



BRITISH LEARNING CENTRE

PEARSON EDEXCEL INTERNATIONAL GCSE · 4MA1

Mathematics

Algebra & Graphs

Fourteen units covering Section 2 (Equations, formulae and identities) and Section 3 (Sequences, functions and graphs) of the Pearson Edexcel International GCSE Mathematics A (4MA1) specification, Foundation and Higher, including calculus.

Name:

Key knowledge · Practice · Exam-style questions · Mark schemes

How to use this workbook

1

Key knowledge

Read the notes, worked examples and common mistakes.

2

Practice

Build the skill on Sheet A.

3

Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [].

4

Check & reflect

Mark your work with the mark scheme, then colour the tracker.

My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

#	Spec ref	Topic · I can...	R	A	G
1	2.1, 2.3 Foundation & Higher	Expressions, substitution and index laws I can simplify expressions, use index laws and substitute into formulae.			
2	2.2 Foundation & Higher	Expanding brackets I can expand and simplify products of brackets, including three brackets (Higher).			
3	2.2 Foundation & Higher	Factorising expressions I can factorise using common factors, quadratics and the difference of two squares.			
4	2.2, 2.4 Higher	Algebraic fractions I can simplify, add, subtract, multiply and divide algebraic fractions and solve equations with them.			
5	2.3, 2.4 Foundation & Higher	Linear equations and rearranging formulae I can solve linear equations, set up equations from problems and change the subject of a formula.			
6	2.6, 2.7 Foundation & Higher	Simultaneous equations I can solve two linear simultaneous equations, and (Higher) one linear with one quadratic.			
7	2.7 Foundation & Higher	Quadratic equations I can solve quadratic equations by factorising and (Higher) by the formula and completing the square.			
8	2.8 Foundation & Higher	Inequalities I can solve linear and (Higher) quadratic inequalities and show regions on a graph.			
9	2.5 Higher	Direct and inverse proportion I can set up and use equations for direct and inverse proportion, including powers and roots.			
10	3.1 Foundation & Higher	Sequences and arithmetic series I can find and use the n th term of a sequence and (Higher) sum an arithmetic series.			
11	3.2 Higher	Function notation I can use function notation, domain and range, and find composite and inverse functions.			
12	3.3 Foundation & Higher	Straight-line graphs I can draw straight-line graphs and find gradients, midpoints and equations of lines.			
13	3.3 Foundation & Higher	Curved graphs and real-life graphs I can plot curves, solve equations graphically and interpret real-life graphs.			
14	3.4 Higher	Calculus I can differentiate integer powers of x and use calculus for gradients, turning points and kinematics.			



CHECK

Starting point check

Starting point check

Name _____ Date _____ Score _____ / 12

Answer these questions before you start the book.

1 Simplify $9p - 4 + 2p + 7$

2 Work out $4a - b^2$ when $a = 3$ and $b = -2$

3 Expand $6(3x + 2)$

4 Factorise $12y - 8$

5 Solve $3x - 8 = 13$

6 Solve $2(x + 4) = 5x - 1$

7 Find the n th term of 9, 13, 17, 21, ...

8 Simplify $y^8 \div y^2$

9 Find the midpoint of (2, 3) and (8, 11)

10 Solve $5x + 2 \geq 17$

11 Expand and simplify $(x + 3)(x + 5)$

12 Solve $2x + y = 10$ and $x - y = 2$

Expressions, substitution and index laws

I can... simplify expressions, use index laws and substitute into formulae.

Spec ref: 2.1, 2.3 · Foundation & Higher

Collect **like terms**: terms with exactly the same letters and powers. When multiplying, deal with the numbers first and then the letters.

The index laws work for letters. Higher tier also uses zero, negative and fractional indices. To **substitute**, write each value in brackets and use the order of operations.

Key words

term

like terms

index

formula

substitute

Key points

- $x^m \times x^n = x^{m+n}$ $x^m \div x^n = x^{m-n}$ $(x^m)^n = x^{mn}$
- (Higher) $x^0 = 1$, $x^{-n} = 1/x^n$, $x^{1/n} = \sqrt[n]{x}$
- $(3a^2b)^2 = 9a^4b^2$: every part is squared.

Worked example 1

Simplify fully $(24a^5b^3)/(8a^2b)$.

Numbers: $24 \div 8 = 3$ · $a^{5-2} = a^3$, $b^{3-1} = b^2$ · Answer: $3a^3b^2$

Worked example 2

Work out $x^2 + 2xy$ when $x = -3$ and $y = 4$.

$(-3)^2 + 2 \times (-3) \times 4 = 9 + (-24) = -15$

Common mistakes

- Adding powers when raising a power to a power: $(k^2)^5 = k^{10}$, not k^7 .
- Forgetting to square the number: $(3a)^2 = 9a^2$, not $3a^2$.
- Squaring a negative without brackets on a calculator.

Exam tip: If a question says 'Simplify fully', your answer must have no further common factors or like terms.

Answer each question. Show your working where it helps.

1 Simplify $8m + 2n - 3m - 5n$

2 Simplify $6y^2 - 2y + y^2 + 9y$

3 Simplify $5a \times 3b \times 2a$

4 Simplify $18x^3y \div 6xy$

5 Simplify $m^4 \times m^6$

6 Simplify $p^{12} \div p^3$

7 Simplify $(k^2)^5$

8 Simplify $(3a^2b)^2$

9 (Higher) Simplify $4x^{-2} \times 3x^5$

10 (Higher) Write $1/x^3$ as a power of x

11 (Higher) Simplify $(27y^6)^{1/3}$

12 (Higher) Simplify $(8x^9)^{2/3}$

13 Work out $3x - 4y$ when $x = -2$ and $y = 5$

14 Work out $x^2 + 2xy$ when $x = -3$ and $y = 4$

15 Work out $(p - q)^2$ when $p = 2.5$ and $q = -1.5$

16 $F = 9C/5 + 32$. Work out F when $C = -15$

17 $A = \pi r^2$. Work out A when $r = 4.5$.
Give 1 d.p.

18 Pens cost 45p each and a ruler costs 80p. Write a formula for the cost, C pence, of n pens and one ruler.

19 Write an expression for the perimeter of a square of side $(x + 3)$ cm.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** (a) Simplify $4p \times 7q$. [1]

Answer

(b) Simplify fully $(24a^5b^3)/(8a^2b)$. [2]

Answer

(c) $x^n \times x^4 = x^{11}$. Find the value of n . [1]Answer $n =$ (d) (Higher) Simplify $(2x^3y^{-1})^4$. [2]

Answer

2 $v^2 = u^2 + 2as$ (a) Work out the positive value of v when $u = 3$, $a = 4$ and $s = 5$. [2]Answer $v =$ (b) Work out the value of s when $v = 10$, $u = 6$ and $a = 2$. [3]Answer $s =$ **3** Hamid buys x bags of rice at £3.20 each and y bottles of oil at £4.50 each. The total cost is £ T .(a) Write down a formula for T in terms of x and y . [2]

Answer

(b) Work out T when $x = 5$ and $y = 3$. [1]

Answer £

4 (Higher)

(a) Work out the value of $8^{2/3} \times 4^{-1/2}$. [2]

Answer

(b) Solve $2^{3x} = 4^{x+2}$. [3]

Answer $x =$

(c) Simplify $(9x^4)^{-1/2}$. [2]

Answer

5 The time, T minutes, needed to cook a turkey of mass w kg is given by $T = 30w + 15$.

(a) Work out the time needed to cook a turkey of mass 4.5 kg. Give your answer in hours and minutes. [3]

Answer hours minutes

(b) A turkey takes 3 hours 30 minutes to cook. Work out its mass. [2]

Answer kg

6 (Higher) $x = 2^p$ and $y = 2^q$. Write each expression in terms of x and/or y .

(a) 2^{p+q} [1]

Answer

(b) 2^{2p} [1]

Answer

(c) 2^{p-q+1} [2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Expanding brackets

I can... expand and simplify products of brackets, including three brackets (Higher).

Spec ref: 2.2 · Foundation & Higher

Multiply every term inside a bracket by the term outside. For two brackets, every term in the first multiplies every term in the second, giving four terms; then collect like terms.

(Higher) For three brackets, expand any two first and then multiply by the third. To **prove** a result, expand, simplify, and then explain why the final form has the property.

Key words

expand

simplify

binomial

product

prove

Key points

- $(a + b)(c + d) = ac + ad + bc + bd$
- $(x - 6)^2 = (x - 6)(x - 6) = x^2 - 12x + 36$
- A multiple of 12 can be written as $12 \times (\text{an integer})$.

Worked example 1

Expand and simplify $(2x - 7)(3x + 2)$.

$$6x^2 + 4x - 21x - 14 = 6x^2 - 17x - 14$$

Worked example 2 (Higher)

Expand and simplify $(x + 2)(x - 3)(x + 4)$.

$$(x + 2)(x - 3) = x^2 - x - 6$$

$$(x^2 - x - 6)(x + 4) = x^3 + 4x^2 - x^2 - 4x - 6x - 24 = x^3 + 3x^2 - 10x - 24$$

Common mistakes

- Missing the middle terms: $(x - 6)^2$ is not $x^2 + 36$.
- Sign errors when a negative number multiplies a bracket: $-2(4y - 7) = -8y + 14$.
- Stopping before collecting like terms.

Exam tip: For 'show that' and 'prove' questions, write every step; a correct final line with no working may score nothing.

Expand, and simplify where needed.

1 Expand $5(3x + 4)$

2 Expand $-2(4y - 7)$

3 Expand $3p(2p + 5)$

4 Expand $4x(x^2 - 3x)$

5 Expand and simplify $2(3x - 1) - 4(x - 5)$

6 Expand and simplify $x(x + 3) + 2x(x - 1)$

7 Expand and simplify $(x + 6)(x + 2)$

8 Expand and simplify $(y - 5)(y - 8)$

9 Expand and simplify $(x + 9)(x - 4)$

10 Expand and simplify $(3x + 1)(x + 5)$

11 Expand and simplify $(2x - 7)(3x + 2)$

12 Expand and simplify $(x - 6)^2$

13 Expand and simplify $(3x + 4)^2$

14 Expand and simplify $(5 - 2x)(5 + 2x)$

15 (Higher) Expand and simplify $(x + 2)(x - 3)(x + 4)$

16 (Higher) Expand and simplify $(2x - 1)^3$

17 (Higher) Expand and simplify $(x + 1)^2(3x - 2)$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** Expand and simplify.

(a) $7(2x + 3) - 3(4x - 1)$

[2]**Answer**

(b) $(m + 4)(m - 9)$

[2]**Answer**

(c) $(4y - 3)(2y + 5)$

[2]**Answer**

(d) $(2x - 5)^2$

[2]**Answer****2** A square of side $(x + 4)$ cm has a square of side x cm cut from one corner. Show that the area that is left is $8(x + 2)$ cm².**[3]****3** (Higher) Expand and simplify $(x - 3)(2x + 1)(x + 5)$.**[3]**

- 4 (Higher) n is an integer. Prove that $(n + 3)^2 - (n - 3)^2$ is a multiple of 12. [3]

- 5 (a) $(x + a)(x + 3) = x^2 + 10x + 21$. Find the value of a . [2]

Answer $a = \dots\dots\dots$

- (b) (Higher) $(2x + p)(x + q) = 2x^2 + 7x - 15$, where p and q are integers. Find p and q . [3]

Answer $p = \dots\dots\dots q = \dots\dots\dots$

- 6 A rectangle has length $(3x - 2)$ cm and width $(x + 4)$ cm.

- (a) Find an expression for the area. Give your answer in its simplest form. [2]

Answer $\dots\dots\dots \text{cm}^2$

- (b) Work out the area when $x = 3$. [1]

Answer $\dots\dots\dots \text{cm}^2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Factorising expressions

I can... factorise using common factors, quadratics and the difference of two squares.

Spec ref: 2.2 · Foundation & Higher

Factorising puts an expression into brackets. Always look for the highest common factor first: $5x^2 - 20x = 5x(x - 4)$.

For $x^2 + bx + c$, look for two numbers with product c and sum b .

(Higher) For $ax^2 + bx + c$, use two numbers with product ac and sum b to split the middle term.

Key words

factorise

highest common factor

quadratic

difference of two squares

prove

Key points

- $a^2 - b^2 = (a + b)(a - b)$
- Expand your answer to check it.
- 'Factorise fully' means take out every common factor.

Worked example 1

Factorise $x^2 - 4x - 21$.

Product -21 , sum -4 : -7 and $+3$ · Answer: $(x - 7)(x + 3)$

Worked example 2 (Higher)

Factorise $4x^2 - 4x - 15$.

$ac = -60$; -10 and 6 · $4x^2 - 10x + 6x - 15 = 2x(2x - 5) + 3(2x - 5)$

Answer: $(2x - 5)(2x + 3)$

Common mistakes

- Leaving a common factor inside: $2x^2 - 32 = 2(x - 4)(x + 4)$, not $(2x - 8)(x + 4)$.
- Writing $x^2 + 25 = (x + 5)(x - 5)$.
- Mixing up the signs in the brackets.

Exam tip: In a proof about consecutive even numbers, use $2n$ and $2n + 2$, not two different letters.

Factorise each expression. Check by expanding.

1 Factorise $7x - 21$

2 Factorise fully $12y^2 + 8y$

3 Factorise $15ab - 10a^2$

4 Factorise fully $4x^2y + 6xy^2$

5 Factorise $x^2 + 9x + 14$

6 Factorise $x^2 - 4x - 21$

7 Factorise $x^2 - 11x + 24$

8 Factorise $y^2 - 81$

9 Factorise $9x^2 - 16$

10 Factorise $25 - x^2$

11 Factorise fully $5x^2 - 20x$

12 Factorise fully $2x^2 - 32$

13 Factorise fully $x^3 - 9x$

14 (Higher) Factorise $3x^2 + 11x + 6$

15 (Higher) Factorise $4x^2 - 4x - 15$

16 (Higher) Factorise $6x^2 - 13x + 6$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

1 (a) Factorise $9p - 12$. [1]

Answer

(b) Factorise fully $10x^3y - 15x^2y^2$. [2]

Answer

(c) Factorise $y^2 + 2y - 35$. [2]

Answer

2 (Higher)

(a) Factorise $3x^2 - 14x + 8$. [2]

Answer

(b) Hence solve $3x^2 - 14x + 8 = 0$. [1]

Answer $x = \dots\dots$ or $x = \dots\dots$

3 (a) Use factorisation to work out $1001^2 - 999^2$. Show your working. [2]

Answer

(b) (Higher) Factorise fully $2x^2 - 50$. [2]

Answer

(c) (Higher) Factorise $(x + 1)^2 - 16$. [2]

Answer

4 Prove that the difference between the squares of two consecutive even numbers is always a multiple of 4. [3]

5 (Higher) The area of a rectangle is $(2x^2 + 7x + 6) \text{ cm}^2$.

(a) Factorise $2x^2 + 7x + 6$.

[2]

Answer

(b) The length of the rectangle is $(x + 2) \text{ cm}$. Write down an expression for the width.

[1]

Answer cm

(c) Find an expression for the perimeter. Simplify your answer.

[2]

Answer cm

6 (Higher) Simplify fully $(x^2 + 5x) \div (x^2 - 25)$.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Algebraic fractions

I can... simplify, add, subtract, multiply and divide algebraic fractions and solve equations with them.

Spec ref: 2.2, 2.4 · Higher

Algebraic fractions follow the rules for number fractions. To simplify, factorise the numerator and the denominator, then cancel common **factors**.

To add or subtract, write each fraction over a common denominator. To multiply, multiply numerators and denominators; to divide, multiply by the reciprocal of the second fraction. To solve an equation, multiply every term by the common denominator.

Key words

numerator

denominator

common denominator

cancel

factor

Key points

- Factorise first, cancel second.
- Use brackets for numerators: $-(x - 3) = -x + 3$.
- $a/b \div c/d = a/b \times d/c$

Worked example

Write $2/(x + 3) + 1/(x - 1)$ as a single fraction.

$$\begin{aligned} \text{Common denominator } (x + 3)(x - 1) &= (2(x - 1) + (x + 3)) / ((x + 3)(x - 1)) \\ &= (3x + 1) / ((x + 3)(x - 1)) \end{aligned}$$

Worked example 2

Simplify $(3x + 6)/(x^2 - 4)$.

$$= 3(x + 2) / ((x + 2)(x - 2)) = 3/(x - 2)$$

Common mistakes

- Cancelling terms instead of factors: $(x + 6)/(x + 2)$ does not simplify to 3.
- Adding the denominators: $1/x + 1/y$ is not $2/(x + y)$.
- Losing the minus sign in front of the second numerator.

Exam tip: When an answer is a quadratic expression over a product of brackets, check whether the top factorises and cancels.

Answer each question. Show your working where it helps.

1 Simplify $10x/15$

2 Simplify $8x^2y/(12xy^3)$

3 Simplify $(3x + 6)/(x^2 - 4)$

4 Simplify $(x^2 - x - 6)/(x^2 - 9)$

5 Simplify $(2x^2 + x - 3)/(x^2 - 1)$

6 Write as a single fraction: $x/2 + x/5$

7 Write as a single fraction: $2x/3 - x/4$

8 Write as a single fraction: $(x + 3)/2 + (x - 1)/3$

9 Write as a single fraction: $4/x - 3/(2x)$

10 Write as a single fraction: $2/(x + 3) + 1/(x - 1)$

11 Write as a single fraction: $5/(x - 2) - 3/(x + 1)$

12 Simplify $(3x/4) \times (8/x^2)$

13 Simplify $(x/6) \div (x^2/9)$

14 Simplify $((x^2 - 1)/(4x)) \times (8/(x + 1))$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** Simplify fully.

(a) $(x^2 + 2x - 15)/(2x^2 - 5x - 3)$

[3]**Answer**

(b) $(6x^2 - 9x)/(4x^2 - 9)$

[3]**Answer****2** Write as a single fraction in its simplest form.

(a) $3/(x + 2) + 4/(x - 5)$

[3]**Answer**

(b) $2x/(x + 3) - (x - 1)/(x - 2)$

[3]**Answer****3** Solve $(x + 2)/4 - (x - 3)/6 = 2$. Show clear algebraic working.**[3]**

- 4 Solve $4/(x + 1) + 3/(x - 2) = 2$. Give your solutions correct to 3 significant figures. [5]

- 5 Show that $((x^2 - 9)/(x^2 + 4x + 3)) \div ((x - 3)/(2x + 2))$ simplifies to 2. [3]

- 6 (a) Show that $1/(x - 1) - 1/(x + 1) = 2/(x^2 - 1)$. [2]

- (b) Hence solve $1/(x - 1) - 1/(x + 1) = 1/12$. [2]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Linear equations and rearranging formulae

I can... solve linear equations, set up equations from problems and change the subject of a formula.

Spec ref: 2.3, 2.4 · Foundation & Higher

Solve an equation by doing the same operation to both sides. Expand brackets, clear fractions by multiplying every term by the common denominator, collect the unknown on one side, then divide.

In a word problem, choose a letter for the unknown, write expressions for each quantity, then form an equation. (Higher) If the new subject appears twice, collect those terms and factorise.

Key words

equation

solve

unknown

subject

rearrange

Key points

- Write one step per line, with the equals signs lined up.
- A formula keeps its letters; an equation has a numerical answer.
- Check your solution by substituting it into the original equation.

Worked example 1

Solve $(x - 4)/2 = (2x + 1)/5$.

Multiply by 10: $5(x - 4) = 2(2x + 1)$ · $5x - 20 = 4x + 2$ · $x = 22$

Worked example 2 (Higher)

Make x the subject of $a(x - b) = cx$.

$ax - ab = cx$ · $ax - cx = ab$ · $x(a - c) = ab$, so $x = ab/(a - c)$

Common mistakes

- Multiplying only the fraction terms by the denominator.
- $-3(x - 2) = -3x - 6$ (it should be $-3x + 6$).
- Answering with x when the question asks for the area or the numbers.

Exam tip: 'Show clear algebraic working' means a trial-and-improvement answer gets no marks.

Answer each question. Show your working where it helps.

1 Solve $4x + 9 = 37$

2 Solve $6(x - 2) = 33$

3 Solve $9x - 4 = 5x + 16$

4 Solve $3(2x + 5) = 4(x - 1)$

5 Solve $x/5 - 2 = 4$

6 Solve $(3x + 2)/4 = 8$

7 Solve $(x - 4)/2 = (2x + 1)/5$

8 Solve $12 - 3x = 2x + 2$

9 Solve $2.5x - 1.5 = 6$

10 Solve $20/x = 8$

11 Make a the subject of $P = 2a + 3b$

12 Make m the subject of $y = mx + c$

13 Make h the subject of $A = \frac{1}{2}bh$

14 Make c the subject of $E = mc^2$ ($c > 0$)

15 Make h the subject of $V = \pi r^2 h$

16 (Higher) Make v the subject of $s = (u + v)t/2$

17 (Higher) Make x the subject of $a(x - b) = cx$

18 (Higher) Make x the subject of $y = \sqrt{(x + 1)/3}$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 Solve. Show clear algebraic working.

(a) $5(2y - 3) = 3y + 13$

[3]

Answer $y = \dots\dots\dots$

(b) $(x + 5)/3 - (x - 2)/4 = 2$

[4]

Answer $x = \dots\dots\dots$

- 2 A rectangle has length $(3x + 1)$ cm and width $(x + 5)$ cm. The perimeter of the rectangle is 60 cm. Work out the area of the rectangle.

[4]

- 3 (Higher) Make y the subject of $x = (3y - 2)/(y + 4)$.

[4]

4 The surface area of a sphere is $A = 4\pi r^2$.

(a) Make r the subject of the formula, where $r > 0$.

[2]

Answer $r =$

(b) Work out the radius of a sphere with surface area 500 cm^2 . Give your answer correct to 3 significant figures.

[2]

Answer cm

5 The sum of three consecutive odd numbers is 111. Use algebra to find the three numbers. [3]

6 Ella has $\text{£}x$. Finn has $\text{£}15$ more than Ella. Ella gives Finn $\text{£}5$. Finn now has twice as much money as Ella. Work out how much money Ella had at the start.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Simultaneous equations

I can... solve two linear simultaneous equations, and (Higher) one linear with one quadratic.

Spec ref: 2.6, 2.7 · Foundation & Higher

Two equations with two unknowns can be solved together. In **elimination**, multiply one or both equations so that one unknown has the same coefficient, then add or subtract to remove it.

(Higher) When one equation contains x^2 or xy , rearrange the linear equation and **substitute** it into the other one. Solve the resulting quadratic and find the matching values.

Key words

simultaneous

elimination

substitution

coefficient

point of intersection

Key points

- The solution is the point where the two graphs cross.
- Find the second unknown by substituting into the simpler equation.
- In problems, define your letters and give the answer in context.

Worked example 1

Solve $3x - 2y = 4$ and $5x + 3y = 13$.

$$(1) \times 3: 9x - 6y = 12 \quad (2) \times 2: 10x + 6y = 26 \quad \cdot \text{ Add: } 19x = 38, \text{ so } x = 2$$

$$3(2) - 2y = 4, \text{ so } y = 1$$

Worked example 2 (Higher)

Solve $y = x + 1$ and $x^2 + y^2 = 25$.

$$x^2 + (x + 1)^2 = 25, \text{ so } 2x^2 + 2x - 24 = 0$$

$$x^2 + x - 12 = 0, \text{ so } (x + 4)(x - 3) = 0 \quad \cdot \quad x = 3, y = 4 \text{ or } x = -4, y = -3$$

Common mistakes

- Adding when the signs are the same (you should subtract).
- Expanding $(x + 1)^2$ as $x^2 + 1$.

Exam tip: Substitute both values into the equation you did not use; if it works, your answer is right.

Solve each pair of simultaneous equations.

1 Solve $x + y = 12$ and $x - y = 2$

2 Solve $3x + y = 14$ and $x + y = 6$

3 Solve $2x + 5y = 24$ and $2x + y = 8$

4 Solve $4x + 3y = 25$ and $2x - 3y = -1$

5 Solve $3x - 2y = 4$ and $5x + 3y = 13$

6 Solve $6x + 5y = 7$ and $4x + 3y = 5$

7 Solve $y = 3x - 2$ and $2x + y = 13$

8 Solve $p + 3q = 1$ and $2p - q = 9$

9 Solve $7x - 2y = 30$ and $3x + 4y = 8$

10 (Higher) Solve $y = x + 1$ and $x^2 + y^2 = 25$

11 (Higher) Solve $y = x^2 - 2$ and $y = 3x + 2$

12 (Higher) Solve $x - y = 1$ and $xy = 12$

13 5 pencils and 2 rulers cost 160p. 3 pencils and 2 rulers cost 120p. Find the cost of each.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** Solve the simultaneous equations. Show clear algebraic working.

$$5x + 2y = 11$$

$$3x - 4y = 4$$

[4]

- 2** At a café, 4 coffees and 3 cakes cost £18.60. 2 coffees and 5 cakes cost £19.80. A coffee costs £ c and a cake costs £ k .

(a) Write down two equations in c and k .

[2]

(b) Work out the cost of one coffee and the cost of one cake.

[3]

Answer coffee £ cake £

- 3** (Higher) Solve the simultaneous equations.

$$y = x^2 - 4x + 5$$

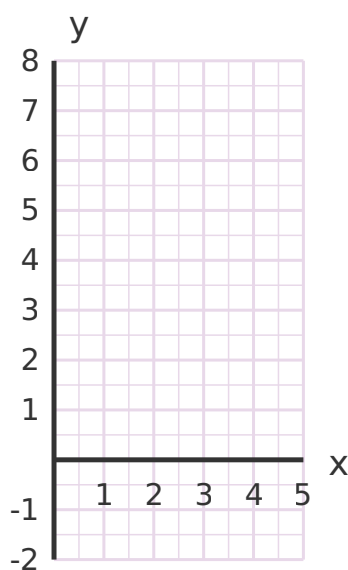
$$y = 2x - 3$$

[4]

4

(a) On the grid, draw the lines $2x + y = 8$ and $y = x + 2$ for $0 \leq x \leq 5$.

[2]



(b) Use your graph to solve the simultaneous equations.

[1]

Answer $x = \dots\dots\dots$ $y = \dots\dots\dots$

- 5 The sum of two numbers is 46. Three times the smaller number is 8 more than twice the larger number. Use algebra to find the two numbers.

[4]

- 6 (Higher) Solve the simultaneous equations.

$$x^2 + y^2 = 41$$

$$y = x - 1$$

[5]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Quadratic equations

I can... solve quadratic equations by factorising and (Higher) by the formula and completing the square.

Spec ref: 2.7 · Foundation & Higher

Rearrange the equation so that one side is 0. If it factorises, use the fact that if $AB = 0$ then $A = 0$ or $B = 0$.

(Higher) If it does not factorise, or you are asked for answers to 3 significant figures, use the quadratic formula. Completing the square gives exact (surd) answers and the turning point.

Key points

- $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$ (given on the formula sheet)
- $x^2 + 10x + 7 = (x + 5)^2 - 25 + 7 = (x + 5)^2 - 18$
- The turning point of $y = a(x + b)^2 + c$ is $(-b, c)$.

Key words

quadratic

factorise

solution

formula

completing the square

Worked example 1

Solve $x^2 - 3x - 40 = 0$.

$$(x - 8)(x + 5) = 0 \quad \cdot \quad x = 8 \text{ or } x = -5$$

Worked example 2 (Higher)

Solve $x^2 - 4x - 3 = 0$, giving answers to 3 s.f.

$$x = (4 \pm \sqrt{(16 + 12)}) / 2 = (4 \pm \sqrt{28}) / 2 \quad \cdot \quad x = -0.646 \text{ or } x = 4.65$$

Common mistakes

- Not rearranging to $= 0$ before factorising.
- Writing -3^2 instead of $(-3)^2$ in the formula.
- Giving a negative length as a final answer: reject it in context.

Exam tip: In a 'show that' question, end with exactly the equation printed in the question.

Answer each question. Show your working where it helps.

1 Solve $x^2 + 8x + 15 = 0$

2 Solve $x^2 - 3x - 40 = 0$

3 Solve $x^2 + 6x = 0$

4 Solve $x^2 - 121 = 0$

5 Solve $x^2 - 12x + 36 = 0$

6 (Higher) Solve $3x^2 + 5x - 2 = 0$

7 (Higher) Solve $4x^2 - 9 = 0$

8 (Higher) Solve $2x^2 - 11x + 15 = 0$

9 (Higher) Solve $x^2 - 4x - 3 = 0$. Give answers to 3 s.f.

10 (Higher) Solve $3x^2 + 7x + 1 = 0$. Give answers to 3 s.f.

11 (Higher) Solve $2x^2 - 3x - 7 = 0$. Give answers to 3 s.f.

12 (Higher) Write $x^2 + 10x + 7$ in the form $(x + b)^2 + c$

13 (Higher) Write $x^2 - 3x + 1$ in the form $(x + b)^2 + c$

14 (Higher) Write $3x^2 - 12x + 20$ in the form $a(x + b)^2 + c$

15 (Higher) Solve $x^2 - 8x + 11 = 0$ by completing the square. Leave surds.

16 Solve $x(x - 4) = 21$

17 Solve $15/x = x + 2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** (a) Factorise $x^2 + 5x - 36$. [2]

Answer

- (b) Hence solve $x^2 + 5x - 36 = 0$. [1]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- 2** (Higher) Solve $2x^2 + 6x - 5 = 0$. Show your working clearly and give your solutions correct to 3 significant figures. [3]

- 3** (Higher)

- (a) Express $2x^2 - 8x + 3$ in the form $a(x + b)^2 + c$. [3]

Answer

- (b) Write down the coordinates of the turning point of $y = 2x^2 - 8x + 3$. [1]

Answer (.....,)

- (c) Hence solve $2x^2 - 8x + 3 = 0$. Give your answers in surd form. [2]

Answer $x = \dots\dots\dots$

- 4 A right-angled triangle has sides of length x cm and $(x + 7)$ cm. Its hypotenuse is $(x + 8)$ cm.

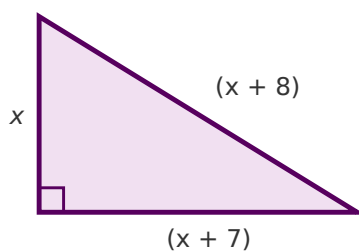


Diagram NOT accurately drawn

- (a) Show that $x^2 - 2x - 15 = 0$.

[3]

- (b) Work out the perimeter of the triangle.

[3]

Answer cm

- 5 (Higher) Solve $\frac{3}{x - 1} - \frac{2}{x + 2} = 1$. Give your answers correct to 3 significant figures.

[4]

- 6 The product of two consecutive positive integers is 132. Use algebra to find the two integers.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Inequalities

I can... solve linear and (Higher) quadratic inequalities and show regions on a graph.

Spec ref: 2.8 · Foundation & Higher

Linear inequalities are solved like equations, except that multiplying or dividing both sides by a **negative** number reverses the sign. On a number line, use ○ for < or > and ● for $\leq$ or $\geq$.

(Higher) For a quadratic inequality, find the **critical values** by solving the equation, sketch the parabola, then read off where it is below (< 0) or above (> 0) the x-axis.

Key words

inequality

integer

number line

critical value

region

Key points

- < 0 for a U-shaped curve gives one interval: $a < x < b$.
- > 0 gives two parts: $x < a$ or $x > b$.
- Region boundaries: solid line for $\leq$, $\geq$; dashed line for <, >.

Worked example 1

Solve $7 - 2x \leq 1$.

$-2x \leq -6$ · Divide by -2 and reverse: $x \geq 3$

Worked example 2 (Higher)

Solve $x^2 + 2x - 8 \geq 0$.

$(x + 4)(x - 2) = 0$ gives critical values -4 and 2

The curve is above the axis outside the roots · $x \leq -4$ or $x \geq 2$

Common mistakes

- Writing $x^2 > 49$ as $x > \pm 7$.
- Forgetting to reverse the sign when dividing by a negative.
- Including end values in a list when the sign is strict (<).

Exam tip: Sketch the parabola for every quadratic inequality; it takes seconds and prevents the most common error.

Answer each question. Show your working where it helps.

1 Solve $4x - 3 < 21$

2 Solve $3x + 8 \geq 2$

3 Solve $7 - 2x \leq 1$

4 Solve $5(x - 1) > 3x + 4$

5 Solve $(x + 3)/2 \geq 5$

6 Solve $-4 \leq 3x + 2 < 14$

7 List the integers n such that $-3 \leq n < 2$

8 List the integers n such that $5 < 2n \leq 14$

9 Find the greatest integer x for which $3x + 1 < 20$

10 Find the smallest integer n for which $4n - 7 > 10$

11 (Higher) Solve $x^2 \leq 16$

12 (Higher) Solve $x^2 > 49$

13 (Higher) Solve $(x - 2)(x + 5) < 0$

14 (Higher) Solve $x^2 + 2x - 8 \geq 0$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

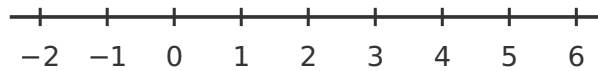
- 1 (a) Solve $3(x - 4) < x + 6$. [2]

Answer

- (b) n is an integer. List the values of n such that $-8 < 3n \leq 6$. [2]

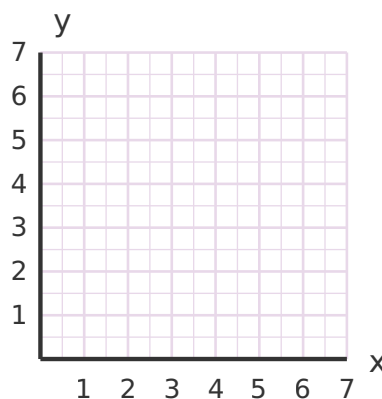
Answer

- (c) Show the inequality $1 < x \leq 4$ on the number line. [2]



- 2 (Higher) Solve $2x^2 - 5x - 3 > 0$. [3]

- 3 (a) On the grid, show the region R that satisfies all three inequalities $x \geq 1$, $y \geq x$, $x + y \leq 6$. Label the region R . [4]



- (b) How many points with integer coordinates lie in R ? Include points on the boundary. [1]

Answer

- 4 Write down the three inequalities that define the region R .

[3]

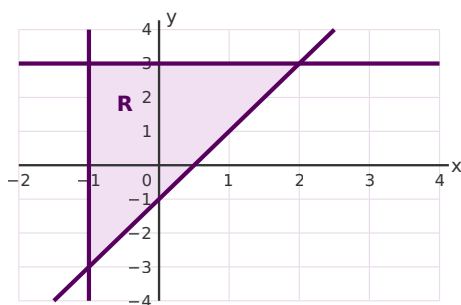


Diagram NOT accurately drawn

- 5 A rectangle has width x cm and length $(x + 5)$ cm. The perimeter is at least 30 cm and less than 50 cm. Find the range of possible values of x .

[3]

- 6 (Higher)

(a) Solve $x^2 < 3x + 10$.

[3]

Answer

(b) Find the integer values of x that satisfy both $x^2 < 3x + 10$ and $2x - 3 > 0$.

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Direct and inverse proportion

I can... set up and use equations for direct and inverse proportion, including powers and roots.

Spec ref: 2.5 · Higher

The symbol $\propto$ means 'is proportional to'. If $y \propto x^2$, then $y = kx^2$ for a constant k . If y is inversely proportional to x^2 , then $y = k/x^2$.

Method: write the equation with k , substitute the pair of values you are given to find k , write the full formula, then use it.

Key points

- Direct: as x increases, y increases. Inverse: as x increases, y decreases.
- $y \propto x$ gives a straight-line graph through the origin.
- Scale factor method: if $y \propto x^3$ and $x \times 2$, then $y \times 2^3 = 8$.

Key words

directly proportional

inversely proportional

constant of proportionality

$\propto$

Worked example

$y \propto 1/\sqrt{x}$. $y = 3$ when $x = 16$. Find y when $x = 4$.

$$y = k/\sqrt{x} \quad 3 = k/4, \text{ so } k = 12 \quad y = 12/\sqrt{4} = 6$$

Common mistakes

- Forgetting to square or square root when finding k .
- Using $y = kx$ for an inverse proportion question.
- Stopping after finding k when the question asks for a formula: write ' $m = 10r^3$ '.

Exam tip: Always write your final formula in full; it usually earns a mark of its own.

Answer each question. Show your working where it helps.

1 $y \propto x$. $y = 18$ when $x = 4$. Find y when $x = 10$.

2 $y \propto x$. $y = 7$ when $x = 2$. Find x when $y = 28$.

3 $y \propto x^2$. $y = 12$ when $x = 2$. Write a formula for y .

4 Using your formula, find y when $x = 5$.

5 $y \propto x^3$. $y = 54$ when $x = 3$. Find y when $x = 2$.

6 $y \propto \sqrt{x}$. $y = 10$ when $x = 4$. Find y when $x = 36$.

7 $y \propto 1/x$. $y = 5$ when $x = 8$. Find y when $x = 20$.

8 $y \propto 1/x^2$. $y = 9$ when $x = 2$. Find y when $x = 3$.

9 $y \propto 1/\sqrt{x}$. $y = 3$ when $x = 16$. Find y when $x = 4$.

10 $y \propto x^2$. x is trebled. What happens to y ?

11 $y \propto 1/x^2$. x is halved. What happens to y ?

12 $d \propto t^2$. $d = 20$ when $t = 2$. Find d when $t = 5$.

13 6 workers take 10 days to build a wall. How long would 4 workers take, at the same rate?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** The mass, m grams, of a metal ball is directly proportional to the cube of its radius, r cm.
When $r = 3$, $m = 270$.

(a) Find a formula for m in terms of r .

[3]

Answer $m = \dots\dots\dots$

(b) Work out the mass when $r = 1.5$.

[1]

Answer $\dots\dots\dots$ g

(c) Work out the radius when $m = 640$.

[2]

Answer $\dots\dots\dots$ cm

- 2** The intensity of light, I , is inversely proportional to the square of the distance, d , from the lamp. When $d = 2$, $I = 50$.

(a) Find a formula for I in terms of d .

[3]

Answer $I = \dots\dots\dots$

(b) Find the value of d when $I = 8$.

[2]

Answer $d = \dots\dots\dots$

3 y is directly proportional to the square root of x . When $x = 9$, $y = 15$.

(a) Find y when $x = 49$.

[3]

Answer $y =$

(b) Find x when $y = 40$.

[2]

Answer $x =$

4 The time, t hours, to complete a job is inversely proportional to the number of machines, n . With 5 machines the job takes 24 hours.

(a) How long does the job take with 8 machines?

[2]

Answer hours

(b) How many machines are needed to finish the job in 10 hours?

[2]

Answer

5 (a) y is directly proportional to x^2 . x is increased by 20%. Work out the percentage increase in y .

[2]

Answer %

(b) w is inversely proportional to v^3 . v is doubled. Describe the effect on w .

[1]

6 F is inversely proportional to x . Complete the table.

[2]

x	2	4	5	
F	30		12	6

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Sequences and arithmetic series

I can... find and use the n th term of a sequence and (Higher) sum an arithmetic series.

Spec ref: 3.1 · Foundation & Higher

In an **arithmetic** sequence the terms go up or down by the same amount, the **common difference** d . The n th term is $a + (n - 1)d$, which simplifies to $dn + (a - d)$.

(Higher) A **series** is the sum of the terms. The sum of the first n terms of an arithmetic series is given on the formula sheet.

Key points

- n th term = $a + (n - 1)d$
- $S_n = n/2 [2a + (n - 1)d]$
- A number is a term only if n is a positive whole number.

Key words

term

 n th term

common difference

arithmetic

series

Worked example 1

Find the n th term of 30, 27, 24, 21, ...

$$d = -3, \text{ so } -3n \cdot 30 - (-3) = 33 \cdot n\text{th term} = 33 - 3n$$

Worked example 2 (Higher)

Find the sum of the first 20 terms of $3 + 7 + 11 + \dots$

$$a = 3, d = 4, n = 20 \cdot S = 20/2 \times (6 + 19 \times 4) = 10 \times 82 = 820$$

Common mistakes

- Writing $n + 4$ instead of $4n + 3$ for 7, 11, 15, ...
- Using n instead of $(n - 1)$ in the formulae.
- Mixing up the n th term and the sum of n terms.

Exam tip: To show a number is not a term, solve n th term = number and state clearly that n is not a whole number.

Answer each question. Show your working where it helps.

1 Write the next two terms: 4, 9, 14, 19, ...

2 Find the n th term: 7, 11, 15, 19, ...

3 Find the n th term: 2, 8, 14, 20, ...

4 Find the n th term: 30, 27, 24, 21, ...

5 Find the n th term: -5, -1, 3, 7, ...

6 Find the 100th term of the sequence with n th term $5n - 8$

7 Is 150 a term of $6n + 4$? Give a reason.

8 Which term of $9n - 2$ is equal to 250?

9 Find the first term of $7n + 5$ that is greater than 200.

10 First term 3, rule 'double and add 1'. Write the next three terms.

11 Write the next term: 1, 8, 27, 64, ...

12 Write the next term: 1, 1, 2, 3, 5, 8, ...

13 (Higher) An arithmetic sequence has $a = 5$ and $d = 4$. Find the 10th term.

14 (Higher) Find the sum of the first 20 terms of $3 + 7 + 11 + \dots$

15 (Higher) Find the sum of the first 50 positive integers.

16 (Higher) Find the sum of the first 15 terms of $40 + 37 + 34 + \dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 Here are the first three patterns in a sequence made from sticks.



Pattern 1



Pattern 2



Pattern 3

- (a) How many sticks are in Pattern 10? [1]

Answer

- (b) Find an expression, in terms of n , for the number of sticks in Pattern n . [2]

Answer

- (c) Kemi has 60 sticks. What is the largest pattern number she can make? How many sticks are left over? [2]

Answer Pattern, left

- 2 The first four terms of an arithmetic sequence are 5, 12, 19, 26

- (a) Find an expression for the n th term. [2]

Answer

- (b) Show that 300 is not a term of the sequence. [2]

- (c) Find the 40th term. [1]

Answer

- 3 Sequence A is 3, 8, 13, 18, ... Sequence B is 101, 98, 95, 92, ... Find the first term number for which the term of A is greater than the term of B . [3]

4 (Higher) An arithmetic series has first term 7 and common difference 3.

(a) Find the 25th term.

[2]

Answer

(b) Find the sum of the first 25 terms.

[2]

Answer

(c) Find the least number of terms for which the sum is greater than 1000.

[3]

Answer

5 (Higher) The 4th term of an arithmetic series is 17 and the 10th term is 41.

(a) Find the first term and the common difference.

[3]

Answer $a = \dots\dots$ $d = \dots\dots$

(b) Find the sum of the first 30 terms.

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Function notation

I can... use function notation, domain and range, and find composite and inverse functions.

Spec ref: 3.2 · Higher

A function maps each input to exactly one output. The **domain** is the set of inputs; the **range** is the set of outputs. Exclude any value that makes you divide by zero or square-root a negative number.

$fg(x)$ means apply g first, then f . To find $f^{-1}(x)$, write $y = f(x)$, rearrange to make x the subject, then write the answer in terms of x .

Key words

function

domain

range

composite

inverse

Key points

- $fg(x) = f(g(x))$
- The range of $(x + a)^2 + b$ is $\geq b$.
- The graph of f^{-1} is the reflection of f in $y = x$.

Worked example 1

$f(x) = 4x + 1$ and $g(x) = x^2 - 2$. Find $fg(3)$.

$$g(3) = 9 - 2 = 7 \quad \cdot \quad f(7) = 28 + 1 = 29$$

Worked example 2

Find $h^{-1}(x)$ when $h(x) = 1/(x - 3)$.

$$y = 1/(x - 3), \text{ so } x - 3 = 1/y \quad \cdot \quad x = 1/y + 3 \quad \cdot \quad h^{-1}(x) = 1/x + 3$$

Common mistakes

- Applying f first in $fg(x)$.
- Thinking $f^{-1}(x) = 1/f(x)$.
- Giving the range using x instead of $f(x)$ or y .

Exam tip: Check an inverse with a number: $f(5) = 21$, so $f^{-1}(21)$ must be 5.

$$f(x) = 4x + 1, g(x) = x^2 - 2 \text{ and } h(x) = 1/(x - 3), x \neq 3.$$

1 Find $f(2)$.

2 Find $f(-3)$.

3 Find $g(4)$.

4 Find $g(-1)$.

5 Find $h(5)$.

6 Find $h(2.5)$.

7 Find $fg(3)$.

8 Find $gf(0)$.

9 Find $fg(x)$.

10 Find $gf(x)$.

11 Find $ff(x)$.

12 Find $f^{-1}(x)$.

13 Find $f^{-1}(21)$.

14 Find $h^{-1}(x)$.

15 Which value must be excluded from the domain of h ?

16 Write down the range of g .

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

1 $f(x) = 3x - 4$ $g(x) = 2/x, x \neq 0$

(a) Find $f(5)$. [1]**Answer****(b)** Find $g(-0.5)$. [1]**Answer****(c)** Find $fg(2)$. [2]**Answer****(d)** Find $gf(x)$. [1]**Answer****(e)** Which value of x must be excluded from any domain of gf ? [1]**Answer****(f)** Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$ [2]**Answer** $f^{-1}(x) = \dots$

2 $f(x) = (2x + 1)/(x - 3), x \neq 3$

(a) Find $f(4)$. [1]**Answer****(b)** Find $f^{-1}(x)$. [3]**Answer** $f^{-1}(x) = \dots$

3 $g(x) = x^2 + 6x + 4$

(a) Write $g(x)$ in the form $(x + a)^2 + b$. [2]

Answer

(b) Write down the range of g . [1]

Answer

(c) $f(x) = x - 3$. Show that $gf(x) = x^2 - 5$. [2]

4 $f(x) = 5 - x^2$ and $g(x) = 2x + 1$. Solve $fg(x) = -4$. [4]

5 $h(x) = \sqrt{x - 2}$

(a) Find $h(11)$. [1]

Answer

(b) State the domain of h . [1]

Answer

(c) Find $h^{-1}(x)$. [2]

Answer $h^{-1}(x) = \dots\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Straight-line graphs

I can... draw straight-line graphs and find gradients, midpoints and equations of lines.

Spec ref: 3.3 · Foundation & Higher

The equation $y = mx + c$ describes a straight line with gradient m and y -intercept $(0, c)$. To draw a line, make a table of values with at least three points.

Parallel lines have the same gradient. (Higher) If two lines are perpendicular, the product of their gradients is -1 . In a real-life graph, the gradient is a rate and the intercept is a starting or fixed value.

Key words

gradient

intercept

midpoint

parallel

perpendicular

Key points

- gradient = change in $y \div$ change in x
- midpoint = (mean of x -coordinates, mean of y -coordinates)
- (Higher) perpendicular gradient = $-1 \div$ gradient

Worked example 1

Find the equation of the line through $(1, 4)$ and $(3, 10)$.

$$m = (10 - 4) \div (3 - 1) = 3 \quad 4 = 3 \times 1 + c, \text{ so } c = 1 \quad y = 3x + 1$$

Worked example 2 (Higher)

Find the line perpendicular to $y = 2x + 1$ through $(4, 3)$.

$$\text{Perpendicular gradient} = -1/2 \quad 3 = -1/2 \times 4 + c, \text{ so } c = 5$$

$$y = -\frac{1}{2}x + 5$$

Common mistakes

- Using change in $x \div$ change in y .
- Reading the gradient of $4y - 8x = 12$ as -8 : rearrange first.
- Using the reciprocal without changing the sign for a perpendicular line.

Exam tip: Use a ruler and plot at least three points; if they do not line up, check your table.

Answer each question. Show your working where it helps.

1 Find the midpoint of (3, 8) and (9, 2).

2 Find the midpoint of (-5, 4) and (1, -6).

3 Find the gradient of the line through (1, -2) and (4, 10).

4 Find the gradient of the line through (-2, 7) and (6, 3).

5 Write down the gradient and y-intercept of $y = 6 - 2x$.

6 Find the gradient and y-intercept of $4y - 8x = 12$.

7 Write the equation of the line with gradient 5 that passes through (0, -4).

8 Find the equation of the line through (1, 4) and (3, 10).

9 Find the equation of the line parallel to $y = -3x + 7$ through (2, -1).

10 Does the point (-2, -7) lie on $y = 4x + 1$? Show why.

11 Where does $3x + 2y = 18$ cross the axes?

12 Find the length of the line from (2, 1) to (8, 9).

13 (Higher) Write down the gradient of a line perpendicular to $y = -4x + 3$.

14 (Higher) Find the line perpendicular to $y = 2x + 1$ through (4, 3).

15 (Higher) Are $y = 3x - 2$ and $6y + 2x = 5$ perpendicular? Explain.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

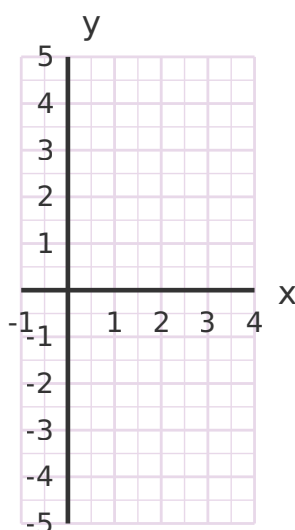
- 1 (a) Complete the table of values for $y = 2x - 3$.

x	-1	0	1	2	3	4
y		-3			3	

[2]

- (b) On the grid, draw the graph of $y = 2x - 3$ for $-1 \leq x \leq 4$.

[2]



- (c) Use your graph to find the value of x when $y = 4$.

[1]

Answer $x = \dots\dots\dots$

- 2 A is the point (1, 7) and B is the point (7, -2).

- (a) Find the coordinates of the midpoint of AB.

[2]

Answer (.....,

- (b) Find the gradient of AB.

[2]

Answer

- (c) Find an equation of the line AB. Give your answer in the form $ax + by = c$, where a , b and c are integers.

[2]

Answer

3 The line L_1 has equation $y = 4x - 5$.

(a) Find an equation of the line L_2 that is parallel to L_1 and passes through (3, 2). [2]

Answer $y = \dots\dots\dots$

(b) (Higher) Find an equation of the line L_3 that is perpendicular to L_1 and passes through (8, 1). [3]

Answer $y = \dots\dots\dots$

(c) Find the coordinates of the point where L_3 crosses the x-axis. [1]

Answer (.....,)

4 $P(-1, -1)$, $Q(2, 5)$ and $R(5, 11)$ are three points.

(a) Show that P , Q and R lie on a straight line. [3]

(b) Find the length of PQ . Give your answer correct to 3 significant figures. [2]

Answer

5 The cost, £ C , of using k units of electricity in a month is $C = 0.15k + 12$.

(a) Explain what the number 12 represents. [1]

(b) Explain what the number 0.15 represents. [1]

(c) Work out the cost of using 300 units. [1]

Answer £

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Curved graphs and real-life graphs

I can... plot curves, solve equations graphically and interpret real-life graphs.

Spec ref: 3.3 · Foundation & Higher

Plot curves from a table of values and join the points with a smooth curve. The solutions of $f(x) = 0$ are where the curve crosses the x -axis; the solutions of $f(x) = k$ are where it meets the line $y = k$.

On a distance–time graph the gradient is the speed and a horizontal section means no movement. (Higher) The gradient of a curve at a point is found by drawing a tangent.

Key words

quadratic

cubic

reciprocal

turning point

tangent

distance–time

Key points

- Quadratic: U or n with a line of symmetry through the turning point.
- Cubic: S-shape. (Higher) Reciprocal $y = k/x$: two branches, never touches the axes.
- average speed = total distance $\div$ total time

Worked example

The graph of $y = x^2 - 4x + 1$ is drawn. Which line solves $x^2 - 5x + 3 = 0$?

Add $x - 2$ to both sides: $x^2 - 4x + 1 = x - 2$

Draw $y = x - 2$ and read the x -coordinates of the intersections.

Worked example 2

A car travels 150 km in 2 hours 30 minutes. Find its average speed.

2 hours 30 minutes = 2.5 hours · $150 \div 2.5 = 60$ km/h

Common mistakes

- Joining plotted points with straight lines.
- Using 2.30 hours for 2 hours 30 minutes.

Exam tip: Label each graph you draw, and draw extra lines long enough to cross the curve.

Answer each question. Show your working where it helps.

1 For $y = x^2 - 4x + 1$, find y when $x = -1$.

2 For $y = 2x^2 - 3$, find y when $x = -2$.

3 For $y = x^3 - 2x$, find y when $x = 2$.

4 For $y = x^3 - 2x$, find y when $x = -1.5$.

5 For $y = 6/x$, find y when $x = -2$.

6 Where does $y = x^2 - 7x + 10$ cross the x -axis?

7 Write down the y -intercept of $y = (x + 2)(x - 6)$.

8 Find the line of symmetry of $y = x^2 + 8x - 1$.

9 Find the turning point of $y = x^2 + 8x - 1$.

10 Describe the shape of the graph of $y = -x^2$.

11 (Higher) Write down the asymptotes of $y = 4/x$.

12 The graph of $y = x^2 - 4x + 1$ is drawn. Which line should be drawn to solve $x^2 - 4x + 1 = 2$?

13 (Higher) The graph of $y = x^2 - 4x + 1$ is drawn. Which line should be drawn to solve $x^2 - 5x + 3 = 0$?

14 A car travels 150 km in 2 hours 30 minutes. Work out its average speed.

15 A distance–time graph is a straight line from (0 min, 0 km) to (20 min, 8 km). Find the speed in km/h.

16 (Higher) A tangent to a curve at P passes through (0, -3) and (2, 5). Estimate the gradient of the curve at P .

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

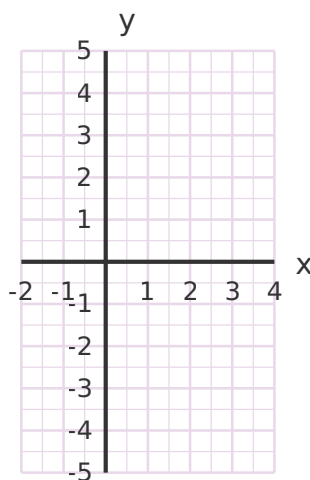
- 1 (a) Complete the table of values for $y = 3 + 2x - x^2$.

x	-2	-1	0	1	2	3	4
y	-5			4	3		-5

[2]

- (b) On the grid, draw the graph of $y = 3 + 2x - x^2$ for $-2 \leq x \leq 4$.

[3]



- (c) Use your graph to solve $3 + 2x - x^2 = 0$.

[2]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- (d) (Higher) By drawing a suitable straight line on your graph, find estimates for the solutions of $x^2 - 3x - 1 = 0$.

[3]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- 2 The graph of $y = x^2 + bx + c$ passes through (0, 6) and (2, 0).

- (a) Find the values of b and c .

[2]

Answer $b = \dots\dots\dots$ $c = \dots\dots\dots$

- (b) Find the coordinates of the other point where the graph crosses the x-axis.

[1]

Answer (.....,)

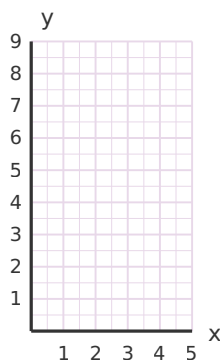
- 3 (Higher) (a) Complete the table of values for $y = x + 4/x$.

x	0.5	1	2	3	4	5
y	8.5		4		5	

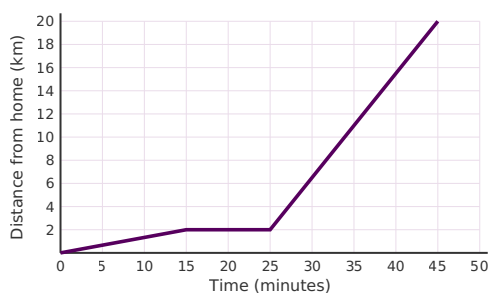
[2]

- (b) On the grid, draw the graph of $y = x + 4/x$ for $0.5 \leq x \leq 5$.

[3]



- 4 Priya walks from home to the station, waits, and then travels by train. The distance–time graph shows her journey.



- (a) Work out Priya's walking speed in km/h.

[2]

Answer km/h

- (b) Describe what happens between 15 and 25 minutes.

[1]

Answer

- (c) Work out the average speed of the train in km/h.

[2]

Answer km/h

- (d) Work out Priya's average speed for the whole journey.

[2]

Answer km/h

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

I can... differentiate integer powers of x and use calculus for gradients, turning points and kinematics.

Spec ref: 3.4 · Higher

Differentiation gives the gradient function. If $y = ax^n$, then $dy/dx = anx^{n-1}$. Expand brackets and write $8/x$ as $8x^{-1}$ first.

Turning points have gradient 0. In kinematics, if s is displacement, then velocity $v = ds/dt$ and acceleration $a = dv/dt$. A particle is **at rest** when $v = 0$.

Key words

differentiate

gradient

turning point

velocity

acceleration

Key points

- $d/dx(k) = 0$ and $d/dx(kx) = k$
- Turning points: solve $dy/dx = 0$, then find y .
- Maximise or minimise a quantity by setting its derivative equal to 0.

Worked example

$s = t^3 - 9t^2 + 24t$. Find when the particle is at rest.

$$v = ds/dt = 3t^2 - 18t + 24 \quad \cdot \quad 3t^2 - 18t + 24 = 0, \text{ so } t^2 - 6t + 8 = 0$$

$$(t - 2)(t - 4) = 0, \text{ so } t = 2 \text{ s and } t = 4 \text{ s}$$

Worked example 2

Differentiate $y = 8/x$.

$$y = 8x^{-1} \quad \cdot \quad dy/dx = -8x^{-2} = -8/x^2$$

Common mistakes

- Reducing the power of a negative index the wrong way: x^{-1} becomes x^{-2} , not x^0 .
- Forgetting the y -coordinate of a turning point.
- Mixing up displacement, velocity and acceleration.

Exam tip: If a question says 'use calculus', you must show the derivative; reading from a graph gets no marks.

Differentiate, then answer each question.

1 Find dy/dx when $y = x^7$

2 Find dy/dx when $y = 5x^3$

3 Find dy/dx when $y = 4x^2 - 9x + 2$

4 Find dy/dx when $y = x^4 - 3x^3 + 6$

5 Find dy/dx when $y = 8/x$

6 Find dy/dx when $y = 3/(2x^3)$

7 Find dy/dx when $y = (x - 4)(x + 1)$

8 Find dy/dx when $y = x^2(2x + 5)$

9 Find the gradient of $y = 3x^2 - 5x$ at $x = 2$.

10 Find the gradient of $y = 2x^3$ at $x = -1$.

11 Find the turning point of $y = 5 + 4x - x^2$.

12 Find the turning points of $y = x^3 - 3x$.

13 $s = 4t^3 - t$. Find the velocity $v = ds/dt$.

14 Using your answer, find v when $t = 2$.

15 Find the acceleration $a = dv/dt$ at $t = 0.5$.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** A curve has equation $y = x^3 - 6x^2 + 5$.

(a) Find dy/dx .

[2]

Answer

(b) Find the coordinates of the turning points of the curve.

[4]

Answer (.....,) and (.....,)

(c) State which turning point is a maximum. Give a reason.

[1]

- 2** A particle moves along a straight line. Its displacement, s metres, from a fixed point O after t seconds is $s = t^3 - 9t^2 + 24t$.

(a) Find an expression for the velocity, v m/s, at time t .

[2]

Answer $v =$

(b) Find the times when the particle is at rest.

[3]

Answer $t =$ and $t =$

(c) Find the acceleration when $t = 5$.

[2]

Answer m/s²

3 A curve has equation $y = 2x^2 - 8x + 3$.

(a) Find the coordinates of the point on the curve where the gradient is 4.

[3]

Answer (.....,)

(b) Find the equation of the tangent to the curve at this point.

[2]

Answer $y = \dots\dots\dots$

4 A farmer uses 40 m of fencing to make three sides of a rectangular pen against a wall. The two sides at right angles to the wall are each x m long.

(a) Show that the area, $A \text{ m}^2$, of the pen is $A = 40x - 2x^2$.

[1]

(b) Use calculus to find the maximum area of the pen.

[4]

Answer m^2

5 Find the coordinates of the turning points of the curve $y = x + 16/x$.

[4]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

End-of-book check

End-of-book check

Name _____ Date _____ Score _____ / 12

Answer these questions to check your progress.

1 Factorise $x^2 + 2x - 35$

2 (Higher) Solve $2x^2 + 7x + 3 = 0$

3 Expand and simplify $(2x - 3)(x + 4)$

4 Simplify $(2a^3)^4$

5 Make t the subject of $v = u + at$

6 Solve $(x + 1)/4 = (x - 2)/3$

7 (Higher) Find the sum of the first 10 terms of $4 + 9 + 14 + \dots$

8 (Higher) $y \propto x^2$. $y = 20$ when $x = 2$. Find y when $x = 3$.

9 Find the equation of the line through $(0, 3)$ parallel to $y = 2x - 7$

10 (Higher) $f(x) = x^2 - 5$. Find $f(-3)$.

11 (Higher) Find dy/dx when $y = 2x^4 - 3x^2$

12 (Higher) Solve $x^2 - 9 < 0$



Practice paper

Name _____ Date _____ Marks _____ / 60

Time: 1 hour 15 minutes
Calculator allowed
Total: 60 marks
Foundation and Higher content

Answer ALL questions. Show all stages in your working. Without sufficient working, correct answers may be awarded no marks.

1

(a) Simplify $3x^2 \times 4x^5$.

[1]

Answer

(b) Expand and simplify $(x + 7)(x - 2)$.

[2]

Answer

(c) Factorise fully $6a^2b - 9ab^2$.

[2]

Answer

2 Solve.

(a) $4(3x - 1) = 2(x + 8)$

[3]

Answer $x =$

(b) $(5 - x)/2 = x + 4$

[3]

Answer $x =$

3 $P = (3a + b)/c$

(a) Work out the value of P when $a = 4.5$, $b = -2.3$ and $c = 0.4$.

[2]

Answer $P = \dots\dots\dots$

(b) Make a the subject of the formula.

[3]

Answer $a = \dots\dots\dots$

4 Here are the first four terms of an arithmetic sequence: 2, 9, 16, 23

(a) Find an expression for the n th term.

[2]

Answer $\dots\dots\dots$

(b) Is 120 a term of the sequence? Give a reason.

[2]

Answer $\dots\dots\dots$

(c) (Higher) Find the sum of the first 20 terms.

[2]

Answer $\dots\dots\dots$

5 Solve the simultaneous equations.

$$3x + 5y = 1$$

$$4x - 3y = 40$$

[4]

6 (a) Solve $7 - 3x > 19$.

[2]

Answer $\dots\dots\dots$

(b) (Higher) Solve $x^2 - 2x - 15 \leq 0$.

[3]

Answer $\dots\dots\dots$

- 7 (Higher) Solve $3x^2 - 4x - 2 = 0$. Give your solutions correct to 3 significant figures. [3]

- 8 (Higher) T is inversely proportional to the square root of L . When $L = 25$, $T = 6$. Find the value of T when $L = 16$. [4]

- 9 (Higher) Simplify fully $(x^2 - 4x)/(x^2 - 16)$. [3]

- 10 (Higher) $f(x) = 3x + 2$ and $g(x) = x/(x - 1)$, $x \neq 1$

(a) Find $gf(1)$. [2]

Answer

(b) Find $g^{-1}(x)$. [3]

Answer $g^{-1}(x) = \dots\dots\dots$

- 11 (Higher) A is the point $(-2, 5)$ and B is the point $(4, 1)$. Find an equation of the perpendicular bisector of AB . [4]

- 12** (Higher) A particle moves so that its displacement, s metres, after t seconds is $s = 2t^3 - 15t^2 + 36t$.

(a) Find an expression for the velocity of the particle.

[2]

Answer $v = \dots\dots\dots$

(b) Find the times when the particle is at rest.

[3]

Answer $t = \dots\dots\dots$ and $t = \dots\dots\dots$

(c) Find the acceleration when $t = 4$.

[2]

Answer $\dots\dots\dots \text{ m/s}^2$

- 13** (Higher) Write $x^2 - 10x + 18$ in the form $(x - p)^2 - q$, and hence write down the coordinates of the turning point of $y = x^2 - 10x + 18$.

[3]



FOR TEACHERS & PARENTS

Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

Starting point check

- 1 $11p + 3$
- 2 8
- 3 $18x + 12$
- 4 $4(3y - 2)$
- 5 $x = 7$
- 6 $x = 3$
- 7 $4n + 5$
- 8 y^6
- 9 (5, 7)
- 10 $x \geq 3$
- 11 $x^2 + 8x + 15$
- 12 $x = 4, y = 2$

Unit 1 · Sheet A — Expressions, substitution and index laws

- 1 $5m - 3n$
- 2 $7y^2 + 7y$
- 3 $30a^2b$
- 4 $3x^2$
- 5 m^{10}
- 6 p^9
- 7 k^{10}
- 8 $9a^4b^2$
- 9 $12x^3$
- 10 x^{-3}
- 11 $3y^2$
- 12 $4x^6$
- 13 -26
- 14 -15
- 15 16
- 16 5
- 17 63.6
- 18 $C = 45n + 80$
- 19 $4x + 12$

Unit 1 · Sheet B — Expressions, substitution and index laws

- 1 (a) 28pq B1 [1] (b) $3a^3b^2$ B2 (B1 for two of 3, a^3 , b^2) [2] (c) 7 B1 [1] (d) $16x^{12}y^{-4}$ or $16x^{12}/y^4$ B2 (B1 for two parts correct) [2]
- 2 (a) $v^2 = 9 + 40 = 49$ M1; $v = 7$ A1 [2] (b) $100 = 36 + 4s$ M1; $4s = 64$ M1; $s = 16$ A1 [3]
- 3 (a) $T = 3.2x + 4.5y$ B2 (B1 for $3.2x + 4.5y$ with no $T =$) [2] (b) £29.50 B1 [1]
- 4 (a) $4 \times 1/2$ M1; 2 A1 [2] (b) $4^x + 2 = 2^{2x} + 4$ M1; $3x = 2x + 4$ M1; $x = 4$ A1 [3] (c) $1/(3x^2)$ or $(1/3)x^{-2}$ B2 (B1 for $3x^2$ or reciprocal seen) [2]
- 5 (a) $30 \times 4.5 + 15$ M1; 150 minutes A1; 2 hours 30 minutes A1 [3] (b) $30w + 15 = 210$ M1; $w = 6.5$ kg A1 [2]
- 6 (a) xy B1 [1] (b) x^2 B1 [1] (c) $2^p \times 2 \div 2^q$ M1; $2x/y$ A1 [2]

Unit 2 · Sheet A — Expanding brackets

- 1 $15x + 20$
- 2 $14 - 8y$
- 3 $6p^2 + 15p$
- 4 $4x^3 - 12x^2$
- 5 $2x + 18$
- 6 $3x^2 + x$
- 7 $x^2 + 8x + 12$
- 8 $y^2 - 13y + 40$
- 9 $x^2 + 5x - 36$
- 10 $3x^2 + 16x + 5$
- 11 $6x^2 - 17x - 14$
- 12 $x^2 - 12x + 36$
- 13 $9x^2 + 24x + 16$
- 14 $25 - 4x^2$
- 15 $x^3 + 3x^2 - 10x - 24$
- 16 $8x^3 - 12x^2 + 6x - 1$
- 17 $3x^3 + 4x^2 - x - 2$

Unit 2 · Sheet B — Expanding brackets

- 1 (a)** $14x + 21 - 12x + 3$ M1; $2x + 24$ A1 **[2]** **(b)** 4 terms with 3 correct M1; $m^2 - 5m - 36$ A1 **[2]** **(c)** $8y^2 + 20y - 6y - 15$ M1; $8y^2 + 14y - 15$ A1 **[2]** **(d)** $(2x - 5)(2x - 5)$ M1; $4x^2 - 20x + 25$ A1 **[2]**
- 2** $(x + 4)^2 - x^2$ M1; $x^2 + 8x + 16 - x^2$ M1; $8x + 16 = 8(x + 2)$ A1 **[3]**
- 3** two brackets expanded correctly M1; all terms of the product M1; $2x^3 + 5x^2 - 28x - 15$ A1 **[3]**
- 4** $n^2 + 6n + 9$ or $n^2 - 6n + 9$ M1; $12n$ A1; $12n = 12 \times n$, so a multiple of 12 A1 **[3]**
- 5 (a)** $a + 3 = 10$ or $3a = 21$ M1; $a = 7$ A1 **[2]** **(b)** $2q + p = 7$ B1; $pq = -15$ B1; $p = -3$, $q = 5$ B1 **[3]**
- 6 (a)** $(3x - 2)(x + 4)$ M1; $3x^2 + 10x - 8$ A1 **[2]** **(b)** 49 cm^2 B1 **[1]**

Unit 3 · Sheet A — Factorising expressions

- 1** $7(x - 3)$
- 2** $4y(3y + 2)$
- 3** $5a(3b - 2a)$
- 4** $2xy(2x + 3y)$
- 5** $(x + 2)(x + 7)$
- 6** $(x - 7)(x + 3)$
- 7** $(x - 8)(x - 3)$
- 8** $(y - 9)(y + 9)$
- 9** $(3x - 4)(3x + 4)$
- 10** $(5 - x)(5 + x)$
- 11** $5x(x - 4)$
- 12** $2(x - 4)(x + 4)$
- 13** $x(x - 3)(x + 3)$
- 14** $(x + 3)(3x + 2)$
- 15** $(2x - 5)(2x + 3)$
- 16** $(2x - 3)(3x - 2)$

Unit 3 · Sheet B — Factorising expressions

- 1 (a)** $3(3p - 4)$ B1 **[1]** **(b)** $5x^2y(2x - 3y)$ B2 (B1 for partial) **[2]** **(c)** $(y - 5)(y + 7)$ B2 (B1 for $(y \pm 7)(y \pm 5)$) **[2]**
- 2 (a)** $(x - 4)(3x - 2)$ B2 **[2]** **(b)** $x = 2/3$ or $x = 4$ B1ft **[1]**
- 3 (a)** $(1001 - 999)(1001 + 999)$ M1; $2 \times 2000 = 4000$ A1 **[2]** **(b)** $2(x^2 - 25)$ M1; $2(x - 5)(x + 5)$ A1 **[2]** **(c)** $(x + 1 - 4)(x + 1 + 4)$ M1; $(x - 3)(x + 5)$ A1 **[2]**
- 4** $(2n + 2)^2 - (2n)^2$ M1; $8n + 4$ A1; $= 4(2n + 1)$, a multiple of 4 A1 **[3]**
- 5 (a)** $(x + 2)(2x + 3)$ B2 **[2]** **(b)** $2x + 3$ B1ft **[1]** **(c)** $2(x + 2) + 2(2x + 3)$ M1; $6x + 10$ A1 **[2]**
- 6** $x(x + 5)$ B1; $(x + 5)(x - 5)$ B1; $x/(x - 5)$ B1 **[3]**

Unit 4 · Sheet A — Algebraic fractions

- 1** $2x/3$
- 2** $2x/(3y^2)$
- 3** $3/(x - 2)$
- 4** $(x + 2)/(x + 3)$
- 5** $(2x + 3)/(x + 1)$
- 6** $7x/10$
- 7** $5x/12$
- 8** $(5x + 7)/6$
- 9** $5/(2x)$
- 10** $(3x + 1)/((x + 3)(x - 1))$
- 11** $(2x + 11)/((x - 2)(x + 1))$
- 12** $6/x$
- 13** $3/(2x)$
- 14** $2(x - 1)/x$

Unit 4 · Sheet B — Algebraic fractions

- 1 (a)** $(x + 5)(x - 3)$ B1; $(2x + 1)(x - 3)$ B1; $(x + 5)/(2x + 1)$ B1 **[3]** **(b)** $3x(2x - 3)$ B1; $(2x - 3)(2x + 3)$ B1; $3x/(2x + 3)$ B1 **[3]**
- 2 (a)** common denominator $(x + 2)(x - 5)$ M1; $3(x - 5) + 4(x + 2)$ M1; $(7x - 7)/((x + 2)(x - 5))$ or $7(x - 1)/((x + 2)(x - 5))$ A1 **[3]** **(b)** $2x(x - 2) - (x - 1)(x + 3)$ M1; $x^2 - 6x + 3$ A1; $(x^2 - 6x + 3)/((x + 3)(x - 2))$ A1 **[3]**
- 3** $3(x + 2) - 2(x - 3) = 24$ M1; $x + 12 = 24$ M1; $x = 12$ A1 **[3]**
- 4** $4(x - 2) + 3(x + 1) = 2(x + 1)(x - 2)$ M1; $7x - 5 = 2x^2 - 2x - 4$ M1; $2x^2 - 9x + 1 = 0$ A1; $(9 \pm \sqrt{73})/4$ M1; 4.39, 0.114 A1 **[5]**
- 5** $(x - 3)(x + 3)$ and $(x + 1)(x + 3)$ B1; multiply by $2(x + 1)/(x - 3)$ M1; all factors cancel to 2 A1 **[3]**
- 6 (a)** $(x + 1 - (x - 1))/((x - 1)(x + 1))$ M1; $= 2/(x^2 - 1)$ A1 **[2]** **(b)** $x^2 - 1 = 24$ M1; $x = 5$ or $x = -5$ A1 **[2]**

Unit 5 · Sheet A — Linear equations and rearranging formulae

- 1** $x = 7$
- 2** $x = 7.5$
- 3** $x = 5$
- 4** $x = -9.5$
- 5** $x = 30$
- 6** $x = 10$
- 7** $x = 22$
- 8** $x = 2$
- 9** $x = 3$
- 10** $x = 2.5$
- 11** $a = (P - 3b)/2$
- 12** $m = (y - c)/x$
- 13** $h = 2A/b$
- 14** $c = \sqrt{(E/m)}$
- 15** $h = \sqrt{(\pi r^2)}$
- 16** $v = 2s/t - u$
- 17** $x = ab/(a - c)$
- 18** $x = 3y^2 - 1$

Unit 5 • Sheet B — Linear equations and rearranging formulae

1 (a) $10y - 15 = 3y + 13$ M1; $7y = 28$ M1; $y = 4$ A1 **[3]** (b) $x \times 12$ M1; $4(x + 5) - 3(x - 2) = 24$ M1; $x + 26 = 24$ M1; $x = -2$ A1 **[4]**

2 $2(3x + 1) + 2(x + 5) = 60$ M1; $8x + 12 = 60$ M1; $x = 6$ A1; area = $19 \times 11 = 209 \text{ cm}^2$ A1 **[4]**

3 $x(y + 4) = 3y - 2$ M1; $xy - 3y = -2 - 4x$ M1; $y(x - 3) = -2 - 4x$ M1; $y = (4x + 2)/(3 - x)$ oe A1 **[4]**

4 (a) $r^2 = A/(4\pi)$ M1; $r = \sqrt{A/(4\pi)}$ A1 **[2]** (b) $\sqrt{(500 \div 4\pi)}$ M1; 6.31 cm A1 **[2]**

5 $n + (n + 2) + (n + 4) = 111$ M1; $3n + 6 = 111$, $n = 35$ A1; 35, 37, 39 A1 **[3]**

6 $x + 20 = 2(x - 5)$ M1 A1; $x = 30$ so £30 A1 **[3]**

Unit 6 • Sheet A — Simultaneous equations

1 $x = 7$, $y = 5$

2 $x = 4$, $y = 2$

3 $x = 2$, $y = 4$

4 $x = 4$, $y = 3$

5 $x = 2$, $y = 1$

6 $x = 2$, $y = -1$

7 $x = 3$, $y = 7$

8 $p = 4$, $q = -1$

9 $x = 4$, $y = -1$

10 $x = -4$, $y = -3$; $x = 3$, $y = 4$

11 $x = -1$, $y = -1$; $x = 4$, $y = 14$

12 $x = -3$, $y = -4$; $x = 4$, $y = 3$

13 pencil 20p, ruler 30p

Unit 6 • Sheet B — Simultaneous equations

1 multiply to match coefficients M1; $13x = 26$ M1; $x = 2$ A1; $y = 1/2$ A1 **[4]**

2 (a) $4c + 3k = 18.60$ B1; $2c + 5k = 19.80$ B1 **[2]** (b) eliminate c: $7k = 21$ M1; $k = 3.00$ A1; $c = 2.40$ A1 **[3]**

3 $x^2 - 4x + 5 = 2x - 3$ M1; $x^2 - 6x + 8 = 0$ A1; $(x - 2)(x - 4) = 0$ M1; $x = 2$, $y = 1$; $x = 4$, $y = 5$ A1 **[4]**

4 (a) $2x + y = 8$ correct B1; $y = x + 2$ correct B1 **[2]** (b) $x = 2$, $y = 4$ B1ft **[1]**

5 $x + y = 46$ B1; $3y = 2x + 8$ B1; elimination or substitution M1; larger 26, smaller 20 A1 **[4]**

6 $x^2 + (x - 1)^2 = 41$ M1; $2x^2 - 2x - 40 = 0$ A1; $(x - 5)(x + 4) = 0$ M1; $x = 5$, $y = 4$ A1; $x = -4$, $y = -5$ A1 **[5]**

Unit 7 • Sheet A — Quadratic equations

1 $x = -5$ or $x = -3$

2 $x = -5$ or $x = 8$

3 $x = -6$ or $x = 0$

4 $x = -11$ or $x = 11$

5 $x = 6$

6 $x = -2$ or $x = 1/3$

7 $x = -3/2$ or $x = 3/2$

8 $x = 5/2$ or $x = 3$

9 $x = -0.646$ or $x = 4.65$

10 $x = -2.18$ or $x = -0.153$

11 $x = -1.27$ or $x = 2.77$

12 $(x + 5)^2 - 18$

13 $(x - 3/2)^2 - 5/4$

14 $3(x - 2)^2 + 8$

15 $x = 4 \pm \sqrt{5}$

16 $x = -3$ or $x = 7$

17 $x = -5$ or $x = 3$

Unit 7 • Sheet B — Quadratic equations

1 (a) $(x - 4)(x + 9)$ B2 **[2]** (b) $x = -9$ or $x = 4$ B1ft **[1]**

2 $(-6 \pm \sqrt{(36 + 40)})/4$ M2 (M1 for one error); 0.679 and -3.68 A1 **[3]**

3 (a) $2(x^2 - 4x)$ M1; $2(x - 2)^2 - 8 + 3$ M1; $2(x - 2)^2 - 5$ A1 **[3]** (b) $(2, -5)$ B1ft **[1]** (c) $(x - 2)^2 = 5/2$ M1; $x = 2 \pm \sqrt{(5/2)}$ or $2 \pm (\sqrt{10})/2$ A1 **[2]**

4 (a) $x^2 + (x + 7)^2 = (x + 8)^2$ M1; $x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$ M1; $x^2 - 2x - 15 = 0$ A1 **[3]** (b) $(x - 5)(x + 3) = 0$ M1; $x = 5$ A1; $5 + 12 + 13 = 30$ cm A1 **[3]**

5 $3(x + 2) - 2(x - 1) = (x - 1)(x + 2)$ M1; $x + 8 = x^2 + x - 2$ M1; $x^2 = 10$ A1; $x = \pm 3.16$ A1 **[4]**

6 $n(n + 1) = 132$ M1; $n^2 + n - 132 = 0$, $(n + 12)(n - 11) = 0$ M1; 11 and 12 A1 **[3]**

Unit 8 • Sheet A — Inequalities

1 $x < 6$

2 $x \geq -2$

3 $x \geq 3$

4 $x > 4.5$

5 $x \geq 7$

6 $-2 \leq x < 4$

7 $-3, -2, -1, 0, 1$

8 3, 4, 5, 6, 7

9 6

10 5

11 $-4 \leq x \leq 4$

12 $x < -7$ or $x > 7$

13 $-5 < x < 2$

14 $x \leq -4$ or $x \geq 2$

Unit 8 · Sheet B — Inequalities

- 1** (a) $3x - 12 < x + 6$ M1; $x < 9$ A1 [2] (b) $-2, -1, 0, 1, 2$ B2 (B1 for one error) [2] (c) open circle at 1 B1; filled circle at 4 with line between B1 [2]
- 2** $(2x + 1)(x - 3)$ M1; critical values $-1/2$ and 3 A1; $x < -1/2$ or $x > 3$ A1 [3]
- 3** (a) $x = 1$ B1; $y = x$ B1; $x + y = 6$ B1; correct region labelled R B1 [4] (b) 9 B1ft [1]
- 4** $x \geq -1$ B1; $y \leq 3$ B1; $y \geq 2x - 1$ B1 [3]
- 5** $30 \leq 4x + 10 < 50$ M1; $20 \leq 4x < 40$ M1; $5 \leq x < 10$ A1 [3]
- 6** (a) $(x - 5)(x + 2) < 0$ M1; critical values $-2, 5$ A1; $-2 < x < 5$ A1 [3] (b) $x > 1.5$ M1; $2, 3, 4$ A1 [2]

Unit 9 · Sheet A — Direct and inverse proportion

- 1** 45
- 2** 8
- 3** $y = 3x^2$
- 4** 75
- 5** 16
- 6** 30
- 7** 2
- 8** 4
- 9** 6
- 10** multiplied by 9
- 11** multiplied by 4
- 12** 125
- 13** 15 days

Unit 9 · Sheet B — Direct and inverse proportion

- 1** (a) $m = kr^3$ M1; $270 = 27k$ M1; $m = 10r^3$ A1 [3] (b) 33.75 g B1ft [1] (c) $r^3 = 64$ M1; $r = 4$ A1 [2]
- 2** (a) $l = k/d^2$ M1; $50 = k/4$ M1; $l = 200/d^2$ A1 [3] (b) $d^2 = 25$ M1; $d = 5$ A1 [2]
- 3** (a) $y = k\sqrt{x}$ M1; $k = 5$ A1; $y = 35$ A1 [3] (b) $\sqrt{x} = 8$ M1; $x = 64$ A1 [2]
- 4** (a) $k = 120$ M1; 15 hours A1 [2] (b) $120 \div 10$ M1; 12 A1 [2]
- 5** (a) $1.2^2 = 1.44$ M1; 44% A1 [2] (b) w is divided by 8 B1 [1]
- 6** 15 B1; 10 B1 [2]

Unit 10 · Sheet A — Sequences and arithmetic series

- 1** 24, 29
- 2** $4n + 3$
- 3** $6n - 4$
- 4** $33 - 3n$
- 5** $4n - 9$
- 6** 492
- 7** No: $6n = 146$ gives n not a whole number
- 8** 28th
- 9** 201
- 10** 7, 15, 31
- 11** 125
- 12** 13
- 13** 41
- 14** 820
- 15** 1275
- 16** 285

Unit 10 · Sheet B — Sequences and arithmetic series

- 1** (a) 39 B1 [1] (b) $4n - 1$ B2 (B1 for $4n + k$) [2] (c) $4n - 1 \leq 60$ M1; Pattern 15, 1 left over A1 [2]
- 2** (a) $7n - 2$ B2 (B1 for $7n + k$) [2] (b) $7n - 2 = 300$ M1; $n = 43.1...$, not an integer A1 [2] (c) 278 B1ft [1]
- 3** $5n - 2$ and $104 - 3n$ B1; $5n - 2 > 104 - 3n$ M1; $n = 14$ A1 [3]
- 4** (a) $7 + 24 \times 3$ M1; 79 A1 [2] (b) $25/2 \times (7 + 79)$ M1; 1075 A1 [2] (c) $n/2(14 + 3(n - 1)) > 1000$ M1; trial or solve $n(3n + 11) > 2000$ M1; $n = 25$ A1 [3]
- 5** (a) $a + 3d = 17$ and $a + 9d = 41$ M1; $d = 4$ A1; $a = 5$ A1 [3] (b) $15(10 + 29 \times 4)$ M1; 1890 A1 [2]

Unit 11 · Sheet A — Function notation

- 1** 9
- 2** -11
- 3** 14
- 4** -1
- 5** $1/2$
- 6** -2
- 7** 29
- 8** -1
- 9** $4x^2 - 7$
- 10** $16x^2 + 8x - 1$
- 11** $16x + 5$
- 12** $(x - 1)/4$
- 13** 5
- 14** $1/x + 3$ or $(3x + 1)/x$
- 15** $x = 3$
- 16** $g(x) \geq -2$

Unit 11 · Sheet B — Function notation

- 1** (a) 11 B1 [1] (b) -4 B1 [1] (c) $g(2) = 1$ M1; -1 A1 [2] (d) $2/(3x - 4)$ B1 [1] (e) $4/3$ B1ft [1] (f) $x = 3y - 4$ rearranged M1; $(x + 4)/3$ A1 [2]
- 2** (a) 9 B1 [1] (b) $y(x - 3) = 2x + 1$ or $x(y - 3) = 2y + 1$ M1; collect terms and factorise M1; $(3x + 1)/(x - 2)$ A1 [3]
- 3** (a) $(x + 3)^2 - 5$ B2 (B1 for $a = 3$ or $b = -5$) [2] (b) $g(x) \geq -5$ B1ft [1] (c) $(x - 3)^2 + 6(x - 3) + 4$ M1; $x^2 - 5$ A1 [2]
- 4** $fg(x) = 5 - (2x + 1)^2$ B1; $(2x + 1)^2 = 9$ M1; $2x + 1 = \pm 3$ M1; $x = 1$ or $x = -2$ A1 [4]
- 5** (a) 3 B1 [1] (b) $x \geq 2$ B1 [1] (c) $x = \sqrt{y - 2}$ so $y = x^2 + 2$ M1; $h^{-1}(x) = x^2 + 2$ ($x \geq 0$) A1 [2]

Unit 12 · Sheet A — Straight-line graphs

- 1** (6, 5)
- 2** (-2, -1)
- 3** 4
- 4** $-1/2$
- 5** -2 and (0, 6)
- 6** 2 and (0, 3)
- 7** $y = 5x - 4$
- 8** $y = 3x + 1$
- 9** $y = -3x + 5$
- 10** Yes: $4(-2) + 1 = -7$
- 11** (6, 0) and (0, 9)
- 12** 10
- 13** $1/4$
- 14** $y = -x/2 + 5$
- 15** Yes: $3 \times (-1/3) = -1$

Unit 12 · Sheet B — Straight-line graphs

- 1** -5, -1, 1, 5 B2 (B1 for 2 or 3 correct) [2] (b) correct line from (-1, -5) to (4, 5) B2 (B1 for 3 correct points plotted) [2] (c) 3.5 B1ft [1]
- 2** (a) (4, 2.5) B2 (B1 for one coordinate) [2] (b) $(-2 - 7)/(7 - 1)$ M1; $-3/2$ A1 [2] (c) $y - 7 = -3/2(x - 1)$ M1; $3x + 2y = 17$ A1 [2]
- 3** (a) $2 = 4 \times 3 + c$ M1; $y = 4x - 10$ A1 [2] (b) gradient $-1/4$ B1; $1 = -1/4 \times 8 + c$ M1; $y = -x/4 + 3$ A1 [3] (c) (12, 0) B1ft [1]
- 4** (a) gradient PQ = 2 B1; gradient QR = 2 B1; equal gradients and common point Q, so collinear A1 [3] (b) $\sqrt{3^2 + 6^2}$ M1; 6.71 A1 [2]
- 5** (a) fixed (standing) charge of £12 B1 [1] (b) cost per unit, 15p (the gradient) B1 [1] (c) £57.00 B1 [1]

Unit 13 · Sheet A — Curved graphs and real-life graphs

- 1** 6
- 2** 5
- 3** 4
- 4** -0.375
- 5** -3
- 6** (2, 0) and (5, 0)
- 7** (0, -12)
- 8** $x = -4$
- 9** (-4, -17)
- 10** n shape with a maximum at (0, 0)
- 11** $x = 0$ and $y = 0$
- 12** $y = 2$
- 13** $y = x - 2$
- 14** 60 km/h
- 15** 24 km/h
- 16** 4

Unit 13 · Sheet B — Curved graphs and real-life graphs

- 1** 0, 3, 0 B2 (B1 for 2 correct) [2] (b) all points plotted B2ft (B1 for 5 correct); smooth curve B1 [3] (c) $x = -1$ B1; $x = 3$ B1 [2] (d) $y = 2 - x$ B1; line drawn B1; $x = -0.3$ and $x = 3.3$ B1 (accept ± 0.1) [3]
- 2** (a) $c = 6$ B1; $4 + 2b + 6 = 0$ so $b = -5$ B1 [2] (b) (3, 0) B1ft [1]
- 3** 5, 4.3, 5.8 B2 (B1 for 2 correct) [2] (b) points plotted B2ft; smooth curve B1 [3]
- 4** (a) $2 \div (15/60)$ M1; 8 km/h A1 [2] (b) Priya waits (is stationary) for 10 minutes B1 [1] (c) $18 \div (20/60)$ M1; 54 km/h A1 [2] (d) $20 \div (45/60)$ M1; 26.7 km/h A1 [2]

Unit 14 · Sheet A — Calculus

- 1** $7x^6$
- 2** $15x^2$
- 3** $8x - 9$
- 4** $4x^3 - 9x^2$
- 5** $-8/x^2$
- 6** $-9/(2x^4)$
- 7** $2x - 3$
- 8** $6x^2 + 10x$
- 9** 7
- 10** 6
- 11** (2, 9)
- 12** (1, -2) and (-1, 2)
- 13** $12t^2 - 1$
- 14** 47
- 15** 12

Unit 14 · Sheet B — Calculus

1 (a) $3x^2 - 12x$ B2 (B1 for one term) **[2]** **(b)** $3x^2 - 12x = 0$ M1; $x = 0$ and $x = 4$ A1; (0, 5) A1; (4, -27) A1 **[4]** **(c)** (0, 5); the curve is a positive cubic so the first turning point is a maximum (or gradient changes + to -) B1 **[1]**

2 (a) $3t^2 - 18t + 24$ B2 **[2]** **(b)** $3t^2 - 18t + 24 = 0$ M1; $t^2 - 6t + 8 = 0$ M1; $t = 2$ and $t = 4$ A1 **[3]** **(c)** $6t - 18$ M1; 12 m/s^2 A1 **[2]**

3 (a) $4x - 8 = 4$ M1; $x = 3$ A1; (3, -3) A1 **[3]** **(b)** $y + 3 = 4(x - 3)$ M1; $y = 4x - 15$ A1 **[2]**

4 (a) length parallel to wall = $40 - 2x$, $A = x(40 - 2x)$ B1 **[1]** **(b)** $dA/dx = 40 - 4x$ M1; $40 - 4x = 0$ M1; $x = 10$ A1; $A = 200 \text{ m}^2$ A1 **[4]**

5 $dy/dx = 1 - 16x^{-2}$ M1; $1 - 16/x^2 = 0$ M1; $x = \pm 4$ A1; (4, 8) and (-4, -8) A1 **[4]**

End-of-book check

1 $(x - 5)(x + 7)$

2 $x = -1/2$ or $x = -3$

3 $2x^2 + 5x - 12$

4 $16a^{12}$

5 $t = (v - u)/a$

6 $x = 11$

7 265

8 45

9 $y = 2x + 3$

10 4

11 $8x^3 - 6x$

12 $-3 < x < 3$

Practice paper

1 (a) $12x^7$ B1 **[1]**

(b) 4 terms with 3 correct M1; $x^2 + 5x - 14$ A1 **[2]** **(c)** $3ab(2a - 3b)$ B2 **[2]**

2 (a) $12x - 4 = 2x + 16$ M1; $10x = 20$ M1; $x = 2$ A1 **[3]** **(b)** $5 - x = 2(x + 4)$ M1; $-3x = 3$ M1; $x = -1$ A1 **[3]**

3 (a) $(13.5 - 2.3) \div 0.4$ M1; 28 A1 **[2]** **(b)** $Pc = 3a + b$ M1; $3a = Pc - b$ M1; $a = (Pc - b)/3$ A1 **[3]**

4 (a) $7n - 5$ B2 (B1 for $7n + k$) **[2]** **(b)** $7n - 5 = 120$ M1; $n = 17.86$, not an integer, so no A1 **[2]** **(c)** $10(4 + 19 \times 7)$ M1; 1370 A1 **[2]**

5 correct method to eliminate M1; $29x = 203$ or $29y = -116$ A1; $x = 7$ A1; $y = -4$ A1 **[4]**

6 (a) $-3x > 12$ M1; $x < -4$ A1 **[2]** **(b)** $(x - 5)(x + 3)$ M1; critical values -3, 5 A1; $-3 \leq x \leq 5$ A1 **[3]**

7 $(4 \pm \sqrt{16 + 24})/6$ M2 (M1 for one error); 1.72 and -0.387 A1 **[3]**

8 $T = k/\sqrt{L}$ M1; $6 = k/5$ M1; $k = 30$ A1; $T = 7.5$ A1 **[4]**

9 $x(x - 4)$ B1; $(x - 4)(x + 4)$ B1; $x/(x + 4)$ B1 **[3]**

10 (a) $f(1) = 5$ M1; $5/4$ A1 **[2]** **(b)** $y(x - 1) = x$ M1; $x(y - 1) = y$ M1; $g^{-1}(x) = x/(x - 1)$ A1 **[3]**

11 midpoint (1, 3) B1; gradient AB = $-2/3$ B1; gradient $3/2$ M1; $y = 1.5x + 1.5$ or $2y = 3x + 3$ A1 **[4]**

12 (a) $6t^2 - 30t + 36$ B2 **[2]** **(b)** $6t^2 - 30t + 36 = 0$ M1; $(t - 2)(t - 3) = 0$ M1; $t = 2$ and $t = 3$ A1 **[3]** **(c)** $12t - 30$ M1; 18 m/s^2 A1 **[2]**

13 $(x - 5)^2$ B1; -7 B1; (5, -7) B1ft **[3]**