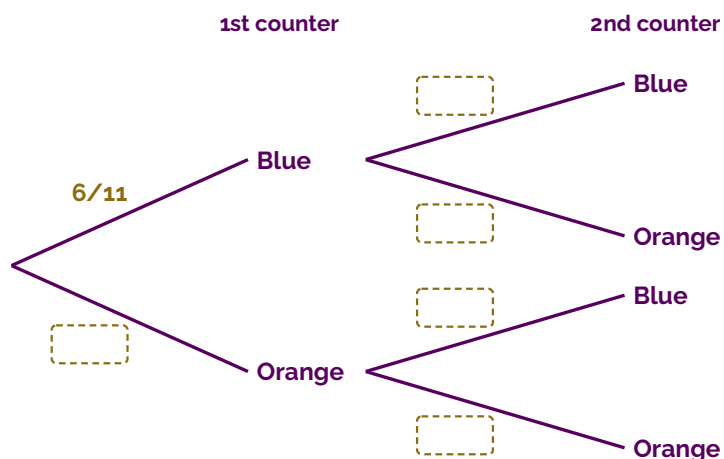
- 4** A drawer has 4 black and 6 white socks. Two are taken at random. Find  $P(\text{a matching pair})$ .

**Answer** .....

**Reflect:**   ☐ Red — not yet   ☐ Amber — nearly   ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 A bag contains 6 blue and 5 orange counters. Two counters are taken at random without replacement.



(a) Complete the probability tree diagram. [2]

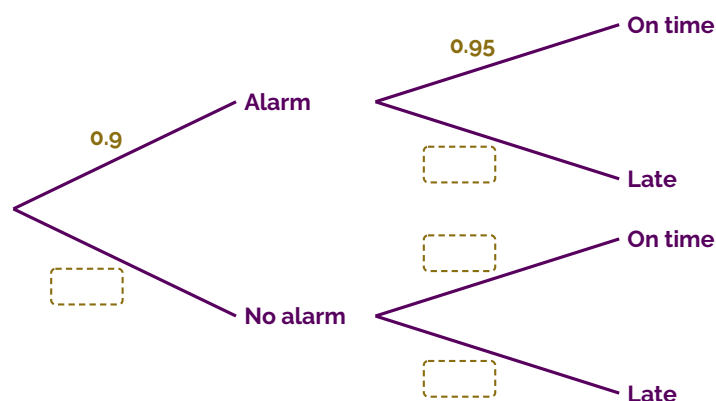
(b) Work out the probability that both counters are the same colour. [3]

Answer .....

(c) Work out the probability that at least one counter is orange. [2]

Answer .....

- 2 The probability that Tom sets his alarm is 0.9. If he sets his alarm, the probability that he is on time is 0.95. If he does not set it, the probability that he is on time is 0.4.



(a) Complete the tree diagram. [2]

(b) Work out the probability that Tom is on time. [3]

Answer .....

(c) Work out the probability that Tom is late. [1]

Answer .....

- 3 A bag has 5 toffees and 7 mints. Ben takes three sweets at random without replacement.

Work out the probability that exactly one of the sweets is a toffee. [4]

Answer .....

- 4 The table shows information about the clubs chosen by 120 students. Each student chose one option.

|         | Sport | Music | None |
|---------|-------|-------|------|
| Year 10 | 24    | 18    | 22   |
| Year 11 | 20    | 15    | 21   |

- (a) A student is chosen at random. Find the probability that the student chose music. [1]

Answer .....

- (b) Given that the student chose sport, find the probability that they are in Year 11. [2]

Answer .....

- (c) Given that the student is in Year 10, find the probability that they chose sport. [2]

Answer .....

- 5 A bag contains  $n$  counters. 3 of the counters are red. Two counters are taken at random without replacement. The probability that both are red is  $1/15$ .

- (a) Show that  $n^2 - n - 90 = 0$ . [3]

- (b) Find the number of counters in the bag. [1]

Answer  $n =$  .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Collecting data and two-way tables

**I can...** organise data in tally charts, frequency tables, grouped tables and two-way tables.

Spec ref: 6.1 · Foundation

A **tally chart** records data as it is collected; |||| |||| means 10. The **frequency** is the total for each value.

Large amounts of data can be grouped into **class intervals** such as  $10 < t \leq 12$ , which means 'more than 10, up to and including 12'.

A **two-way table** shows two kinds of information together. Rows and columns add up to their totals.

## Key words

tally

frequency table

class interval

two-way table

discrete

continuous

## Key points

- Discrete data are counted; continuous data are measured
- Class intervals must not overlap
- Two-way tables: check every row and column total

## Worked example

**50 students: 28 are girls. 15 girls and 9 boys have a pet. Draw a two-way table.**

$$\text{Boys} = 50 - 28 = 22$$

$$\text{Girls without a pet} = 28 - 15 = 13; \text{ boys without} = 22 - 9 = 13$$

$$\text{Pet total} = 24; \text{ no pet total} = 26; \text{ grand total } 50 \checkmark$$

## Common mistakes

- Putting a value on the boundary in the wrong class.
- Forgetting to check that the totals add up.

**Exam tip:** Cross off each value as you tally it, then check that your frequencies add up to the number of values.

Answer each question. Show your working where it helps.

- 1 24 students chose their favourite fruit: A = apple, B = banana, M = mango, O = orange.  
A, B, M, A, O, M, M, B, A, M, O, A, B, M, M, A, O, B, M, A, M, B, A, M

(a) How many students chose mango?

Answer .....

(b) Which fruit is the mode?

Answer .....

(c) What fraction of the students chose apple? Give your answer in its simplest form.

Answer .....

- 2 80 people were asked where they would like to go on holiday. Complete the two-way table, then answer the questions.

|          | Beach | Mountains | City | Total |
|----------|-------|-----------|------|-------|
| Adults   | 18    | 9         |      | 40    |
| Children |       | 7         | 11   |       |
| Total    | 40    |           |      | 80    |

(a) How many children chose the beach?

Answer .....

(b) How many adults were asked?

Answer .....

(c) How many people chose the city?

Answer .....

- 3 The heights, in cm, of 16 plants are:  
142, 155, 161, 148, 152, 167, 159, 144, 171, 150, 163, 158, 149, 166, 154, 160

(a) How many heights are in the class  $150 < h \leq 160$ ?

Answer .....

(b) Which class has the highest frequency?

Answer .....

- 4 Is 'the time taken to run 400 m' discrete or continuous data?

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 The shoe sizes of 20 students are:  
5, 6, 4, 7, 6, 5, 6, 8, 5, 6, 7, 4, 6, 5, 6, 9, 7, 6, 5, 6

| Shoe size | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------|---|---|---|---|---|---|
| Tally     |   |   |   |   |   |   |
| Frequency |   |   |   |   |   |   |

(a) Complete the frequency table. [3]

(b) Write down the mode. [1]

Answer .....

(c) How many students have a shoe size of 6 or more? [1]

Answer .....

- 2 200 students each study one language. The two-way table shows some information.

|         | French | Spanish | Mandarin | Total |
|---------|--------|---------|----------|-------|
| Year 10 | 34     | 28      |          | 95    |
| Year 11 |        | 31      | 47       |       |
| Total   | 61     |         |          | 200   |

(a) Complete the two-way table. [4]

(b) What fraction of the Year 11 students study Mandarin? [1]

Answer .....

(c) What percentage of all the students study Spanish? [2]

Answer ..... %

- 3 The times, in seconds, taken by 30 students to run 100 m are:  
12.4, 15.8, 13.1, 18.6, 14.9, 16.2, 11.7, 13.8, 17.3, 15.1, 12.9, 14.2, 16.8, 19.4, 15.5, 13.6, 14.7, 16.0, 12.2, 17.9, 15.0, 14.4, 13.3, 16.5, 18.1, 14.0, 15.7, 13.9, 17.0, 14.8

| Time ( $t$ s) | $10 < t \leq 12$ | $12 < t \leq 14$ | $14 < t \leq 16$ | $16 < t \leq 18$ | $18 < t \leq 20$ |
|---------------|------------------|------------------|------------------|------------------|------------------|
| Frequency     |                  |                  |                  |                  |                  |

(a) Complete the grouped frequency table. [3]

(b) Write down the modal class. [1]

Answer .....

(c) A student's time of exactly 16.0 seconds was put in the class  $16 < t \leq 18$ . Explain the mistake. [1]

---

- 4 The table shows the distances, in km, between four towns.

|         | Arton | Bexford | Calby | Dunmore |
|---------|-------|---------|-------|---------|
| Arton   | —     | 48      | 112   | 75      |
| Bexford | 48    | —       | 86    | 39      |
| Calby   | 112   | 86      | —     | 64      |
| Dunmore | 75    | 39      | 64    | —       |

(a) Write down the distance between Calby and Dunmore. [1]

Answer ..... km

(b) Kai drives from Arton to Bexford and then from Bexford to Calby. How far does he drive? [2]

Answer ..... km

(c) Which two towns are closest together? [1]

Answer .....

- 5 There are 32 students in a class. 18 of them are girls. 11 of the girls and 5 of the boys walk to school.

(a) Draw a two-way table to show this information. [3]

(b) How many boys do not walk to school? [1]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Pictograms and bar charts

**I can...** draw and interpret pictograms, bar charts, comparative bar charts and composite bar charts.

Spec ref: 6.1 · Foundation

A **pictogram** uses symbols; the **key** tells you what one symbol represents. Part of a symbol shows part of that amount.

In a **bar chart**, bars have equal widths and the height shows the frequency. Label both axes and use an even scale.

A **comparative bar chart** puts bars for two groups side by side.

A **composite bar chart** stacks the parts of a total in one bar.

## Key words

pictogram

key

bar chart

comparative bar chart

composite bar chart

## Key points

- Always read the key of a pictogram
- Leave equal gaps between the bars of a bar chart
- In a composite bar, find each part by subtracting the readings

## Worked example

In a composite bar chart, the Year 9 bar has Walk from 0 to 12, Bus from 12 to 22 and Car from 22 to 28. How many travel by bus? How many students are in Year 9?

Bus =  $22 - 12 = 10$  students

Total = top of the bar = **28** students

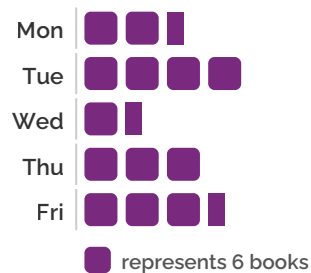
## Common mistakes

- Reading the top of a stacked section instead of its height.
- Ignoring the key in a pictogram.

**Exam tip:** In a composite bar chart, each section's value is the reading at its top minus the reading at its bottom.

Answer each question. Show your working where it helps.

- 1 The pictogram shows the number of books borrowed from a library each day.



- (a) How many books were borrowed on Tuesday?

Answer .....

- (b) How many books were borrowed on Wednesday?

Answer .....

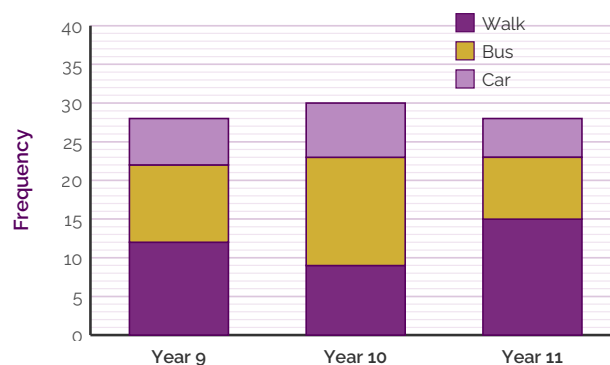
- (c) How many more books were borrowed on Friday than on Monday?

Answer .....

- (d) How many books were borrowed altogether?

Answer .....

- 2 The composite bar chart shows how students in three year groups travel to school.



- (a) How many Year 10 students walk?

Answer .....

- (b) How many Year 9 students are there altogether?

Answer .....

- (c) Which year group has the most bus users?

Answer .....

- (d) How many students travel by car altogether?

Answer .....

3 Why should all the bars in a bar chart have the same width?

Answer .....

4 In a pictogram, one symbol represents 8 people. How many people do  $3\frac{1}{2}$  symbols represent?

Answer .....

5 In a pictogram, one symbol represents 20 cars. How many symbols show 50 cars?

Answer .....

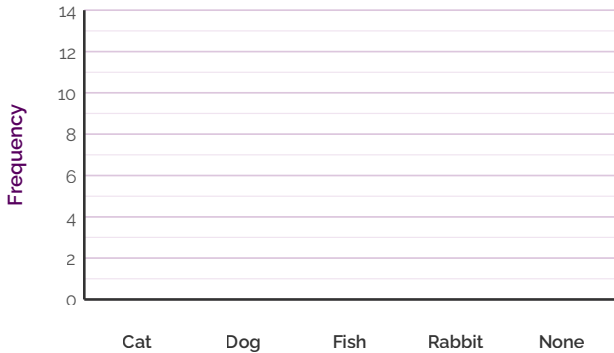
6 What is the difference between a comparative bar chart and a composite bar chart?

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 The table shows the pets owned by some students. Draw a bar chart to show this information.

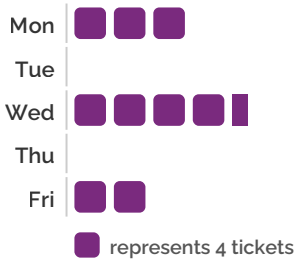


| Pet       | Cat | Dog | Fish | Rabbit | None |
|-----------|-----|-----|------|--------|------|
| Frequency | 9   | 12  | 4    | 6      | 7    |

- (a) Draw the bar chart on the axes. [3]
- (b) Write down the mode and work out how many students were asked. [2]

Answer mode ..... total .....

- 2 The pictogram shows the number of tickets sold for a school play. It is not complete.



- (a) How many tickets were sold on Wednesday? [1]
- Answer .....
- (b) 10 tickets were sold on Tuesday and 6 on Thursday. Complete the pictogram. [2]
- (c) Work out the total number of tickets sold from Monday to Friday. [2]

Answer .....

- 3 The comparative bar chart shows the number of phones sold by two shops.



- (a) How many phones did Shop B sell in February? [1]

Answer .....

- (b) In which month was the difference between the two shops the greatest? [1]

Answer .....

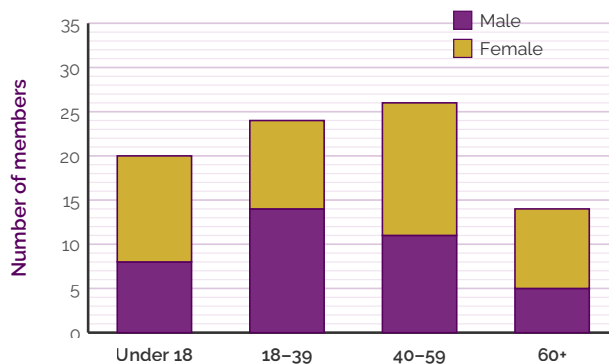
- (c) Work out the total number of phones sold by Shop A. [2]

Answer .....

- (d) Ali says Shop B sold more phones than Shop A in these four months. Is he correct? Explain. [1]

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- 4 The composite bar chart shows the ages of the members of a sports club.



- (a) How many members are aged 40-59? [2]

Answer .....

- (b) What fraction of the members aged 40-59 are female? [2]

Answer .....

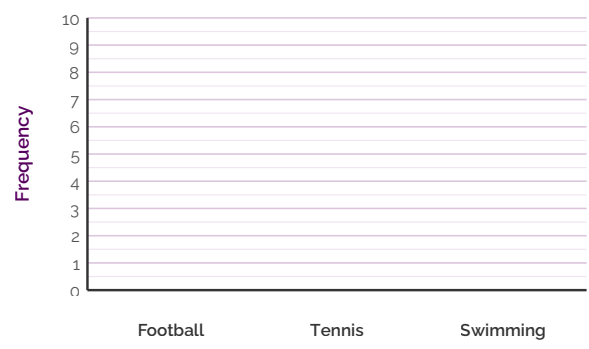
- (c) What percentage of all the members are female? [2]

Answer ..... %

**5** The table shows the clubs chosen by some boys and girls.

| Club  | Football | Tennis | Swimming |
|-------|----------|--------|----------|
| Boys  | 8        | 4      | 6        |
| Girls | 5        | 7      | 9        |

**(a)** Draw a comparative bar chart to show this information. **[3]**



**(b)** Make one comparison between the clubs chosen by boys and by girls. **[1]**

\_\_\_\_\_

Reflect:    ☐ Red — not yet    ☐ Amber — nearly    ☐ Green — secure

# Pie charts

**I can...** draw pie charts and use them to find frequencies, fractions and percentages.

Spec ref: 6.1 · Foundation

A **pie chart** shows how a total is split into parts. The whole circle,  $360^\circ$ , represents the total frequency.

To draw one, work out the number of degrees for one item ( $360 \div \text{total}$ ), then multiply by each frequency. Draw each angle with a protractor.

To read one, use  $\text{frequency} = \text{angle} \div 360 \times \text{total}$ .

## Key words

**pie chart**

**sector**

**angle**

**proportion**

**total**

## Key points

- Angle = frequency  $\div$  total  $\times 360^\circ$
- Frequency = angle  $\div 360^\circ \times$  total
- Pie charts show proportions, not actual numbers

## Worked example 1

**40 students: football 14, basketball 9, tennis 6, swimming 11. Find the football angle.**

$$360 \div 40 = 9^\circ \text{ per student}$$

$$\text{Football: } 14 \times 9 = \mathbf{126^\circ}$$

## Worked example 2

**A sector of  $54^\circ$  represents 12 people. How many people are in the whole pie chart?**

$$54 \div 12 = 4.5^\circ \text{ per person}$$

$$\text{Total} = 360 \div 4.5 = \mathbf{80 \text{ people}}$$

## Common mistakes

- Forgetting to check that the angles add up to  $360^\circ$ .
- Comparing the sizes of sectors in pie charts with different totals.

**Exam tip:** Show the calculation for each angle; method marks are given even if one angle is drawn inaccurately.

Answer each question. Show your working where it helps.

1 40 students chose their favourite sport. Work out the angle for each sport in a pie chart.

| Sport     | Football | Basketball | Tennis | Swimming |
|-----------|----------|------------|--------|----------|
| Frequency | 14       | 9          | 6      | 11       |

(a) Football

Answer .....°

(b) Basketball

Answer .....°

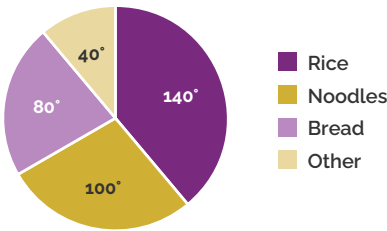
(c) Tennis

Answer .....°

(d) Swimming

Answer .....°

2 The pie chart shows what 90 people eat for lunch.



(a) How many people eat rice?

Answer .....

(b) How many people eat bread?

Answer .....

(c) What fraction of the people eat noodles?

Answer .....

(d) What percentage of the people eat 'Other'?

Answer ..... %

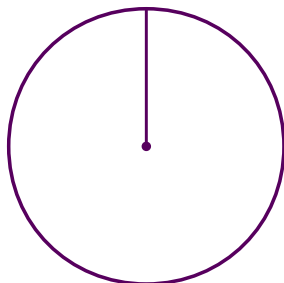
3 In a pie chart, 54° represents 12 people. How many people does the whole pie chart represent?

Answer .....

Reflect:    ☐ Red — not yet    ☐ Amber — nearly    ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 60 people were asked how they travel to work.



| Travel    | Car | Bus | Walk | Cycle |
|-----------|-----|-----|------|-------|
| Frequency | 16  | 22  | 12   | 10    |

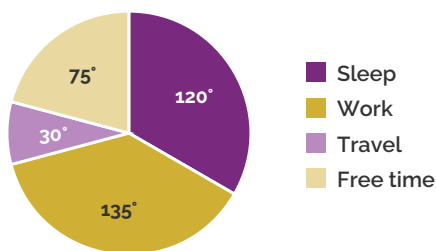
- (a) Work out the angle for each type of travel. [2]

- (b) Draw an accurate pie chart. Label each sector. [2]

- (c) What fraction of the people walk? Give your answer in its simplest form. [1]

Answer .....

- 2 The pie chart shows how Mei spent a 24-hour day.



- (a) How many hours did Mei spend sleeping? [2]

Answer ..... hours

- (b) How many more hours did Mei spend working than travelling? [2]

Answer ..... hours

- (c) What percentage of the day did Mei spend on free time? [2]

Answer ..... %

- 3 In a pie chart, the sector for 'red' has an angle of  $72^\circ$  and represents 18 cars.

- (a) Work out the total number of cars. [2]

Answer .....

- (b) The sector for 'silver' has an angle of  $108^\circ$ . How many cars are silver? [2]

Answer .....

- 4 A pie chart shows the favourite colours of 150 people. The angles of the four sectors are  $108^\circ$ ,  $x^\circ$ ,  $2x^\circ$  and  $36^\circ$ .

- (a) Find the value of  $x$ . [3]

Answer  $x =$  .....

- (b) How many people are represented by the sector of angle  $2x^\circ$ ? [2]

Answer .....

- 5 Two schools each drew a pie chart to show how their students travel. School P has 480 students and its 'bus' sector is  $90^\circ$ . School Q has 300 students and its 'bus' sector is  $120^\circ$ .

(a) Work out the number of students who travel by bus at each school.

[2]

Answer P ..... Q .....

(b) Leila says, 'The bus sector is bigger for School Q, so more students at Q travel by bus.' Explain why she is wrong.

[1]

- 6 45 people answered a question with 'yes' or 'no'. In a pie chart of the results, the angle for 'yes' is  $168^\circ$ .

How many people answered 'yes'?

[2]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Mean, median, mode and range

**I can...** calculate and use the mean, median, mode and range for lists and frequency tables.

Spec ref: 6.2 · Foundation

The **mean** = total  $\div$  number of values. The **median** is the middle value of the ordered data. The **mode** is the most common value. The **range** = largest – smallest measures **spread**.

For a frequency table, mean =  $\Sigma fx \div \Sigma f$ . Find the median by counting through the frequencies to the middle position.

If you know the mean, then total = mean  $\times$  number of values.

## Key words

mean

median

mode

range

average

spread

## Key points

- Mean from a table =  $\Sigma fx \div \Sigma f$
- Median position =  $(n + 1) \div 2$
- Total = mean  $\times$  number of values

## Worked example

**Goals 0, 1, 2, 3 have frequencies 5, 9, 4, 2. Find the mean and the median.**

$\Sigma f = 20$ ,  $\Sigma fx = 0 + 9 + 8 + 6 = 23$ ; mean =  $23 \div 20 = 1.15$

Median = 10.5th value: the 6th to 14th values are 1, so median = **1**

## Common mistakes

- Dividing  $\Sigma fx$  by the number of different values instead of  $\Sigma f$ .
- Finding the median of unordered data.

**Exam tip:** For 'missing number' mean questions, always work out the total first.

Answer each question. Show your working where it helps.

1 Find the mean of 14, 9, 12, 17, 9, 15, 8.

2 Find the median of 14, 9, 12, 17, 9, 15, 8.

3 Find the mode of 14, 9, 12, 17, 9, 15, 8.

4 Find the range of 14, 9, 12, 17, 9, 15, 8.

5 Find the median of 3.2, 4.5, 2.8, 5.1, 4.5, 3.9.

6 Find the mean of 3.2, 4.5, 2.8, 5.1, 4.5, 3.9.

7 Six numbers have a mean of 15. Five of them are 12, 18, 9, 14 and 20. Find the sixth.

8 The mean of 5 numbers is 8. A sixth number, 14, is added. Find the new mean.

9 The range of a set of numbers is 12. The smallest number is 7. Find the largest.

10 Write down three numbers with a mode of 4 and a mean of 5.

11 A frequency table has  $\Sigma f = 40$  and  $\Sigma fx = 132$ . Find the mean.

12 In a frequency table of 25 values, which value gives the median?

13 Which average should a shoe shop use to decide which size to stock most of?

14 Why might the median be better than the mean for house prices in a town?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 The table shows the number of goals scored by a hockey team in 30 matches.

| Goals     | 0 | 1 | 2  | 3 | 4 | 5 |
|-----------|---|---|----|---|---|---|
| Frequency | 3 | 8 | 10 | 6 | 2 | 1 |

(a) Write down the mode.

[1]

Answer .....

(b) Find the median.

[1]

Answer .....

(c) Work out the mean number of goals. Give your answer to 1 decimal place.

[3]

Answer .....

(d) Write down the range.

[1]

Answer .....

- 2 Seven numbers have a mean of 9. Six of the numbers are 8, 11, 6, 9, 7 and 5.

Work out the seventh number.

[3]

Answer .....

- 3 Amy and Ben each took five tests. Their scores are shown.

Amy: 62, 70, 58, 75, 65

Ben: 80, 45, 72, 50, 78

(a) Work out the mean and the range of each student's scores.

[4]

(b) Who should be chosen for a team that needs a reliable student? Give a reason.

[2]

- 4 In a class, the mean mark of 15 girls is 64 and the mean mark of 10 boys is 59.

Work out the mean mark of the whole class.

[3]

Answer .....

- 5 Four positive whole numbers have a mode of 5, a median of 6, a mean of 7 and a range of 6.

Find the four numbers.

[3]

Answer .....

- 6 The mean of the four numbers  $x$ ,  $x + 3$ ,  $2x$  and  $3x - 1$  is 11.

(a) Find the value of  $x$ .

[3]

Answer  $x =$  .....

(b) Find the median of the four numbers.

[1]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

## Grouped data: estimating the mean

**I can...** estimate the mean of grouped data and identify the modal class and the class containing the median.

Spec ref: 6.2 · Foundation & Higher

When data are grouped, the exact values are lost. Use the **mid-point** of each class as the value for every item in it.

Estimated mean =  $\frac{\sum fx}{\sum f}$ , where  $x$  is the mid-point. The **modal class** has the highest frequency.

To find the class with the median, add the frequencies until you pass the middle position.

### Key words

grouped frequency

mid-point

estimate

modal class

class interval

### Key points

- Mid-point =  $(\text{lower} + \text{upper class limit}) \div 2$
- Estimated mean =  $\frac{\sum fx}{\sum f}$
- The answer must lie inside the range of the data

### Worked example

**Times:**  $0 < t \leq 20$ : 8;  $20 < t \leq 40$ : 14;  $40 < t \leq 60$ : 18. Estimate the mean.

Mid-points 10, 30, 50;  $\sum f = 40$

$\sum fx = 80 + 420 + 900 = 1400$

Estimated mean =  $1400 \div 40 = \mathbf{35 \text{ minutes}}$

### Common mistakes

- Using the upper class limit instead of the mid-point.
- Dividing by the number of classes.

**Exam tip:** Check that your estimated mean lies in a sensible class — a mean outside the data range is always wrong.

Answer each question. Show your working where it helps.

- 1 The table shows the waiting times,  $w$  minutes, of 40 customers.

| Time ( $w$ min) | $0 < w \leq 10$ | $10 < w \leq 20$ | $20 < w \leq 30$ | $30 < w \leq 40$ | $40 < w \leq 50$ |
|-----------------|-----------------|------------------|------------------|------------------|------------------|
| Frequency       | 6               | 11               | 14               | 7                | 2                |

- (a) Write down the modal class.

Answer .....

- (b) Write down the mid-point of the class  $10 < w \leq 20$ .

Answer .....

- (c) Work out  $\Sigma fx$  using the mid-points.

Answer .....

- (d) Work out an estimate of the mean waiting time.

Answer ..... minutes

- (e) Which class contains the median?

Answer .....

- (f) How many customers waited more than 30 minutes?

Answer .....

- 2 The table shows the distances,  $d$  km, that 40 people travel to work.

| Distance ( $d$ km) | $0 < d \leq 5$ | $5 < d \leq 10$ | $10 < d \leq 15$ | $15 < d \leq 20$ |
|--------------------|----------------|-----------------|------------------|------------------|
| Frequency          | 9              | 15              | 12               | 4                |

- (a) Write down the modal class.

Answer .....

- (b) Work out an estimate of the mean distance.

Answer ..... km

- (c) Which class contains the median?

Answer .....

- 3 Why is a mean calculated from grouped data only an estimate?

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

**Answer all questions. Show your working clearly.**

- 1 The table shows the marks,  $m$ , of 40 students in a test.

| Mark ( $m$ ) | $50 < m \leq 60$ | $60 < m \leq 70$ | $70 < m \leq 80$ | $80 < m \leq 90$ | $90 < m \leq 100$ |
|--------------|------------------|------------------|------------------|------------------|-------------------|
| Frequency    | 4                | 9                | 15               | 8                | 4                 |

- (a) Write down the modal class.

**[1]****Answer .....**

- (b) Write down the class that contains the median.

**[1]****Answer .....**

- (c) Work out an estimate of the mean mark.

**[4]****Answer .....**

- 2 The table shows the times,  $t$  minutes, that 50 people spent in a museum shop. One frequency is missing.

| Time ( $t$ min) | $0 < t \leq 6$ | $6 < t \leq 10$ | $10 < t \leq 16$ | $16 < t \leq 20$ |
|-----------------|----------------|-----------------|------------------|------------------|
| Frequency       | 5              |                 | 12               | 3                |

- (a) Find the missing frequency.

**[1]****Answer .....**

- (b) Work out an estimate of the mean time.

**[4]****Answer ..... min**

- 3 Two groups of 20 students did a puzzle. The tables show their times.

| Time (min) | $10 < t \leq 15$ | $15 < t \leq 20$ | $20 < t \leq 25$ | $25 < t \leq 30$ |
|------------|------------------|------------------|------------------|------------------|
| Group A    | 3                | 8                | 6                | 3                |
| Group B    | 1                | 5                | 9                | 5                |

- (a) Work out an estimate of the mean time for each group. [4]

Answer A ..... B .....

- (b) Which group was faster on average? Give a reason. [1]

- 4 The table shows the ages of 50 people at a party.

| Age ( $a$ years) | $0 < a \leq 20$ | $20 < a \leq 40$ | $40 < a \leq 60$ | $60 < a \leq 80$ |
|------------------|-----------------|------------------|------------------|------------------|
| Frequency        | 7               | 18               | 21               | 4                |

- (a) Write down the modal class. [1]

Answer .....

- (b) Work out an estimate of the mean age. [3]

Answer ..... years

- (c) Explain why your answer to (b) is an estimate. [1]

- 5 The table shows the number of text messages sent by 30 students in one day.

| Messages  | 0–9 | 10–19 | 20–29 | 30–39 |
|-----------|-----|-------|-------|-------|
| Frequency | 6   | 11    | 9     | 4     |

- (a) Work out an estimate of the mean number of messages. [3]

Answer .....

- (b) Jo says the range of the data is 39. Explain why she might be wrong. [1]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

## Cumulative frequency, median and IQR

**I can...** draw cumulative frequency graphs and estimate the median and interquartile range.

Spec ref: 6.1, 6.2 · Higher

**Cumulative frequency** is a running total. Plot each total at the **upper bound** of its class and join the points with a smooth curve (or straight lines).

For  $n$  values, read across from  $n/2$  for the median,  $n/4$  for the lower quartile and  $3n/4$  for the upper quartile. **IQR** = UQ – LQ.

For a list of  $n$  ordered values, LQ is the  $((n + 1)/4)$ th value and UQ is the  $(3(n + 1)/4)$ th value.

## Key words

cumulative frequency

upper bound

median

quartile

interquartile range

## Key points

- Plot at upper class bounds, starting from 0 at the lowest bound
- IQR = upper quartile – lower quartile
- A smaller IQR means the middle half of the data is more consistent

## Worked example

**Find the IQR of 4, 7, 9, 10, 13, 15, 18, 20, 24, 25, 29.**

$n = 11$ : LQ is the 3rd value = 9; UQ is the 9th value = 24

IQR =  $24 - 9 = 15$

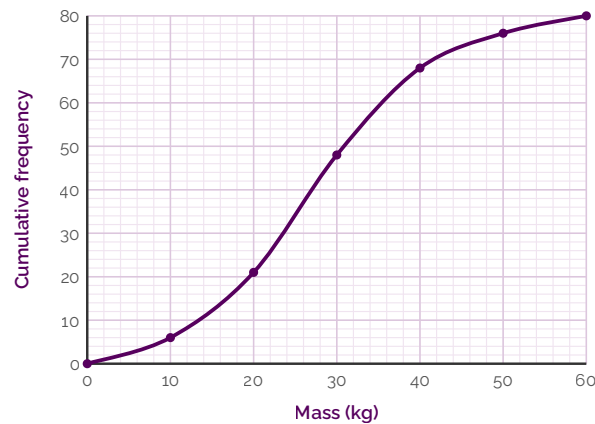
## Common mistakes

- Plotting cumulative frequencies at the mid-points of the classes.
- Giving the cumulative frequency, not the value on the horizontal axis.

**Exam tip:** Draw the reading lines on the graph in pencil and write down both quartiles before you subtract.

Answer each question. Draw lines on the graph to show your readings.

- 1 The cumulative frequency graph shows the masses of 80 suitcases.



- (a) Estimate the median mass.

Answer ..... kg

- (b) Estimate the lower quartile.

Answer ..... kg

- (c) Estimate the upper quartile.

Answer ..... kg

- (d) Estimate the interquartile range.

Answer ..... kg

- (e) Estimate how many suitcases have a mass of 25 kg or less.

Answer .....

- (f) Estimate how many suitcases have a mass of more than 45 kg.

Answer .....

- 2 Here are 11 numbers in order: 3, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21

- (a) Find the median.

Answer .....

- (b) Find the lower quartile.

Answer .....

- (c) Find the upper quartile.

Answer .....

- (d) Find the interquartile range.

Answer .....

- 3 Find the interquartile range of: 22, 25, 28, 30, 31, 35, 36, 40, 41, 44, 45, 48, 50, 52, 55

Answer .....

- 4 For a cumulative frequency graph of 200 values, at what cumulative frequency do you read the upper quartile?

**Answer** .....

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**Reflect:**   ☐ Red — not yet   ☐ Amber — nearly   ☐ Green — secure

Answer all questions. Show your working clearly.

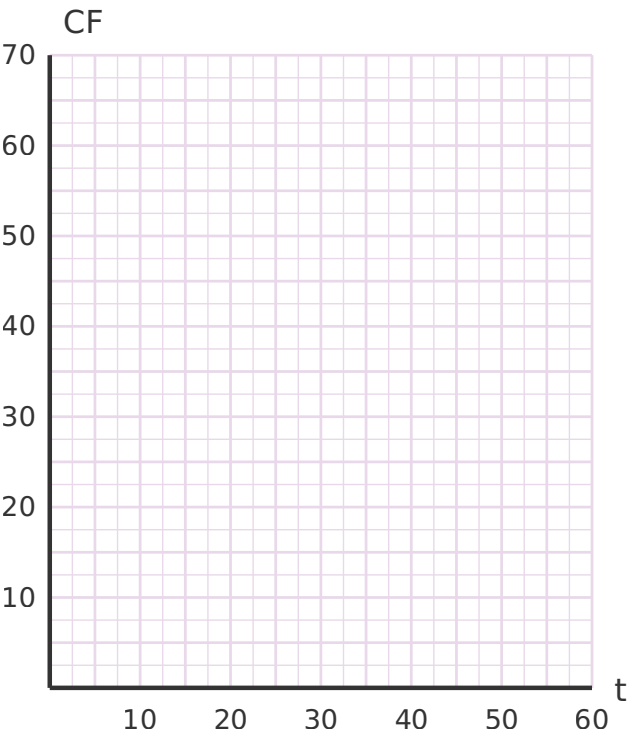
1 The table shows the times,  $t$  minutes, that 70 people waited at a clinic.

| Time ( $t$ min) | $0 < t \leq 10$ | $10 < t \leq 20$ | $20 < t \leq 30$ | $30 < t \leq 40$ | $40 < t \leq 50$ | $50 < t \leq 60$ |
|-----------------|-----------------|------------------|------------------|------------------|------------------|------------------|
| Frequency       | 4               | 9                | 18               | 24               | 11               | 4                |

(a) Complete the cumulative frequency table. [1]

| Time ( $t$ min)      | $t \leq 10$ | $t \leq 20$ | $t \leq 30$ | $t \leq 40$ | $t \leq 50$ | $t \leq 60$ |
|----------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Cumulative frequency |             |             |             |             |             |             |

(b) On the grid, draw a cumulative frequency graph for your table. [2]



(c) Use your graph to find an estimate for the median time. [1]

Answer ..... min

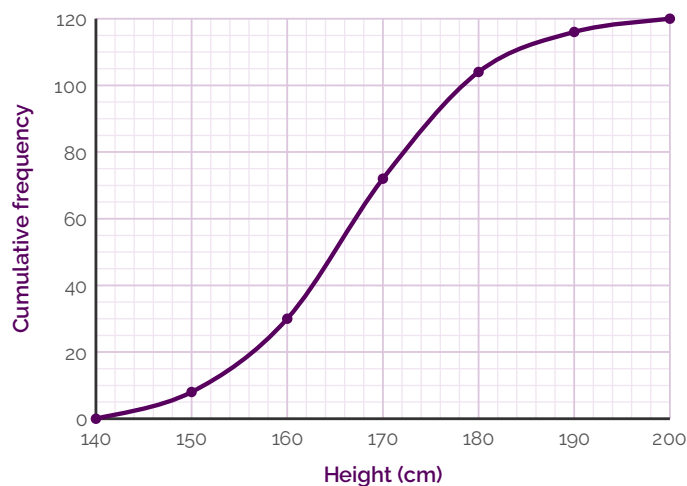
(d) Use your graph to find an estimate for the interquartile range. [2]

Answer ..... min

(e) Use your graph to estimate the number of people who waited more than 45 minutes. [2]

Answer .....

- 2 The cumulative frequency graph shows the heights of 120 boys.



- (a) Find an estimate for the median height. [1]

Answer ..... cm

- (b) Find an estimate for the interquartile range. [2]

Answer ..... cm

- (c) Estimate the number of boys shorter than 155 cm. [2]

Answer .....

- (d) For 120 girls, the median height is 162 cm and the interquartile range is 11 cm. Compare the heights of the boys and the girls. [2]

- (e) Estimate the percentage of the boys who are taller than 180 cm. [2]

Answer ..... %

**3** Here are the numbers of emails received by 15 people in one day, in order:

2, 4, 4, 5, 6, 7, 7, 8, 9, 11, 12, 12, 13, 15, 18

**(a)** Find the lower quartile and the upper quartile.

**[2]**

**Answer** LQ ..... UQ .....

**(b)** Work out the interquartile range.

**[1]**

**Answer** .....

**(c)** Find the median.

**[1]**

**Answer** .....

**(d)** Explain why the interquartile range can be a better measure of spread than the range. **[1]**

**4** The table shows the numbers of pets owned by 23 students.

| Number of pets | 0 | 1 | 2 | 3 | 4 |
|----------------|---|---|---|---|---|
| Frequency      | 4 | 7 | 6 | 4 | 2 |

**(a)** Find the median.

**[1]**

**Answer** .....

**(b)** Find the interquartile range.

**[2]**

**Answer** .....

Reflect:    ☐ Red — not yet    ☐ Amber — nearly    ☐ Green — secure

# Histograms

**I can...** construct and interpret histograms for continuous data with unequal class intervals.

Spec ref: 6.1 · Higher

A **histogram** shows grouped continuous data. The bars touch, and the class intervals can be **unequal**.

The height of each bar is the **frequency density**, so the **area** of each bar represents its frequency.

To estimate the number in part of a class, use the same fraction of the bar.

## Key words

histogram

frequency density

class width

continuous

area

## Key points

- Frequency density = frequency  $\div$  class width
- Frequency = frequency density  $\times$  class width
- Bars must touch; no gaps for continuous data

## Worked example

The class  $40 < x \leq 60$  has frequency 28, and the bar for  $60 < x \leq 100$  has height 0.3. Find the frequency density of the first class and the frequency of the second.

$$\text{Frequency density} = 28 \div 20 = \mathbf{1.4}$$

$$\text{Frequency} = 0.3 \times 40 = \mathbf{12}$$

## Common mistakes

- Plotting frequency instead of frequency density.
- Reading the class width wrongly from the inequality.

**Exam tip:** Add a 'class width' column and a 'frequency density' column to the table before you draw anything.

**Answer each question. Show your working where it helps.****1** Work out the frequency density for each class.**(a)**  $0 < x \leq 5$ , frequency 20

Answer .....

**(b)**  $5 < x \leq 15$ , frequency 18

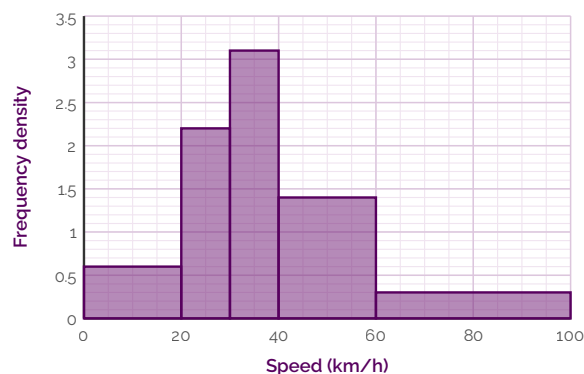
Answer .....

**(c)**  $15 < x \leq 40$ , frequency 30

Answer .....

**(d)**  $40 < x \leq 100$ , frequency 9

Answer .....

**2** The histogram shows the speeds of some cars.**(a)** How many cars had a speed of 20 to 30 km/h?

Answer .....

**(b)** How many cars had a speed of 40 to 60 km/h?

Answer .....

**(c)** How many cars were there altogether?

Answer .....

**(d)** Estimate the number of cars with a speed of more than 50 km/h.

Answer .....

**(e)** Estimate the number of cars with a speed between 30 and 35 km/h.

Answer .....

**3** A class  $2 < t \leq 2.5$  has frequency density 24. Find the frequency.

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

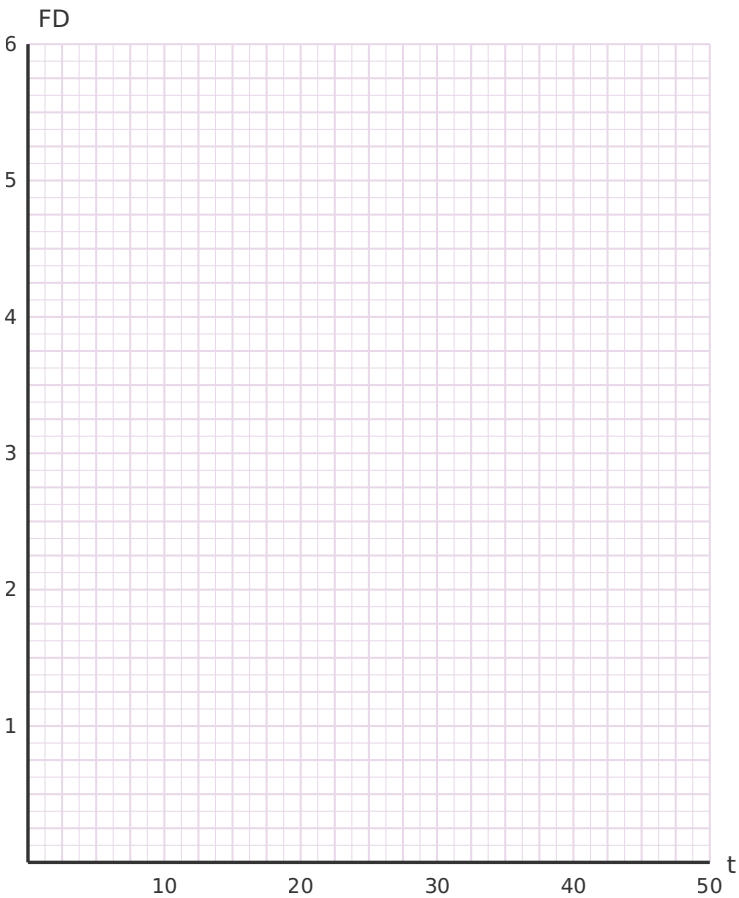
Answer all questions. Show your working clearly.

1 The table shows the lengths of time, in minutes, that 97 people spent at a gym.

| Time ( $t$ min)   | $0 < t \leq 10$ | $10 < t \leq 15$ | $15 < t \leq 20$ | $20 < t \leq 30$ | $30 < t \leq 50$ |
|-------------------|-----------------|------------------|------------------|------------------|------------------|
| Frequency         | 15              | 24               | 28               | 22               | 8                |
| Frequency density |                 |                  |                  |                  |                  |

(a) Complete the frequency density row. [2]

(b) On the grid, draw a histogram for this information. [3]



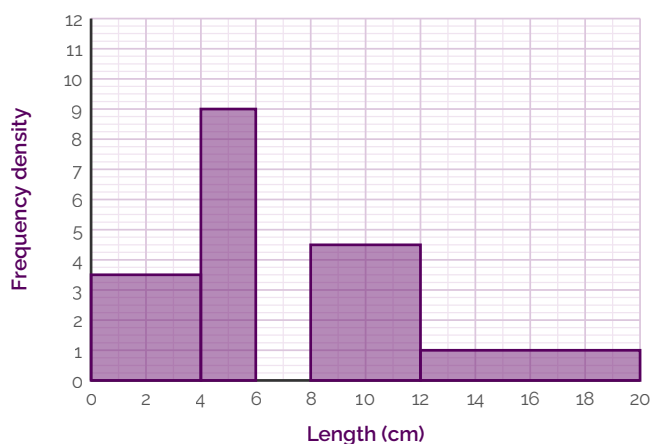
(c) Estimate the number of people who spent more than 25 minutes at the gym. [2]

Answer .....

(d) Write down the modal class. [1]

Answer .....

- 2 The histogram and the table give information about the lengths of 80 leaves. Both are incomplete.



| Length ( $l$ cm) | $0 < l \leq 4$ | $4 < l \leq 6$ | $6 < l \leq 8$ | $8 < l \leq 12$ | $12 < l \leq 20$ |
|------------------|----------------|----------------|----------------|-----------------|------------------|
| Frequency        |                |                | 22             |                 |                  |

- (a) Use the histogram to complete the table. [2]  
 (b) Use the table to complete the histogram. [1]  
 (c) Work out an estimate of the mean length. [4]

Answer ..... cm

- (d) Estimate the median length. [2]

Answer ..... cm

- 3 A histogram has no scale on the frequency density axis. The bar for the class  $10 < w \leq 20$  is 4 cm high and represents 30 people. The bar for  $20 < w \leq 40$  is 3 cm high.

Work out how many people the bar for  $20 < w \leq 40$  represents. [3]

Answer .....

- 4 The table gives the frequency densities for the heights,  $h$  cm, of some seedlings.

| Height ( $h$ cm)  | $0 < h \leq 10$ | $10 < h \leq 15$ | $15 < h \leq 25$ | $25 < h \leq 40$ |
|-------------------|-----------------|------------------|------------------|------------------|
| Frequency density | 1.2             | 3.4              | 2.6              | 0.8              |

- (a) Work out the frequency of each class. [2]

- (b) Work out the total number of seedlings. [1]

Answer .....

- (c) Estimate the number of seedlings taller than 20 cm. [2]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

# End-of-book check

## End-of-book check

Name \_\_\_\_\_ Date \_\_\_\_\_ Marks \_\_\_\_\_ / 21

Answer all questions. Show your working. Compare your score with your starting point check.

- 1 A bag contains 3 green, 5 red and 12 blue beads. One bead is taken at random. Find  $P(\text{green or red})$ . [1]

Answer .....

- 2 A biased dice has  $P(1) = 0.12$ ,  $P(2) = 0.15$ ,  $P(3) = 0.18$ ,  $P(4) = 0.2$  and  $P(5) = 0.15$ . Find  $P(6)$ . [2]

Answer .....

- 3 The probability that it rains on any day in the rainy season is 0.35. On how many days out of 60 would you expect it to rain? [1]

Answer .....

- 4 Two fair dice are rolled and the scores are added. Find the probability that the total is 10 or more. [2]

Answer .....

- 5 A Venn diagram shows  $P(A \text{ only}) = 0.25$ ,  $P(A \cap B) = 0.2$  and  $P(B \text{ only}) = 0.3$ . Find  $P(A' \cap B')$ . [1]

Answer .....

- 6 **(Higher)** A and B are independent.  $P(A) = 0.7$  and  $P(B) = 0.4$ . Find the probability that neither A nor B happens. [2]

Answer .....

- 7 **(Higher)** A bag has 4 red and 6 blue balls. Two are taken without replacement. Find  $P(\text{both blue})$ . [2]

Answer .....

- 8 Work out the mean. [2]

| Value     | 2 | 3 | 4 | 5 |
|-----------|---|---|---|---|
| Frequency | 4 | 6 | 7 | 3 |

Answer .....

- 9 Work out an estimate of the mean. [3]

| $x$       | $0 < x \leq 4$ | $4 < x \leq 8$ | $8 < x \leq 12$ |
|-----------|----------------|----------------|-----------------|
| Frequency | 5              | 9              | 6               |

Answer .....

- 10 In a pie chart, a sector of  $80^\circ$  represents 16 people. How many people does the pie chart represent? [2]

Answer .....

- 11 **(Higher)** Find the interquartile range of 3, 5, 6, 9, 10, 12, 14, 17, 20, 21, 25. [2]

Answer .....

- 12 **(Higher)** In a histogram, the bar for  $10 < x \leq 25$  has frequency density 2.4. Find the frequency. [1]

Answer .....



## Practice paper

Name \_\_\_\_\_ Date \_\_\_\_\_ Marks \_\_\_\_\_ / 60

Time: 1 hour 15 minutes  
Calculator allowed  
Total: 60 marks  
Questions marked (Higher) are for Higher tier students.

Answer all questions. Show all your working. Write your answers in the spaces provided.

- 1 90 students chose their favourite subject, Maths or Science.

|         | Maths | Science | Total |
|---------|-------|---------|-------|
| Year 10 | 22    |         | 48    |
| Year 11 |       | 19      |       |
| Total   | 45    |         | 90    |

(a) Complete the two-way table. [3]

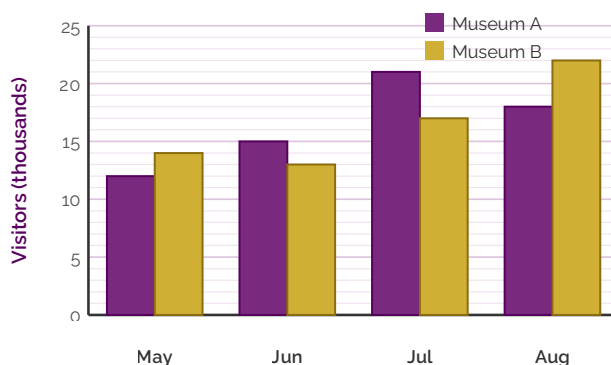
(b) A student is chosen at random. Find the probability that the student is in Year 11 and chose Maths. [1]

Answer .....

(c) (Higher) A student who chose Science is chosen at random. Find the probability that this student is in Year 10. [2]

Answer .....

- 2 The comparative bar chart shows the numbers of visitors, in thousands, to two museums.



- (a) How many visitors did Museum B have in July? [1]

Answer .....

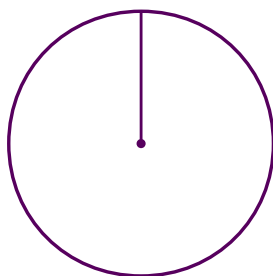
- (b) In which month did Museum A have the most visitors compared with Museum B? [1]

Answer .....

- (c) Work out the total number of visitors to Museum A in the four months. [2]

Answer .....

- 3 The table shows the colours of 36 cars in a car park.



| Colour    | White | Black | Silver | Red |
|-----------|-------|-------|--------|-----|
| Frequency | 11    | 9     | 10     | 6   |

- (a) Work out the angle for each colour in a pie chart. [2]

- (b) Draw an accurate pie chart. [2]

- (c) What fraction of the cars are red? Give your answer in its simplest form. [1]

Answer .....

- 4 The table shows the number of people living in each of 40 houses.

| Number of people | 1 | 2  | 3 | 4 | 5 | 6 |
|------------------|---|----|---|---|---|---|
| Frequency        | 7 | 12 | 9 | 8 | 3 | 1 |

(a) Write down the mode.

[1]

Answer .....

(b) Find the median.

[1]

Answer .....

(c) Work out the mean. Give your answer to 1 decimal place.

[3]

Answer .....

- 5 The table shows the heights of 60 sunflowers.

| Height ( $h$ cm) | $100 < h \leq 120$ | $120 < h \leq 140$ | $140 < h \leq 160$ | $160 < h \leq 180$ |
|------------------|--------------------|--------------------|--------------------|--------------------|
| Frequency        | 8                  | 17                 | 22                 | 13                 |

(a) Write down the modal class.

[1]

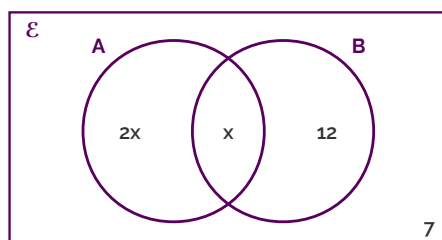
Answer .....

(b) Work out an estimate of the mean height.

[4]

Answer ..... cm

- 6 40 people were asked if they like apples (A) or bananas (B). The Venn diagram shows the numbers of people.



- (a) Find the value of  $x$ .

[2]

Answer  $x = \dots\dots$

- (b) A person is chosen at random. Find  $P(A)$ .

[1]

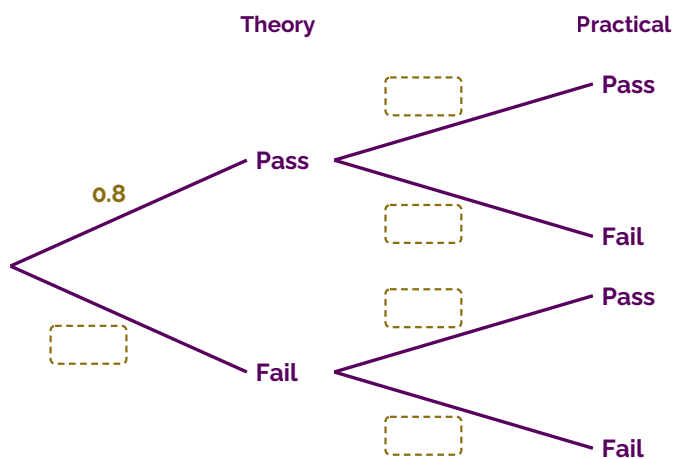
Answer  $\dots\dots$

- (c) (Higher) A person who likes apples is chosen at random. Find the probability that they also like bananas.

[2]

Answer  $\dots\dots$

- 7 (Higher) The probability that Jo passes her theory test is 0.8. The probability that she passes her practical test is 0.65. The two events are independent.



- (a) Complete the probability tree diagram.

[2]

- (b) Work out the probability that Jo passes both tests.

[2]

Answer  $\dots\dots$

- (c) Work out the probability that Jo passes exactly one of the tests.

[2]

- 8 **(Higher)** A box contains 4 dark, 5 milk and 3 white chocolates. Ali takes two chocolates at random without replacement.

Work out the probability that both chocolates are the same type.

[4]

Answer .....

9

**(Higher)** The table shows the lengths,  $t$  minutes, of 100 phone calls.

| Length ( $t$ min) | $0 < t \leq 5$ | $5 < t \leq 10$ | $10 < t \leq 15$ | $15 < t \leq 20$ | $20 < t \leq 25$ | $25 < t \leq 30$ |
|-------------------|----------------|-----------------|------------------|------------------|------------------|------------------|
| Frequency         | 12             | 28              | 31               | 18               | 8                | 3                |

**(a)** Complete the cumulative frequency table. [1]

| Length ( $t$ min)    | $t \leq 5$ | $t \leq 10$ | $t \leq 15$ | $t \leq 20$ | $t \leq 25$ | $t \leq 30$ |
|----------------------|------------|-------------|-------------|-------------|-------------|-------------|
| Cumulative frequency |            |             |             |             |             |             |

**(b)** On the grid, draw a cumulative frequency graph. [2]



**(c)** Use your graph to find an estimate for the median. [1]

**Answer** ..... min

**(d)** Use your graph to find an estimate for the interquartile range. [2]

**Answer** ..... min

**(e)** Estimate the number of calls that lasted more than 18 minutes. [2]

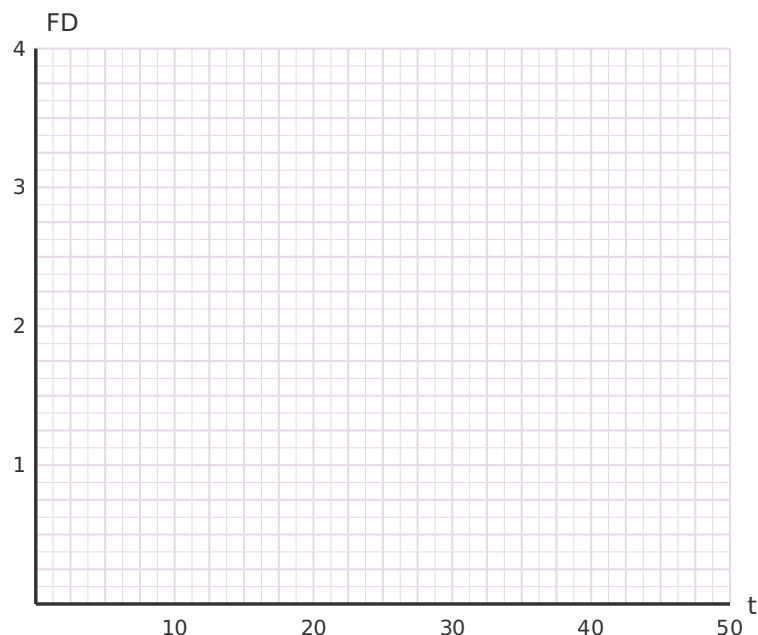
**Answer** .....

- 10 (Higher)** The table shows the times,  $t$  seconds, taken by 65 people to solve a puzzle.

| Time ( $t$ s) | $0 < t \leq 10$ | $10 < t \leq 15$ | $15 < t \leq 20$ | $20 < t \leq 30$ | $30 < t \leq 50$ |
|---------------|-----------------|------------------|------------------|------------------|------------------|
| Frequency     | 5               | 14               | 18               | 16               | 12               |

- (a)** On the grid, draw a histogram for this information.

[3]



- (b)** Estimate the number of people who took more than 25 seconds.

[2]

Answer .....

- (c)** Write down the class that contains the median.

[1]

Answer .....

- 11** Leo dropped a drawing pin 200 times. It landed point up 74 times.

- (a)** Work out the relative frequency of the pin landing point up.

[1]

Answer .....

- (b)** The pin is dropped 1500 times. Estimate the number of times it lands point up.

[2]

Answer .....

- (c)** Mia dropped the same pin 50 times and it landed point up 25 times. Whose results give the better estimate? Give a reason.

[1]

---

- 12** Spinner A is numbered 1, 2, 3, 4 and spinner B is numbered 1, 2, 3. Both are fair. They are spun and the numbers are multiplied.

Work out the probability that the product is an even number.

**[2]**

**Answer** .....



FOR TEACHERS & PARENTS

## Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

### Starting point check

- 1  $3/10$  B1 [1]
- 2 0.38 B1 [1]
- 3 0.19 B1 [1]
- 4 10 B1 [1]
- 5 10 B1 [1]
- 6 0.75 B1 [1]
- 7  $14 + 11 - 5$  M1; 20 A1 [2]
- 8 10 B1 [1]
- 9 mode 8 B1; range 9 B1 [2]
- 10  $120^\circ$  B1 [1]
- 11 25 B1 [1]
- 12 3.2 B1 [1]

### Unit 1 · Sheet A — Probability and the probability scale

- 1  $6/15 = 2/5$
- 2  $3/5$
- 3 0
- 4 certain
- 5 evens
- 6  $6/12 = 1/2$
- 7  $4/12 = 1/3$
- 8  $4/12 = 1/3$
- 9  $2/11$
- 10  $7/11$
- 11 0.65
- 12 0.35
- 13 14
- 14 the outcomes are not equally likely

### Unit 1 · Sheet B — Probability and the probability scale

- 1 (a)  $0.3 + 0.25 + x + 2x = 1$  M1;  $3x = 0.45$  so  $x = 0.15$  M1;  $P(4) = 0.3$  A1 [3] (b)  $0.3 + 0.15$  M1; 0.45 A1 ft [2] (c) the probabilities are not all equal (fair would be 0.25 each) B1 [1]
- 2 (a)  $8/20 (= 2/5)$  B1 [1] (b)  $13/20$  B1 [1] (c) cross at 0.35 B1 [1] (d)  $(5 + n)/(20 + n) = 1/2$  M1;  $10 + 2n = 20 + n$  M1;  $n = 10$  A1 [3]
- 3 (a) 0.35 B1 [1] (b)  $12 \div 0.2$  M1; 60 A1 [2] (c) 21 B1 ft [1]
- 4 (a)  $2/8 = 1/4$  B1 [1] (b)  $7/8$  B1 [1] (c) primes 2, 3, 5, 7 listed M1;  $4/8 = 1/2$  so yes A1 [2] (d)  $P(\text{even}) = 1/2$  but  $P(\text{less than } 9) = 1$  (certain) B1 [1]
- 5 (a)  $0.035 \times 400$  M1; 14 A1 [2] (b)  $42/400$  M1;  $21/200$  A1 [2]

### Unit 2 · Sheet A — Relative frequency and expected frequency

- 1 0.4
- 2 0.46
- 3 21
- 4 50
- 5 7.2
- 6  $3/4$
- 7 150
- 8 155
- 9 1000
- 10 No — too few throws
- 11 8
- 12 180
- 13 0.25
- 14 4

### Unit 2 · Sheet B — Relative frequency and expected frequency

- 1 (a)  $24/120 = 0.2$  B1 [1] (b)  $22 + 20 + 24 = 66$  M1;  $66/120 = 0.55$  A1 [2] (c) Yes — each frequency is close to the expected 20 B1 (accept 'no, 6 appears most often' only with reasoning about bias) [1] (d)  $0.2 \times 900$  M1; 180 A1 ft [2]
- 2 (a)  $45/250 = 0.18$  B1 [1] (b)  $20/250 \times 1000$  M1; 80 A1 [2] (c) e.g. traffic varies with time of day / day of week; sample from one time only B1 [1]
- 3 (a)  $0.15 \times 300$  M1; 45 A1 [2] (b)  $300 \times 2 = \$600$  M1;  $45 \times 10 = \$450$  M1; \$150 A1 [3]
- 4 (a) Amir 0.35 B1; Bella 0.4375 B1 [2] (b)  $84/200$  M1; 0.42 A1 [2] (c) Bella's — more throws B1 [1]
- 5 (a)  $0.32 \times 250$  M1; 80 A1 [2] (b) Not necessarily — results vary; the expected value is only an average B1 [1] (c)  $48 \div 0.32$  M1; 150 A1 [2]

### Unit 3 · Sheet A — Sample spaces and mutually exclusive events

- 1 12
- 2 24
- 3  $4/36 = 1/9$
- 4  $9/36 = 1/4$
- 5  $6/36 = 1/6$
- 6  $4/16 = 1/4$
- 7  $3/16$
- 8 0.75
- 9 0.35
- 10 No
- 11  $4/10 = 2/5$
- 12 8
- 13  $7/8$
- 14 0.25

### Unit 3 · Sheet B — Sample spaces and mutually exclusive events

- 1 (a) row 1: 3, 4, 5, 6; row 3: 5, 6, 7, 8; row 5: 7, 8, 9, 10 B2 (B1 for at least 8 correct) [2] (b)  $2/12 = 1/6$  B1 ft [1] (c) 6 outcomes M1;  $6/12 = 1/2$  A1 ft [2] (d) 0 B1 [1]
- 2 (a) VSp, VSa, StSp, StSa, CSp, CSa B2 (B1 for at least 4 correct, no repeats) [2] (b)  $1/6$  B1 ft [1]
- 3 (a)  $1 - (0.15 + 0.3 + 0.2)$  M1; 0.35 A1 [2] (b) 0.35 B1 [1] (c)  $0.3 \times 80$  M1; 24 A1 [2]
- 4 (a) 16 totals correct B2 (B1 for at least 12) [2] (b) 5 B1;  $4/16 = 1/4$  B1 [2] (c)  $1 + 2 + 4 + 2 = 9$  outcomes M1;  $9/16$  A1 [2]
- 5 (a) No — 10 and 20 are in both B1 [1] (b) 10 even + 5 and 15 = 12 numbers M1;  $12/20 = 3/5$  A1 [2] (c)  $P(A) + P(B) = 14/20$  B1; 10 and 20 are counted twice B1 [2]

### Unit 4 · Sheet A — Probability from Venn diagrams

- 1 (a)  $3/25$  (b)  $2/5$  (c)  $3/5$  (d)  $17/25$  (e)  $7/25$  (f)  $2/5$
- 2 (a) 0.3 (b) 0.45 (c) 0.75
- 3 {4, 12}
- 4  $7/12$
- 5 7
- 6  $9/25$

### Unit 4 · Sheet B — Probability from Venn diagrams

- 1 (a)  $34 + 28 + 9 = 60$  M1; 11 A1 [2] (b) 23, 11, 17 and 9 B2 (B1 for 2 or 3 correct) ft [2] (c)  $23/60$  B1 ft [1] (d)  $9/60 = 3/20$  B1 [1]
- 2 (a)  $A \cap B = \{1, 4, 16\}$  B1; A only {9}, B only {2, 8} B1; the other 10 numbers outside B1 [3] (b)  $3/16$  B1 ft [1] (c)  $2/16 = 1/8$  B1 ft [1]
- 3 (a)  $0.4 + 0.1 + y + 0.25 = 1$  M1;  $y = 0.25$  A1 [2] (b) 0.35 B1 ft [1] (c) 0.75 B1 [1]
- 4 (a)  $28 + x + x + 4 = 50$  M1;  $x = 9$  A1 [2] (b)  $8 + 6 + 5 = 19$  M1;  $19/50$  A1 [2] (c)  $21/50$  B1 ft [1]
- 5 (a)  $17 + 13 - 6 = 24$  M1; 6 A1 [2] (b)  $11/30$  B1 [1] (c)  $24/30$  M1;  $4/5$  A1 [2]

### Unit 5 · Sheet A — Tree diagrams and independent events

- 1 0.3
- 2  $1/36$
- 3  $25/36$
- 4  $11/36$
- 5 0.21
- 6 0.42
- 7 0.008
- 8 0.512
- 9 0.488
- 10  $4/25$
- 11  $12/25$
- 12 0.675
- 13 0.325
- 14 0.3

### Unit 5 · Sheet B — Tree diagrams and independent events

- 1 (a) 0.3 on first 'miss' B1; 0.7, 0.3, 0.7, 0.3 on second shot B1 [2] (b)  $0.7 \times 0.7$  M1; 0.49 A1 [2] (c)  $0.7 \times 0.3$  M1;  $2 \times 0.21$  or  $0.21 + 0.21$  M1; 0.42 A1 [3]
- 2 (a) structure B1;  $3/8, 5/8$  on first branches B1;  $4/6, 2/6$  on each second pair B1 [3] (b)  $3/8 \times 4/6$  M1;  $12/48 = 1/4$  A1 [2] (c)  $1 - 1/4$  M1;  $3/4$  A1 ft [2]
- 3 (a)  $0.6^3$  M1; 0.216 A1 [2] (b)  $0.6 \times 0.6 \times 0.4$  M1;  $\times 3$  M1; 0.432 A1 [3]
- 4 (a)  $p^2 = 0.36$  M1;  $p = 0.6$  A1 [2] (b)  $0.4^2 = 0.16$  B1 ft [1] (c) 0.84 B1 ft [1]
- 5  $5/6 \times 5/6 \times 1/6$  M1;  $25/216$  A1 [2]

### Unit 6 · Sheet A — Conditional probability

- 1 (a)  $6/9 = 2/3$  (b)  $42/90 = 7/15$  (c)  $6/90 = 1/15$  (d)  $42/90 = 7/15$
- 2 (a)  $18/50 = 9/25$  (b)  $4/10 = 2/5$  (c)  $4/18 = 2/9$  (d)  $26/32 = 13/16$
- 3 0.3
- 4  $7/15$

### Unit 6 · Sheet B — Conditional probability

- 1 (a) first:  $6/11, 5/11$  B1; second:  $5/10, 5/10$  and  $6/10, 4/10$  B1 [2] (b)  $6/11 \times 5/10$  or  $5/11 \times 4/10$  M1; sum of both M1;  $50/110 = 5/11$  A1 [3] (c)  $1 - 30/110$  M1;  $80/110 = 8/11$  A1 [2]
- 2 (a) 0.1 B1; 0.05, 0.4, 0.6 B1 [2] (b)  $0.9 \times 0.95$  or  $0.1 \times 0.4$  M1;  $0.855 + 0.04$  M1; 0.895 A1 [3] (c) 0.105 B1 ft [1]
- 3  $5/12 \times 7/11 \times 6/10$  M1;  $= 210/1320$  A1;  $\times 3$  M1;  $21/44$  A1 [4]
- 4 (a)  $33/120 = 11/40$  B1 [1] (b)  $20/44$  M1;  $5/11$  A1 [2] (c)  $24/64$  M1;  $3/8$  A1 [2]
- 5 (a)  $3/n \times 2/(n-1)$  M1;  $= 1/15$  M1;  $90 = n(n-1)$  leading to  $n^2 - n - 90 = 0$  A1 [3] (b)  $(n-10)(n+9) = 0$  so  $n = 10$  B1 [1]

### Unit 7 · Sheet A — Collecting data and two-way tables

- 1 (a) 9 (b) mango (c)  $7/24$
- 2 (a) 22 (b) 40 (c) 24
- 3 (a) 6 (b)  $150 < h \leq 160$
- 4 continuous

### Unit 7 · Sheet B — Collecting data and two-way tables

- 1 (a) 4: 2, 5: 5, 6: 8, 7: 3, 8: 1, 9: 1 B3 (B2 for 4 or 5 correct, B1 for 2 or 3 correct) [3] (b) 6 B1 ft [1] (c) 13 B1 ft [1]
- 2 (a) Y10 Mandarin 33; Y11 total 105; Y11 French 27; Spanish total 59; Mandarin total 80 B4 (B3 for 4 correct, B2 for 3, B1 for 2) [4] (b) 47/105 B1 ft [1] (c)  $59/200 \times 100$  M1; 29.5% A1 ft [2]
- 3 (a) 1, 9, 11, 6, 3 B3 (B2 for 4 correct, B1 for 3 correct) [3] (b)  $14 < t \leq 16$  B1 ft [1] (c)  $16 < t$  means more than 16, so 16.0 belongs in  $14 < t \leq 16$  B1 [1]
- 4 (a) 64 km B1 [1] (b)  $48 + 86$  M1; 134 km A1 [2] (c) Bexford and Dunmore (39 km) B1 [1]
- 5 (a) table with Girls/Boys and Walk/Do not walk B1; 11, 7, 5, 9 B1; totals 18, 14, 16, 16, 32 B1 [3] (b) 9 B1 ft [1]

### Unit 8 · Sheet A — Pictograms and bar charts

- 1 (a) 24 (b) 9 (c) 6 (d) 87
- 2 (a) 9 (b) 28 (c) Year 10 (d) 18
- 3 so the height alone shows the frequency
- 4 28
- 5  $2\frac{1}{2}$
- 6 comparative: bars side by side; composite: parts stacked in one bar

### Unit 8 · Sheet B — Pictograms and bar charts

- 1 (a) all bars correct heights B2 (B1 for 3 or 4 correct); equal widths and labelled B1 [3] (b) Dog B1; 38 B1 [2]
- 2 (a) 18 B1 [1] (b)  $2\frac{1}{2}$  symbols for Tue B1;  $1\frac{1}{2}$  symbols for Thu B1 [2] (c)  $12 + 10 + 18 + 6 + 8$  M1; 54 A1 [2]
- 3 (a) 25 B1 [1] (b) March B1 [1] (c)  $24 + 18 + 30 + 26$  M1; 98 A1 [2] (d) No — both sold 98 B1 [1]
- 4 (a)  $11 + 15$  M1; 26 A1 [2] (b)  $15/26$  B2 (B1 for 15 seen) [2] (c)  $46/84 \times 100$  M1; 54.8% A1 [2]
- 5 (a) pairs of bars for each club B1; heights correct B1; key/labels B1 [3] (b) e.g. more girls than boys chose swimming; football was most popular with boys B1 [1]

### Unit 9 · Sheet A — Pie charts

- 1 (a)  $126^\circ$  (b)  $81^\circ$  (c)  $54^\circ$  (d)  $99^\circ$
- 2 (a) 35 (b) 20 (c)  $100/360 = 5/18$  (d) 11.1%
- 3 80

### Unit 9 · Sheet B — Pie charts

- 1 (a)  $360 \div 60 = 6$  M1;  $96^\circ$ ,  $132^\circ$ ,  $72^\circ$ ,  $60^\circ$  A1 [2] (b) all sectors  $\pm 2^\circ$  B1; labels B1 [2] (c)  $12/60 = 1/5$  B1 [1]
- 2 (a)  $120/360 \times 24$  M1; 8 hours A1 [2] (b)  $9 - 2$  M1; 7 hours A1 [2] (c)  $75/360 \times 100$  M1; 20.8% A1 [2]
- 3 (a)  $360 \div 72 \times 18$  M1; 90 A1 [2] (b)  $108/360 \times 90$  M1; 27 A1 ft [2]
- 4 (a)  $108 + x + 2x + 36 = 360$  M1;  $3x = 216$  M1;  $x = 72$  A1 [3] (b)  $144/360 \times 150$  M1; 60 A1 ft [2]
- 5 (a) P: 120 B1; Q: 100 B1 [2] (b) the pie charts show proportions of different totals; P has more bus users ( $120 > 100$ ) B1 [1]
- 6  $168/360 \times 45$  M1; 21 A1 [2]

### Unit 10 · Sheet A — Mean, median, mode and range

- 1 12
- 2 12
- 3 9
- 4 9
- 5 4.2
- 6 4
- 7 17
- 8 9
- 9 19
- 10 e.g. 4, 4, 7
- 11 3.3
- 12 the 13th
- 13 mode
- 14 a few very expensive houses raise the mean

### Unit 10 · Sheet B — Mean, median, mode and range

- 1 (a) 2 B1 [1] (b) 2 B1 [1] (c)  $\Sigma fx = 0 + 8 + 20 + 18 + 8 + 5$  M1;  $59 \div 30$  M1; 2.0 (1.97) A1 [3] (d) 5 B1 [1]
- 2  $7 \times 9 = 63$  M1;  $63 - 46$  M1; 17 A1 [3]
- 3 (a) Amy mean 66 B1, range 17 B1; Ben mean 65 B1, range 35 B1 [4] (b) Amy B1; her range is smaller so her scores are more consistent (and mean slightly higher) B1 [2]
- 4  $15 \times 64 + 10 \times 59$  M1;  $1550 \div 25$  M1; 62 A1 [3]
- 5 5, 5 (mode) and 7 (median 6) M1; sum = 28 M1; 5, 5, 7, 11 A1 [3]
- 6 (a)  $x + x + 3 + 2x + 3x - 1 = 44$  M1;  $7x + 2 = 44$  M1;  $x = 6$  A1 [3] (b) 6, 9, 12, 17  $\rightarrow 10.5$  B1 ft [1]

### Unit 11 · Sheet A — Grouped data: estimating the mean

- 1 (a)  $20 < w \leq 30$  (b) 15 (c) 880 (d) 22 minutes (e)  $20 < w \leq 30$  (f) 9
- 2 (a)  $5 < d \leq 10$  (b)  $355 \div 40 = 8.875$ , about 8.9 km (c)  $5 < d \leq 10$
- 3 the exact values are not known

### Unit 11 · Sheet B — Grouped data: estimating the mean

- 1 (a)  $70 < m \leq 80$  B1 [1] (b)  $70 < m \leq 80$  B1 [1] (c) mid-points 55, 65, 75, 85, 95 M1;  $\Sigma fx = 220 + 585 + 1125 + 680 + 380$  M1;  $2990 \div 40$  M1; 74.75 A1 [4]
- 2 (a)  $50 - 20 = 30$  B1 [1] (b) mid-points 3, 8, 13, 18 M1;  $\Sigma fx = 15 + 240 + 156 + 54$  M1;  $465 \div 50$  M1; 9.3 min A1 [4]
- 3 (a) Group A:  $\Sigma fx = 395$  M1, 19.75 min A1; Group B:  $\Sigma fx = 440$  M1, 22 min A1 [4] (b) Group A — smaller estimated mean B1 ft [1]
- 4 (a)  $40 < a \leq 60$  B1 [1] (b)  $\Sigma fx = 70 + 540 + 1050 + 280$  M1;  $\div 50$  M1; 38.8 years A1 [3] (c) mid-points used; actual ages unknown B1 [1]
- 5 (a) mid-points 4.5, 14.5, 24.5, 34.5 M1;  $\Sigma fx = 545$  M1; 18.2 A1 [3] (b) the actual smallest and largest values are not known; 39 is only the greatest possible range B1 [1]

### Unit 12 · Sheet A — Cumulative frequency, median and IQR

- 1 (a) 27.0 kg (accept  $\pm 1$ ) (b) 19.5 kg (c) 35.4 kg (d) 15.9 kg (e) 34 (f) 7
- 2 (a) 12 (b) 7 (c) 18 (d) 11
- 3  $48 - 30 = 18$
- 4 150

## Unit 12 · Sheet B — Cumulative frequency, median and IQR

- 1** (a) 4, 13, 31, 55, 66, 70 B1 **[1]** (b) points plotted at upper bounds B1 (ft); smooth curve or line segments B1 **[2]** (c) 31.7 min (accept 31–33) B1 ft **[1]** (d)  $UQ \approx 38.7$  and  $LQ \approx 23.0$  M1;  $\approx 15.7$  (accept 14–18) A1 ft **[2]** (e) reading at 45  $\approx 62$  M1;  $\approx 8$  (accept 5–11) A1 ft **[2]**
- 2** (a) 167.1 cm (accept  $\pm 1$ ) B1 **[1]** (b)  $UQ \approx 175.0$ ,  $LQ \approx 160.0$  M1;  $\approx 15.0$  cm A1 **[2]** (c) reading at 155 M1;  $\approx 17$  (accept  $\pm 3$ ) A1 **[2]** (d) boys taller on average (higher median) B1; boys' heights more spread out / girls more consistent (compare IQRs) B1 ft **[2]** (e)  $120 - 104 = 16$  M1;  $16/120 \times 100 = 13.3\%$  A1 **[2]**
- 3** (a)  $LQ = 5$  B1;  $UQ = 12$  B1 **[2]** (b) 7 B1 ft **[1]** (c) 8 B1 **[1]** (d) it is not affected by extreme values B1 **[1]**
- 4** (a) 12th value = 2 B1 **[1]** (b)  $LQ = 6$ th value = 1,  $UQ = 18$ th value = 3 M1; 2 A1 **[2]**

## Unit 13 · Sheet A — Histograms

- 1** (a) 4 (b) 1.8 (c) 1.2 (d) 0.15
- 2** (a) 22 (b) 28 (c) 105 (d)  $14 + 12 = 26$  (e) 15.5
- 3** 12

## Unit 13 · Sheet B — Histograms

- 1** (a) 1.5, 4.8, 5.6, 2.2, 0.4 B2 (B1 for 3 or 4 correct) **[2]** (b) correct widths B1; heights B2 (B1 for 3 or 4 correct) ft **[3]** (c)  $\frac{1}{2} \times 22 = 11$  M1;  $11 + 8 = 19$  A1 **[2]** (d)  $15 < t \leq 20$  (highest frequency density) B1 **[1]**
- 2** (a) 14, 18 B1; 18, 8 B1 **[2]** (b) bar  $6 < l \leq 8$  at height 11 B1 **[1]** (c) mid-points 2, 5, 7, 10, 16 M1;  $\Sigma fx = 580$  M1;  $\div 80$  M1; 7.25 cm A1 **[4]** (d) 40th value: 8 more into  $6 < l \leq 8$  M1;  $6 + 8/22 \times 2 = 6.7$  cm A1 **[2]**
- 3**  $FD = 30 \div 10 = 3$  is shown by 4 cm, so 1 cm = 0.75 M1;  $FD = 3 \times 0.75 = 2.25$  M1;  $2.25 \times 20 = 45$  A1 **[3]**
- 4** (a) 12, 17, 26, 12 B2 (B1 for 2 or 3 correct) **[2]** (b) 67 B1 ft **[1]** (c)  $\frac{1}{2} \times 26 = 13$  M1;  $13 + 12 = 25$  A1 ft **[2]**

## End-of-book check

- 1**  $8/20 = 2/5$  B1 **[1]**
- 2**  $1 - 0.8$  M1; 0.2 A1 **[2]**
- 3** 21 B1 **[1]**
- 4** 6 outcomes M1;  $6/36 = 1/6$  A1 **[2]**
- 5** 0.25 B1 **[1]**
- 6**  $0.3 \times 0.6$  M1; 0.18 A1 **[2]**
- 7**  $6/10 \times 5/9$  M1;  $1/3$  A1 **[2]**
- 8**  $\Sigma fx = 69$  M1; 3.45 A1 **[2]**
- 9** mid-points 2, 6, 10 M1;  $\Sigma fx = 124$  M1; 6.2 A1 **[3]**
- 10**  $360 \div 80 \times 16$  M1; 72 A1 **[2]**
- 11**  $LQ = 6$  and  $UQ = 20$  M1; 14 A1 **[2]**
- 12** 36 B1 **[1]**

## Practice paper

- 1** (a) Y10 Science 26; Y11 Maths 23; Y11 total 42; Science total 45 B3 (B2 for 3 correct, B1 for 2) **[3]** (b)  $23/90$  B1 ft **[1]** (c)  $26/45$  M1;  $26/45$  A1 ft **[2]**
- 2** (a) 17 thousand (17 000) B1 **[1]** (b) July B1 **[1]** (c)  $12 + 15 + 21 + 18$  M1; 66 thousand A1 **[2]**
- 3** (a)  $360 \div 36 = 10$  M1;  $110^\circ$ ,  $90^\circ$ ,  $100^\circ$ ,  $60^\circ$  A1 **[2]** (b) sectors  $\pm 2^\circ$  B1; labels B1 **[2]** (c)  $6/36 = 1/6$  B1 **[1]**
- 4** (a) 2 B1 **[1]** (b) 3 B1 **[1]** (c)  $\Sigma fx = 111$  M1;  $111 \div 40$  M1; 2.8 A1 **[3]**
- 5** (a)  $140 < h \leq 160$  B1 **[1]** (b) mid-points 110, 130, 150, 170 M1;  $\Sigma fx = 8600$  M1;  $\div 60$  M1; 143.3 cm A1 **[4]**
- 6** (a)  $2x + x + 12 + 7 = 40$  M1;  $x = 7$  A1 **[2]** (b)  $21/40$  B1 ft **[1]** (c)  $7/21$  M1;  $1/3$  A1 ft **[2]**
- 7** (a) 0.2 B1; 0.65, 0.35 on both pairs B1 **[2]** (b)  $0.8 \times 0.65$  M1; 0.52 A1 **[2]** (c)  $0.8 \times 0.35 + 0.2 \times 0.65$  M1; 0.41 A1 **[2]**
- 8**  $4/12 \times 3/11$  or  $5/12 \times 4/11$  or  $3/12 \times 2/11$  M1; all three products M1; sum M1;  $38/132 = 19/66$  A1 **[4]**
- 9** (a) 12, 40, 71, 89, 97, 100 B1 **[1]** (b) points at upper bounds B1; curve B1 **[2]** (c) 11.6 min (accept  $\pm 0.5$ ) B1 ft **[1]** (d)  $UQ \approx 15.9$ ,  $LQ \approx 7.5$  M1;  $\approx 8.3$  min A1 ft **[2]** (e) reading at 18  $\approx 83$  M1;  $\approx 17$  (accept  $\pm 3$ ) A1 ft **[2]**
- 10** (a) frequency densities 0.5, 2.8, 3.6, 1.6, 0.6 M1; correct widths B1; correct heights A1 **[3]** (b)  $\frac{1}{2} \times 16 = 8$  M1;  $8 + 12 = 20$  A1 **[2]** (c)  $15 < t \leq 20$  B1 **[1]**
- 11** (a) 0.37 B1 **[1]** (b)  $0.37 \times 1500$  M1; 555 A1 ft **[2]** (c) Leo's — more trials B1 **[1]**
- 12** 12 outcomes with 8 even M1;  $8/12 = 2/3$  A1 **[2]**