



BRITISH LEARNING CENTRE

CAMBRIDGE IGCSE · 0580

Mathematics

Trigonometry, Transformations & Vectors

Covers Topic 6 (Trigonometry) and Topic 7 (Transformations and vectors) of Cambridge IGCSE Mathematics 0580 at Core and Extended tier, with clear notes, practice, exam-style questions and full mark schemes. In mark schemes, a column vector is written as (top over bottom).

Name:

Key knowledge · Practice · Exam-style questions · Mark schemes

How to use this workbook

1

Key knowledge

Read the notes, worked examples and common mistakes.

2

Practice

Build the skill on Sheet A.

3

Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [].

4

Check & reflect

Mark your work with the mark scheme, then colour the tracker.

My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

#	Syllabus ref	Topic · I can...	R	A	G
1	C6.1, E6.1 Core & Extended	Pythagoras' theorem I can use Pythagoras' theorem to find a missing side in a right-angled triangle.			
2	C6.2, E6.2 Core & Extended	Trigonometry in right-angled triangles I can use sine, cosine and tangent to find missing sides and angles.			
3	C6.2, E6.2 Core & Extended	Angles of elevation and depression I can solve problems involving angles of elevation and depression.			
4	C4.3, C6.2, E4.3, E6.2 Core & Extended	Bearings and trigonometry I can use three-figure bearings and trigonometry to solve navigation problems.			
5	E6.3 Extended	Exact trigonometric values I can recall and use exact values of sin, cos and tan for 0°, 30°, 45°, 60° and 90°.			
6	E6.4 Extended	Trigonometric graphs and equations I can sketch trig graphs and solve trig equations for 0° to 360°.			
7	E6.5 Extended	The sine rule I can use the sine rule to find missing sides and angles in any triangle.			
8	E6.5 Extended	The cosine rule and area of a triangle I can use the cosine rule and $\frac{1}{2}ab \sin C$ to solve problems in any triangle.			
9	E6.6 Extended	Pythagoras and trigonometry in 3D I can find lengths and angles in 3D shapes, including the angle between a line and a plane.			
10	C7.1, E7.1 Core & Extended	Reflections, rotations and translations I can carry out and fully describe reflections, rotations and translations.			
11	C7.1, E7.1 Core & Extended	Enlargement I can enlarge shapes by positive, fractional and negative scale factors and describe enlargements fully.			
12	C7.2, E7.2, E7.3 Core & Extended	Vectors and column vectors I can add, subtract and multiply column vectors and find the magnitude of a vector.			
13	E7.4 Extended	Vector geometry I can use vectors to find routes, midpoints and ratios, and prove that lines are parallel or collinear.			



CHECK

Starting point check

Starting point check

Name _____ Date _____ Score _____ / 12

Answer these questions to check what you already know. Use a calculator where needed.

- 1 Find the length of the hypotenuse of a right-angled triangle whose other sides are 6 cm and 8 cm.

- 2 Round 23.4567 to 3 significant figures.

- 3 Use your calculator to find $\cos 60^\circ$.

- 4 Two angles of a triangle are 47° and 85° . Find the third angle.

- 5 Write down the three-figure bearing of due west.

- 6 Solve $12 \div x = 4$.

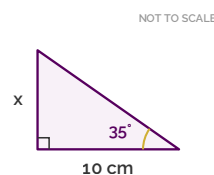
- 7 Reflect the point (2, 5) in the y-axis.

- 8 The point (1, 3) moves 4 right and 2 down. Where does it end?

- 9 Find the area of a triangle with base 9 cm and perpendicular height 4 cm.

Find x.

10



Answer $x = \dots\dots\dots$ cm

- 11 A 3 cm by 5 cm rectangle is enlarged by scale factor 2. Write down the new length and width.

- 12 Simplify $\sqrt{50}$.

1

UNIT 1 · KEY KNOWLEDGE

Pythagoras' theorem

I can... use Pythagoras' theorem to find a missing side in a right-angled triangle.

Syllabus ref: C6.1, E6.1 · Core & Extended

The **hypotenuse** is the longest side of a right-angled triangle, opposite the right angle. If you know two sides, Pythagoras' theorem gives the third.

Key points

- $c^2 = a^2 + b^2$, where c is the hypotenuse.
- Hypotenuse: **add** the squares, then square root.
- Shorter side: **subtract** the squares, then square root.

Key words

hypotenuse

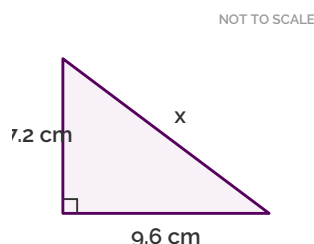
right angle

Pythagoras' theorem

square root

Worked example 1

Find the length x .



$$x^2 = 7.2^2 + 9.6^2 = 144$$

$$x = \sqrt{144} = \mathbf{12 \text{ cm}}$$

Worked example 2

A ladder 15.5 m long leans against a wall. Its foot is 8.3 m from the wall. How high up the wall does it reach?

The ladder is the hypotenuse, so subtract: $h^2 = 15.5^2 - 8.3^2 = 171.36$

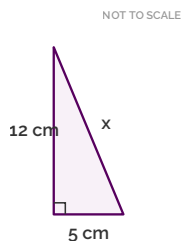
$$h = \sqrt{171.36} = 13.0905\dots = \mathbf{13.1 \text{ m}} \text{ (3 s.f.)}$$

Common mistakes

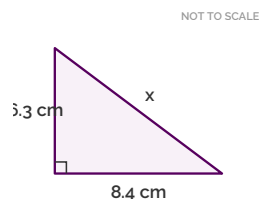
- Adding the squares when the hypotenuse is already given.
- Forgetting the square root, or rounding too early.

Exam tip: Check that the hypotenuse is the longest side in your answer.

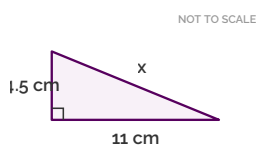
Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Find x .

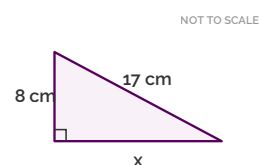
1

Answer $x = \dots\dots\dots$ cmFind x .

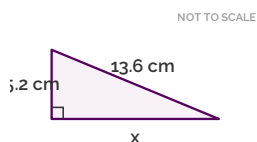
2

Answer $x = \dots\dots\dots$ cmFind x .

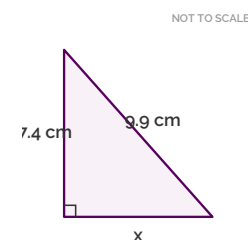
3

Answer $x = \dots\dots\dots$ cmFind x .

4

Answer $x = \dots\dots\dots$ cmFind x .

5

Answer $x = \dots\dots\dots$ cmFind x .

6

Answer $x = \dots\dots\dots$ cm

7

A rectangle measures 24 cm by 10 cm. Find the length of its diagonal.

8

A square has a diagonal of 10 cm. Find the side length.

9

Is a triangle with sides 9 cm, 12 cm and 15 cm right-angled? Show why.

10

Find the distance between the points (1, 2) and (7, 10).

11

An isosceles triangle has sides 13 cm, 13 cm and base 10 cm. Find its height.

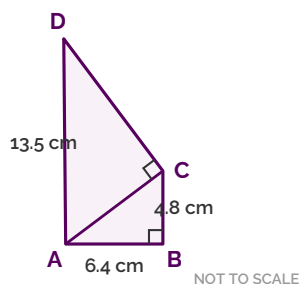
12

A ship sails 6.5 km north and then 4.2 km east. How far is it from its start?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 The diagram shows two right-angled triangles ABC and ACD. $AB = 6.4$ cm, $BC = 4.8$ cm and $AD = 13.5$ cm.



- (a) Calculate AC. [2]

Answer AC = cm

- (b) Calculate CD. [3]

Answer CD = cm

- (c) Calculate the perimeter of the shape ABCD. [1]

Answer cm

- 2 A ladder 6.5 m long leans against a vertical wall on horizontal ground. The foot of the ladder is 1.8 m from the wall.

- (a) Calculate how far up the wall the ladder reaches. [3]

Answer m

- (b) The top of the ladder slides 0.5 m down the wall. Calculate the new distance of the foot of the ladder from the wall. [3]

Answer m

- 3 A is the point $(-2, 3)$ and B is the point $(6, -3)$.

(a) Calculate the length of AB.

[3]

Answer units

(b) C is the point $(6, 3)$. Show that triangle ABC is right-angled and find its area.

[3]

Answer units²

- 4 The diagonals of a rhombus are 16 cm and 30 cm long. The diagonals of a rhombus cross at right angles and bisect each other.

(a) Calculate the length of one side of the rhombus.

[3]

Answer cm

(b) Calculate the perimeter of the rhombus.

[1]

Answer cm

- 5 An isosceles triangle has two sides of length 11 cm and a base of 14 cm.

(a) Calculate the perpendicular height of the triangle.

[3]

Answer cm

(b) Calculate the area of the triangle.

[2]

Answer cm²

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

2

UNIT 2 · KEY KNOWLEDGE

Trigonometry in right-angled triangles

I can... use sine, cosine and tangent to find missing sides and angles.

Syllabus ref: C6.2, E6.2 · Core & Extended

Label the sides from the angle: **hypotenuse** (H) opposite the right angle, **opposite** (O) facing the angle, **adjacent** (A) next to it. Pick the ratio that uses the two sides in the question.

Key points

- $\sin x = O/H$, $\cos x = A/H$, $\tan x = O/A$ (SOH CAH TOA)
- Side on top: multiply, e.g. $O = H \sin x$. Side on the bottom: divide, e.g. $H = O \div \sin x$.
- Angle: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$. Calculator in **degrees**.

Key words

opposite

adjacent

hypotenuse

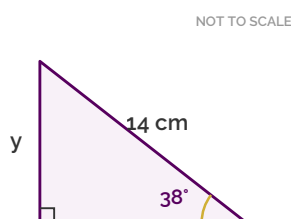
sine

cosine

tangent

Worked example 1

Find y .



O and H, so use sin: $y = 14 \sin 38^\circ = 8.6193... = \mathbf{8.62 \text{ cm}}$

Worked example 2

In triangle PQR, angle Q = 90° , PQ = 9.1 cm and QR = 5.6 cm. Find angle P.

From P: QR is opposite, PQ is adjacent, so $\tan P = 5.6 \div 9.1$

$P = \tan^{-1}(5.6 \div 9.1) = 31.608... = \mathbf{31.6^\circ}$

Common mistakes

- Labelling O and A from the wrong angle.
- Calculator in radians mode, or using $\sin^{-1}$ when finding a side.

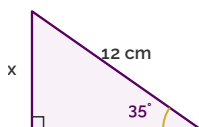
Exam tip: Write the ratio with numbers substituted (e.g. $\cos 52^\circ = x/8$) — this earns the method mark.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Find x .

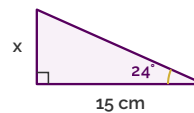
NOT TO SCALE

1

Answer $x = \dots\dots\dots$ cmFind x .

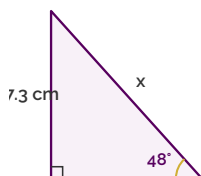
NOT TO SCALE

2

Answer $x = \dots\dots\dots$ cmFind x .

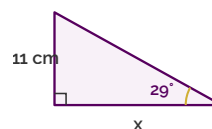
NOT TO SCALE

3

Answer $x = \dots\dots\dots$ cmFind x .

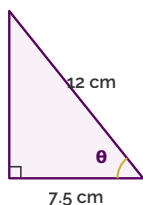
NOT TO SCALE

4

Answer $x = \dots\dots\dots$ cmFind angle θ .

NOT TO SCALE

5

Answer $\theta = \dots\dots\dots^\circ$ Find angle θ .

NOT TO SCALE

6

Answer $\theta = \dots\dots\dots^\circ$

7 Find $\tan^{-1}(0.8)$ to 1 d.p.

8 Work out $7.5 \sin 63^\circ$ to 3 s.f.

9 Find $\cos^{-1}(0.35)$ to 1 d.p.

10 Work out $25 \cos 40^\circ$ to 3 s.f.

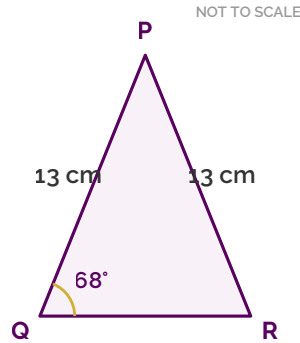
11 Work out $18 \div \tan 55^\circ$ to 3 s.f.

12 In triangle ABC, angle B = 90° , AB = 4 cm and AC = 9 cm. Find angle C.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 Triangle PQR is isosceles with $PQ = PR = 13$ cm. Angle $PQR = 68^\circ$. M is the midpoint of QR.



- (a) Calculate the height PM. [2]

Answer cm

- (b) Calculate the length QR. [3]

Answer cm

- (c) Calculate the area of triangle PQR. [2]

Answer cm²

- 2 A straight ramp is 7.5 m long. It rises a vertical height of 0.9 m.

- (a) Calculate the angle the ramp makes with the horizontal. [2]

Answer°

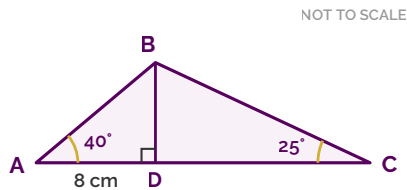
- (b) Safety rules say the angle must be less than 5° . A new ramp rises 0.9 m at an angle of 4.5° . Calculate the length of the new ramp. [3]

Answer m

- (c) Calculate the horizontal distance covered by the original ramp. [2]

Answer m

- 3 In the diagram, ABD and BCD are right-angled triangles with the right angle at D. $AD = 8$ cm, angle $BAD = 40^\circ$ and angle $BCD = 25^\circ$. A, D and C lie on a straight line.



- (a) Calculate BD.

[2]

Answer BD = cm

- (b) Calculate DC.

[2]

Answer DC = cm

- (c) Calculate BC.

[2]

Answer BC = cm

- 4 A rectangle ABCD has $AB = 14.2$ cm and $BC = 6.5$ cm.

- (a) Calculate the angle between the diagonal AC and the side AB.

[2]

Answer $^\circ$

- (b) Calculate the obtuse angle between the two diagonals.

[2]

Answer $^\circ$

- 5 A straight path AB starts at point A and makes an angle of 32° with a straight road AC. Point B is 50 m from A along the path.

- (a) Calculate the shortest distance from B to the road.

[2]

Answer m

- (b) Explain why this is the shortest distance.

[1]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

3

UNIT 3 · KEY KNOWLEDGE

Angles of elevation and depression

I can... solve problems involving angles of elevation and depression.

Syllabus ref: C6.2, E6.2 · Core & Extended

An **angle of elevation** is measured **up** from the horizontal to a line of sight; an **angle of depression** is measured **down** from the horizontal. Then use right-angled trigonometry.

Key points

- Both angles are measured from the **horizontal**, never the vertical.
- Elevation of B from A = depression of A from B (alternate angles).

Key words

elevation

depression

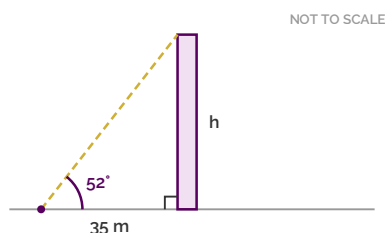
horizontal

line of sight

alternate angles

Worked example 1

From a point 35 m from the foot of a tower, the angle of elevation of the top is 52° . Find the height.



$$\tan 52^\circ = h/35, \text{ so } h = 35 \tan 52^\circ = \mathbf{44.8 \text{ m}}$$

Worked example 2

From the top of an 80 m cliff, the angle of depression of a boat is 23° . How far is the boat from the foot of the cliff?

The angle at the boat is also 23° (alternate angles).

$$\tan 23^\circ = 80/d, \text{ so } d = 80 \div \tan 23^\circ = \mathbf{188 \text{ m}}$$

Common mistakes

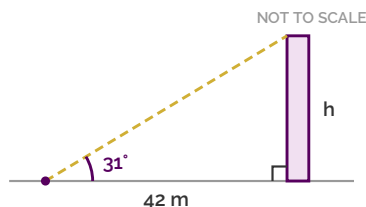
- Measuring the angle of depression from the vertical cliff face.
- Forgetting to add the observer's eye height.

Exam tip: Draw the horizontal as a dashed line before marking an angle of depression.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Find the height h .

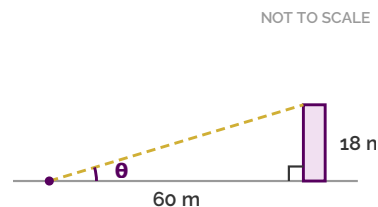
1



Answer $h = \dots\dots\dots$ m

Find the angle of elevation θ .

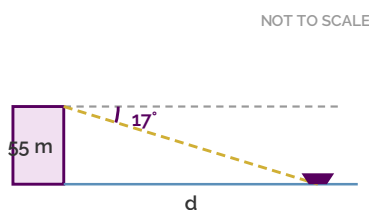
2



Answer $\theta = \dots\dots\dots^\circ$

Find the distance d .

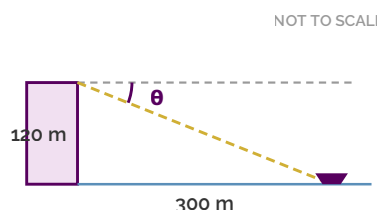
3



Answer $d = \dots\dots\dots$ m

Find the angle of depression θ .

4



Answer $\theta = \dots\dots\dots^\circ$

A kite string is 40 m long and makes an angle of 58° with the ground. How high is the kite?

5

From a boat, the angle of elevation of the top of a 65 m lighthouse is 9° . How far away is the boat?

6

A man whose eyes are 1.6 m above the ground stands 25 m from a tree. The angle of elevation of the top of the tree is 21° . How tall is the tree?

7

The angle of depression from a window to a car is 34° . The car is 22 m from the building. How high is the window?

8

A plane is 900 m above the ground. The angle of depression to an airport is 12° . Find the horizontal distance to the airport.

9

A pole 6 m tall casts a shadow 4.5 m long. Find the angle of elevation of the sun.

10

The angle of elevation of B from A is 28° . What is the angle of depression of A from B?

11

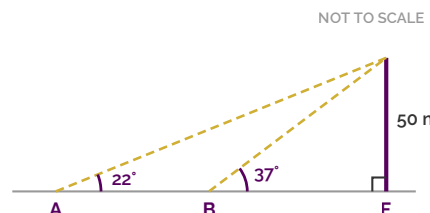
A 12 m ladder reaches 10.5 m up a wall. Find the angle it makes with the ground.

12

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 Points A and B are on level ground in a straight line with the foot F of a vertical mast. The mast is 50 m tall. From A the angle of elevation of the top of the mast is 22° . From B the angle of elevation is 37° .



- (a) Calculate AF. [2]

Answer AF = m

- (b) Calculate the distance AB. [3]

Answer AB = m

- 2 From the top of a cliff 95 m high, two boats P and Q are seen directly out to sea. The angles of depression of P and Q are 26° and 14° .

- (a) Which boat is closer to the cliff? Give a reason. [1]

- (b) Calculate the distance between the two boats. [4]

Answer m

- 3 Sam stands at a window 28 m from a tall building on the other side of a street. From the window, the angle of elevation of the top of the building is 40° and the angle of depression of the foot of the building is 18° . The ground is horizontal.

(a) Calculate the height of Sam's window above the ground.

[2]

Answer m

(b) Calculate the height of the building.

[3]

Answer m

- 4 A balloon is held by a straight rope 150 m long. The other end of the rope is fixed to the ground at point R. The rope makes an angle of 63° with the horizontal.

(a) Calculate the height of the balloon.

[2]

Answer m

(b) Lina stands 40 m from R, directly below the line of the rope. Calculate the angle of elevation of the balloon from Lina.

[3]

Answer $^\circ$

- 5 Point A is on level ground. The angle of elevation of the top T of a tree from A is 25° . Omar walks 20 m from A directly towards the tree to point B. The angle of elevation of T from B is 40° . The distance from B to the foot of the tree is x metres and the height of the tree is h metres.

(a) Write down two different expressions for h , one using angle 40° and one using angle 25° .

[2]

(b) Calculate the value of x and the height of the tree.

[3]

Answer $x = \dots\dots\dots$ m, $h = \dots\dots\dots$ m

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Bearings and trigonometry

I can... use three-figure bearings and trigonometry to solve navigation problems.

Syllabus ref: C4.3, C6.2, E4.3, E6.2 · Core & Extended

A **bearing** is measured **clockwise from north** and written with three figures (065°, 230°). Draw a north line at each point and split journeys into right-angled triangles.

Key points

- Bearing of B **from** A: the north line goes at A.
- Back bearing: bearing of A from B = bearing of B from A $\pm$ 180°.
- For d km on bearing θ (000°–090°): east = $d \sin \theta$, north = $d \cos \theta$.

Key words

bearing

clockwise

north line

back bearing

three-figure

Worked example 1

A ship sails 12 km on a bearing of 065°. How far east and how far north of its start is it?



East = $12 \sin 65^\circ = \mathbf{10.9 \text{ km}}$

North = $12 \cos 65^\circ = \mathbf{5.07 \text{ km}}$

Worked example 2

B is 7 km east and 4 km south of A. Find the bearing of B from A, and of A from B.

Angle below east = $\tan^{-1}(4/7) = 29.7^\circ$, so bearing of B from A = $90^\circ + 29.7^\circ = \mathbf{119.7^\circ}$

Bearing of A from B = $119.7^\circ + 180^\circ = \mathbf{299.7^\circ}$

Common mistakes

- Measuring anticlockwise, or from east.
- Writing 65° instead of 065°.

Exam tip: The word after *from* tells you where to draw the north line.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 Write the bearing of a direction exactly south-west.

- 2 The bearing of Q from P is 072° . Find the bearing of P from Q.

- 3 The bearing of X from Y is 305° . Find the bearing of Y from X.

- 4 A boat sails 20 km on a bearing of 040° . How far north has it travelled?

- 5 A plane flies 350 km on a bearing of 120° . How far east has it travelled?

- 6 A hiker walks 3.5 km on a bearing of 210° . How far south has she travelled?

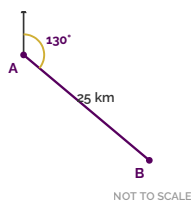
- 7 B is 9 km north and 5 km east of A. Find the bearing of B from A.

- 8 D is 6 km south and 8 km west of C. Find the bearing of D from C.

- 9 T is 15 km west and 4 km north of S. Find the bearing of T from S.

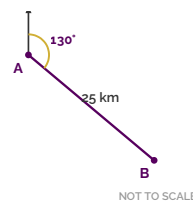
- 10 In question 9, find the distance ST.

- 11 B is on a bearing of 130° from A, 25 km away. How far east of A is B?



Answer km

- 12 For the same diagram, how far south of A is B?

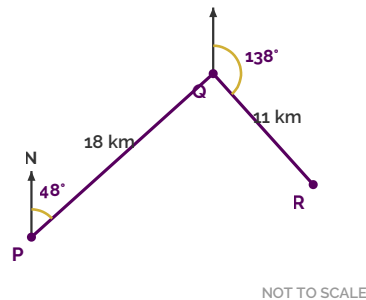


Answer km

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 A ship leaves port P and sails 18 km on a bearing of 048° to Q. It then sails 11 km on a bearing of 138° to R.



- (a) Show that angle $PQR = 90^\circ$. [2]

- (b) Calculate the distance PR. [2]

Answer km

- (c) Calculate the bearing of R from P. [3]

Answer°

- 2 A plane flies 64 km from airport A on a bearing of 110° to point B.

- (a) Calculate how far east of A the plane is. [2]

Answer km

- (b) Calculate how far south of A the plane is. [2]

Answer km

- (c) Write down the bearing of A from B. [1]

Answer°

- 3 A lighthouse L is 6.2 km due north of a harbour H. A boat B is due east of the lighthouse and on a bearing of 034° from the harbour.

(a) Calculate the distance LB.

[2]

Answer km

(b) Calculate the distance HB.

[2]

Answer km

(c) Find the bearing of H from B.

[2]

Answer $^\circ$

- 4 Amira walks 2.4 km due north from her house H to a shop S. She then walks 3.1 km on a bearing of 115° to a park K.

(a) Calculate how far east of H the park is.

[2]

Answer km

(b) Calculate how far north of H the park is.

[2]

Answer km

(c) Calculate the distance and bearing of K from H.

[3]

Answer km on $^\circ$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Exact trigonometric values

I can... recall and use exact values of sin, cos and tan for 0° , 30° , 45° , 60° and 90° .

Syllabus ref: E6.3 · Extended

The exact values of sin, cos and tan of 30° , 45° and 60° come from two triangles: half of an equilateral triangle with side 2, and a right-angled isosceles triangle with shorter sides 1. Leave answers as fractions and surds.

Key points

- $\sin 30^\circ = 1/2$, $\cos 30^\circ = \sqrt{3}/2$, $\tan 30^\circ = 1/\sqrt{3} = \sqrt{3}/3$
- $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$, $\tan 45^\circ = 1$
- $\sin 60^\circ = \sqrt{3}/2$, $\cos 60^\circ = 1/2$, $\tan 60^\circ = \sqrt{3}$; $\sin 0^\circ = \cos 90^\circ = 0$, $\sin 90^\circ = \cos 0^\circ = 1$, $\tan 0^\circ = 0$

Key words

exact value

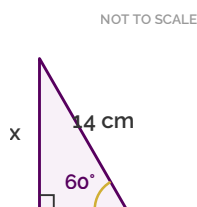
surd

equilateral

isosceles

Worked example 1

Find the exact length x .



x is opposite 60° and 14 cm is the hypotenuse: $x = 14 \sin 60^\circ = 14 \times \sqrt{3}/2 = 7\sqrt{3} \text{ cm}$

Worked example 2

Find the exact value of $4 \sin 30^\circ + \tan 45^\circ \times \cos 60^\circ$.

$= 4 \times 1/2 + 1 \times 1/2 = 2 + 1/2 = 5/2$

Common mistakes

- Mixing up $\sin 30^\circ$ ($1/2$) and $\sin 60^\circ$ ($\sqrt{3}/2$): sin increases from 0 to 1 between 0° and 90° .
- Giving a rounded decimal when an exact answer is asked for.

Exam tip: Sketch the two triangles in the margin at the start of the exam — then you can read off every exact value.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

1 $\sin 30^\circ =$

2 $\cos 45^\circ =$

3 $\tan 60^\circ =$

4 $\cos 30^\circ =$

5 $\tan 30^\circ =$

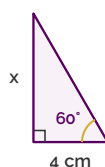
6 $\sin 90^\circ + \cos 90^\circ =$

7 Find the exact value of $2 \sin 60^\circ \times \cos 30^\circ$.

8 Find the exact value of $\tan 60^\circ \times \tan 30^\circ$.

Find the exact value of x .

NOT TO SCALE

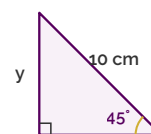


9

Answer $x = \dots\dots\dots$ cm

Find the exact value of y .

NOT TO SCALE

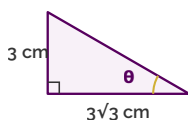


10

Answer $y = \dots\dots\dots$ cm

Find angle θ without a calculator.

NOT TO SCALE

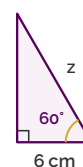


11

Answer $\theta = \dots\dots\dots^\circ$

Find the exact length z .

NOT TO SCALE



12

Answer $z = \dots\dots\dots$ cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

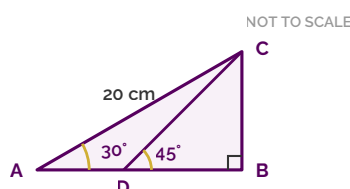
Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 Complete the table of exact values.

[4]

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$		1
$\cos \theta$	1		$\sqrt{2}/2$	$1/2$	
$\tan \theta$	0			$\sqrt{3}$	—

- 2 Triangle ABC has a right angle at B. AC = 20 cm and angle BAC = 30° . D is the point on AB such that angle BDC = 45° .



- (a) Find the exact length of BC.

[2]

Answer BC = cm

- (b) Find the exact length of AB.

[2]

Answer AB = cm

- (c) Show that $AD = 10(\sqrt{3} - 1)$ cm.

[3]

- 3 Do not use a calculator in this question. Show your working.

- (a) Show that $\sin 60^\circ \times \tan 30^\circ = 1/2$.

[2]

- (b) Find the exact value of $\cos^2 30^\circ - \sin^2 30^\circ$.

[2]

Answer

- (c) Find the exact value of $\tan 60^\circ \div \cos 30^\circ$.

[2]

Answer

4 An equilateral triangle has sides of length 8 cm.

(a) Show that its height is $4\sqrt{3}$ cm. [2]

(b) Find the exact area of the triangle. [2]

Answer cm^2

5 Triangle PRS has a right angle at R. $PR = 6$ cm and $RS = 6\sqrt{3}$ cm.

(a) Find angle RPS. Do not use a calculator. [2]

Answer $^\circ$

(b) Find the exact length of PS. [2]

Answer $PS =$ cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Trigonometric graphs and equations

I can... sketch trig graphs and solve trig equations for 0° to 360° .

Syllabus ref: E6.4 · Extended

The graphs of $\sin x$ and $\cos x$ repeat every 360° and lie between -1 and 1 . $\tan x$ repeats every 180° and has asymptotes at 90° and 270° . Their symmetry gives a second solution to an equation.

Key points

- $\sin x = k$: $x = \sin^{-1}k$ and $180^\circ - \sin^{-1}k$
- $\cos x = k$: $x = \cos^{-1}k$ and $360^\circ - \cos^{-1}k$
- $\tan x = k$: $x = \tan^{-1}k$ and $\tan^{-1}k + 180^\circ$. Add 360° to any negative answer.

Key words

period

amplitude

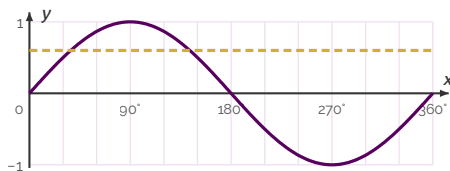
asymptote

symmetry

solution

Worked example 1

Solve $\sin x = 0.6$ for $0^\circ \leq x \leq 360^\circ$.



$\sin^{-1}0.6 = 36.9^\circ$, and $180^\circ - 36.9^\circ = 143.1^\circ$. **$x = 36.9^\circ, 143.1^\circ$**

Worked example 2

Solve $2 \cos x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

$\cos x = -1/2$, so $\cos^{-1}(-0.5) = 120^\circ$ and $360^\circ - 120^\circ = 240^\circ$. **$x = 120^\circ, 240^\circ$**

Common mistakes

- Giving only the calculator answer and missing the second solution.
- Rearranging wrongly: $2 \cos x + 1 = 0$ gives $\cos x = -1/2$, not -2 .

Exam tip: Sketch the graph with a horizontal line $y = k$: the number of crossings is the number of solutions.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

1 Write down the maximum value of $\cos x$.

2 Write down the period of $\tan x$.

3 For $0^\circ \leq x \leq 360^\circ$, where is $\cos x = 0$?

4 $\sin 140^\circ = \sin$ $^\circ$

5 $\cos 300^\circ = \cos$ $^\circ$

6 Solve $\sin x = 0.5$ for $0^\circ \leq x \leq 360^\circ$.

7 Solve $\cos x = 0.3$ for $0^\circ \leq x \leq 360^\circ$.

8 Solve $\tan x = 2$ for $0^\circ \leq x \leq 360^\circ$.

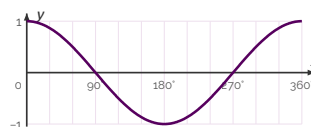
9 Solve $\sin x = -0.4$ for $0^\circ \leq x \leq 360^\circ$.

10 Solve $\cos x = -0.8$ for $0^\circ \leq x \leq 360^\circ$.

11 Solve $3 \sin x = 2$ for $0^\circ \leq x \leq 360^\circ$.

12

This is the graph of $y = f(x)$. Is $f(x)$ $\sin x$, $\cos x$ or $\tan x$?

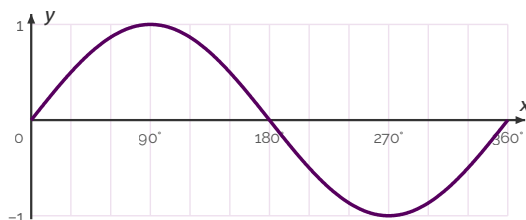


Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 The diagram shows the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$.



(a) On the same axes, sketch the graph of $y = \cos x$. [2]

(b) Write down the number of solutions of $\sin x = \cos x$ for $0^\circ \leq x \leq 360^\circ$. [1]

Answer

(c) Solve $\sin x = -0.5$ for $0^\circ \leq x \leq 360^\circ$. [3]

Answer $x = \dots\dots$

- 2 Solve these equations for $0^\circ \leq x \leq 360^\circ$.

(a) $4 \sin x = 3$ [3]

Answer $x = \dots\dots$

(b) $\tan x + 3 = 0$ [3]

Answer $x = \dots\dots$

(c) $5 \cos x - 2 = 0$ [3]

Answer $x = \dots\dots$

- 3 You are given that $\cos 70^\circ = 0.342$, correct to 3 significant figures.

(a) Write down another angle between 0° and 360° whose cosine is 0.342. [1]

Answer°

(b) Find two angles between 0° and 360° whose cosine is -0.342 . [2]

Answer° ;°

4 This question is about the graph of $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$.

(a) Write down the equations of the two asymptotes.

[2]

Answer and

(b) Solve $\tan x = 0.7$.

[2]

Answer $x =$

5 The equation $\sin x = k$ has exactly one solution for $0^\circ < x < 360^\circ$.

(a) Write down the two possible values of k .

[2]

Answer $k =$

(b) For each value, write down the solution.

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

7

UNIT 7 · KEY KNOWLEDGE

The sine rule

I can... use the sine rule to find missing sides and angles in any triangle.

Syllabus ref: E6.5 · Extended

The **sine rule** works in any triangle. Use it when you know a side and the angle **opposite** it, plus one more side or angle. Label sides with small letters opposite the matching capital-letter angles.

Key points

- $a/\sin A = b/\sin B = c/\sin C$ (finding a side)
- $\sin A/a = \sin B/b = \sin C/c$ (finding an angle)
- $\sin x = \sin(180^\circ - x)$, so an angle found from $\sin$ may have an obtuse alternative (the ambiguous case).

Key words

sine rule

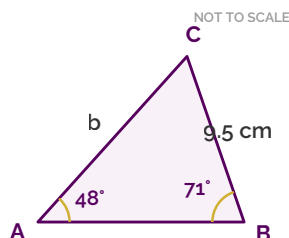
opposite

obtuse

ambiguous case

Worked example 1

Find b .



$$b/\sin 71^\circ = 9.5/\sin 48^\circ, \text{ so } b = 9.5 \sin 71^\circ \div \sin 48^\circ = \mathbf{12.1 \text{ cm}}$$

Worked example 2

In triangle PQR, PR = 7 cm, QR = 5.2 cm and angle P = 40°. Find the two possible sizes of angle Q.

$$\sin Q/7 = \sin 40^\circ/5.2, \text{ so } \sin Q = 0.8653\dots$$

$$Q = \mathbf{59.9^\circ} \text{ or } Q = 180^\circ - 59.9^\circ = \mathbf{120.1^\circ}$$

Common mistakes

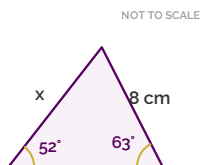
- Pairing a side with an angle that is not opposite it.
- Using the sine rule when no side–opposite angle pair is known (use the cosine rule).

Exam tip: Put the unknown on the top: use $a/\sin A$ for a side and $\sin A/a$ for an angle.

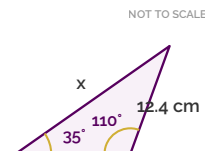
Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Find x .

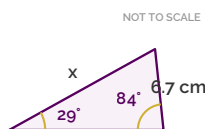
1

Answer $x = \dots\dots\dots$ cmFind x .

2

Answer $x = \dots\dots\dots$ cmFind x .

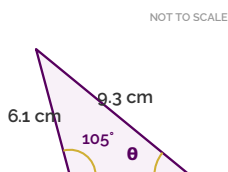
3

Answer $x = \dots\dots\dots$ cmFind angle θ .

4

Answer $\theta = \dots\dots\dots^\circ$ Find angle θ .

5

Answer $\theta = \dots\dots\dots^\circ$

6 $\sin 130^\circ = \sin \boxed{}^\circ$

7

Find the obtuse angle whose sine is 0.8.

8

In triangle ABC, $a = 15$ cm, $A = 72^\circ$ and $B = 39^\circ$. Find b .

9

In triangle PQR, $p = 6$ cm, $q = 9$ cm and $P = 30^\circ$. Find the two possible values of angle Q.

10

In triangle XYZ, $X = 45^\circ$, $Y = 75^\circ$ and $z = 10$ cm. Find x .

11

In triangle DEF, $d = 8$ cm, $D = 50^\circ$ and $E = 60^\circ$. Find e .

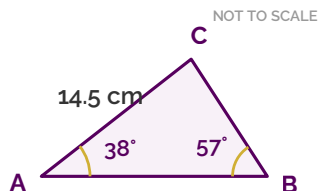
12

Explain why the sine rule cannot be used directly when you know three sides only.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 In triangle ABC, AC = 14.5 cm, angle BAC = 38° and angle ABC = 57° .



- (a) Calculate BC.

[3]

Answer BC = cm

- (b) Calculate AB.

[3]

Answer AB = cm

- 2 A ship sails 40 km from port A on a bearing of 060° to point B. A lighthouse L is on a bearing of 020° from A and on a bearing of 310° from B.

- (a) Show that angle LAB = 40° and angle ABL = 70° .

[2]

- (b) Calculate the distance BL.

[3]

Answer km

- (c) Calculate the shortest distance from L to the line AB.

[2]

Answer km

- 3 In triangle PQR, $PR = 8.4$ cm, $QR = 5.6$ cm and angle $QPR = 35^\circ$.

(a) Show that there are two possible triangles and find the two possible sizes of angle PQR.

[4]

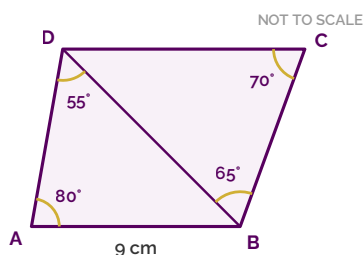
Answer $^\circ$ or $^\circ$

(b) Angle PQR is obtuse. Find angle PRQ.

[1]

Answer $^\circ$

- 4 ABCD is a quadrilateral. $AB = 9$ cm, angle $BAD = 80^\circ$, angle $ADB = 55^\circ$, angle $BCD = 70^\circ$ and angle $DBC = 65^\circ$.



(a) Calculate BD.

[2]

Answer BD = cm

(b) Calculate CD.

[3]

Answer CD = cm

(c) Calculate the area of ABCD.

[3]

Answer cm²

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

The cosine rule and area of a triangle

I can... use the cosine rule and $\frac{1}{2}ab \sin C$ to solve problems in any triangle.

Syllabus ref: E6.5 · Extended

Use the **cosine rule** when you know two sides and the angle between them, or all three sides. The area of any triangle can be found from two sides and the included angle.

Key points

- $a^2 = b^2 + c^2 - 2bc \cos A$
- $\cos A = (b^2 + c^2 - a^2) \div 2bc$ (a negative value means A is obtuse)
- Area = $\frac{1}{2}ab \sin C$ (C is the angle between sides a and b)

Key words

cosine rule

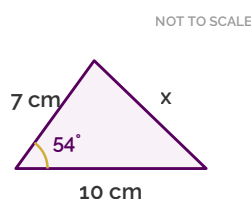
included angle

area

obtuse

Worked example 1

Find x and the area of the triangle.



$$x^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 54^\circ = 66.710\dots, \text{ so } x = \mathbf{8.17 \text{ cm}}$$

$$\text{Area} = \frac{1}{2} \times 7 \times 10 \times \sin 54^\circ = \mathbf{28.3 \text{ cm}^2}$$

Worked example 2

A triangle has sides 6 cm, 8 cm and 11 cm. Find the largest angle.

$$\text{The largest angle is opposite 11 cm: } \cos \theta = (36 + 64 - 121) \div 96 = -0.2188\dots, \theta = \mathbf{102.6^\circ}$$

Common mistakes

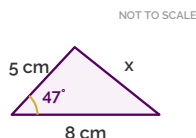
- Working out $(b^2 + c^2 - 2bc)$ first and then multiplying by $\cos A$ — follow the order of operations.
- Forgetting to square root at the end.

Exam tip: Before you start, ask: do I know an angle and its opposite side? Yes → sine rule. No → cosine rule.

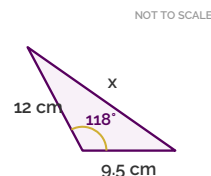
Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Find x .

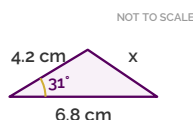
1

Answer $x = \dots\dots\dots$ cmFind x .

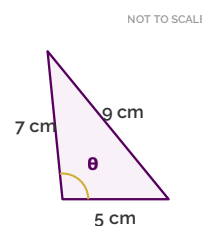
2

Answer $x = \dots\dots\dots$ cmFind x .

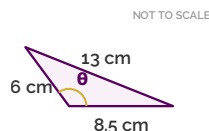
3

Answer $x = \dots\dots\dots$ cmFind angle θ .

4

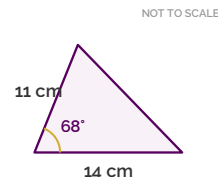
Answer $\theta = \dots\dots\dots^\circ$ Find angle θ .

5

Answer $\theta = \dots\dots\dots^\circ$

Find the area.

6

Answer $\dots\dots\dots$ cm²

Find the area of triangle ABC with $a = 8$ cm, $b = 15$ cm and $C = 150^\circ$.

7

A triangle has area 30 cm², and two sides 8 cm and 10 cm. Find the acute included angle.

8

In triangle PQR, $p = 7$ cm, $q = 9$ cm and $R = 100^\circ$. Find r .

9

A triangle has sides 4 cm, 5 cm and 8 cm. Is its largest angle acute or obtuse?

10

An equilateral triangle has side 10 cm. Use $\frac{1}{2}ab \sin C$ to find its area.

11

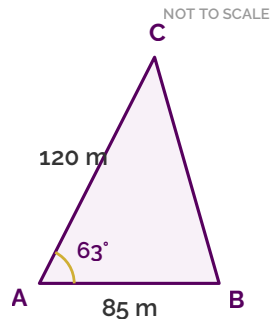
Two sides of a parallelogram are 6 cm and 9 cm with an angle of 72° between them. Find its area.

12

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 A triangular field ABC has $AB = 85$ m, $AC = 120$ m and angle $BAC = 63^\circ$.



- (a) Calculate the length of BC. [3]

Answer BC = m

- (b) Calculate the area of the field. [2]

Answer m^2

- (c) Calculate the shortest distance from A to BC. [2]

Answer m

- 2 Triangle PQR has $PQ = 7$ cm, $QR = 12$ cm and $PR = 9$ cm.

- (a) Calculate angle PRQ. [3]

Answer $^\circ$

- (b) Calculate the area of triangle PQR. [2]

Answer cm^2

- 3 A boat sails 30 km from harbour H on a bearing of 040° to buoy B. It then sails 45 km on a bearing of 110° to island I.

(a) Show that angle $HBI = 110^\circ$.

[2]

(b) Calculate the distance HI.

[3]

Answer km

- 4 The area of triangle ABC is 40 cm^2 . $AB = 9 \text{ cm}$ and angle $ABC = 48^\circ$.

(a) Calculate BC.

[3]

Answer BC = cm

(b) Calculate AC.

[3]

Answer AC = cm

- 5 Parallelogram ABCD has $AB = 11 \text{ cm}$, $AD = 7 \text{ cm}$ and angle $DAB = 58^\circ$.

(a) Calculate the length of the diagonal BD.

[3]

Answer BD = cm

(b) Calculate the area of the parallelogram.

[2]

Answer cm^2

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Pythagoras and trigonometry in 3D

I can... find lengths and angles in 3D shapes, including the angle between a line and a plane.

Syllabus ref: E6.6 · Extended

To solve a 3D problem, find a right-angled triangle inside the shape and draw it flat. The angle between a line and a plane is the angle between the line and its projection (its 'shadow') on the plane.

Key points

- Diagonal of a cuboid: $d = \sqrt{l^2 + w^2 + h^2}$
- Find a base diagonal first, then use it as one side of a vertical triangle.
- Angle between line AG and base ABCD: angle GAC, where C is directly below G.

Key words

cuboid

pyramid

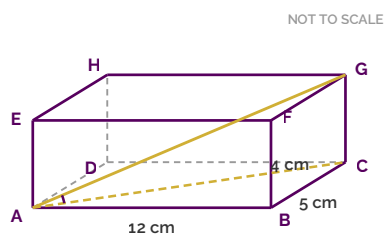
plane

projection

diagonal

Worked example 1

The cuboid is 12 cm by 5 cm by 4 cm. Find AG and the angle between AG and the base ABCD.



$$AC = \sqrt{12^2 + 5^2} = 13; AG = \sqrt{13^2 + 4^2} = \mathbf{13.6 \text{ cm}}$$

$$\tan GAC = 4/13, \text{ so angle} = \mathbf{17.1^\circ}$$

Common mistakes

- Using a side of the cuboid instead of the base diagonal.
- Measuring the angle with the plane from the wrong vertex.

Exam tip: Redraw each right-angled triangle flat, with the right angle marked, before calculating.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 A cuboid measures 8 cm by 6 cm by 5 cm. Find the length of its longest diagonal.

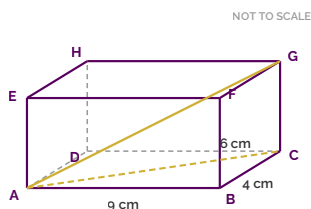
- 2 A cuboid measures 10 cm by 4 cm by 7 cm. Find the length of its longest diagonal.

- 3 Find the longest rod that fits in a cube of side 20 cm.

- 4 A cuboid has diagonal 15 cm, length 10 cm and width 6 cm. Find its height.

The cuboid has $AB = 9$ cm, $BC = 4$ cm and $BF = 6$ cm. Find AC .

5



6

For the cuboid in question 5, find AG .

Answer $AC = \dots\dots\dots$ cm

7

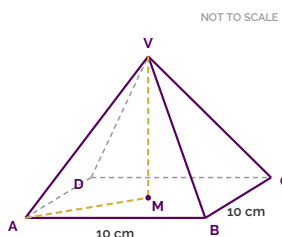
Find angle GAC (the angle between AG and the base).

8

Find angle FAB .

$VABCD$ is a square-based pyramid with base side 10 cm and vertical height $VM = 12$ cm. Find AM .

9



10

For the pyramid in question 9, find the length of the edge VA .

Answer $AM = \dots\dots\dots$ cm

11

For the same pyramid, find the angle between VA and the base.

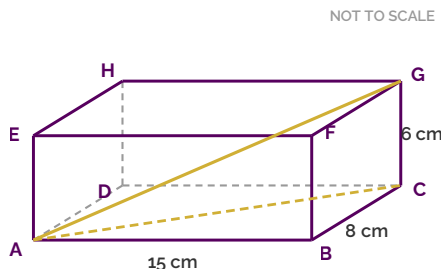
12

A flagpole 9 m tall stands at a corner of a 20 m by 15 m rectangular field. Find the angle of elevation of its top from the opposite corner.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 The diagram shows a cuboid ABCDEFGH. $AB = 15$ cm, $BC = 8$ cm and $CG = 6$ cm.



- (a) Calculate the length AC. [2]

Answer AC = cm

- (b) Calculate the length AG. [2]

Answer AG = cm

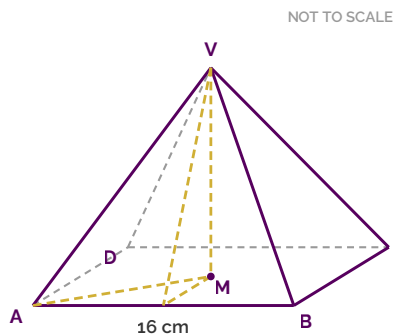
- (c) Calculate the angle between AG and the base ABCD. [3]

Answer°

- (d) Calculate the angle between AG and the face BCGF. [3]

Answer°

- 2 VABCD is a pyramid on a square base ABCD of side 16 cm. The vertex V is vertically above the centre M of the base, and $VM = 15$ cm. N is the midpoint of AB.



- (a) Calculate the length VA.

[3]

Answer $VA = \dots\dots\dots$ cm

- (b) Calculate the angle between VA and the base.

[2]

Answer $\dots\dots\dots^\circ$

- (c) Calculate the angle between the face VAB and the base.

[3]

Answer $\dots\dots\dots^\circ$

- 3 A vertical mast TP of height 24 m stands at the corner P of a horizontal rectangular yard PQRS. $PQ = 40$ m and $QR = 30$ m.

- (a) Calculate the angle of elevation of T from R.

[3]

Answer $\dots\dots\dots^\circ$

- (b) Calculate the distance TR.

[2]

Answer $\dots\dots\dots$ m

- 4 A closed box is a cuboid measuring 40 cm by 30 cm by 20 cm.

- (a) Will a thin rod of length 55 cm fit inside the box? Show your working.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Reflections, rotations and translations

I can... carry out and fully describe reflections, rotations and translations.

Syllabus ref: C7.1, E7.1 · Core & Extended

Reflections, rotations and translations give a **congruent** image.

To describe one **fully**, give every detail below.

Key points

- Reflection: the equation of the mirror line, e.g. $x = -1$, $y = x$.
- Rotation: centre, angle and direction, e.g. 90° clockwise about $(0, 2)$.
- Translation: a column vector — top number across, bottom number up (3 right, 2 down has 3 on top, -2 below).

Key words

reflection

rotation

translation

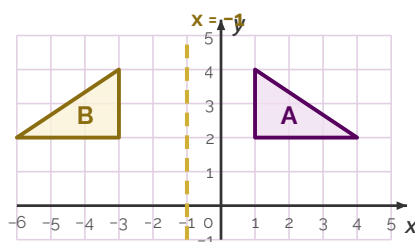
mirror line

centre

congruent

Worked example 1

Describe fully the single transformation that maps A onto B.



Each point and its image are the same distance from $x = -1$: **reflection in the line $x = -1$**

Worked example 2

Triangle T has vertices $(1, 1)$, $(3, 1)$ and $(3, 2)$. Rotate T through 90° clockwise about $(0, 0)$.

$(x, y) \rightarrow (y, -x)$, so the image is **$(1, -1)$, $(1, -3)$, $(2, -3)$**

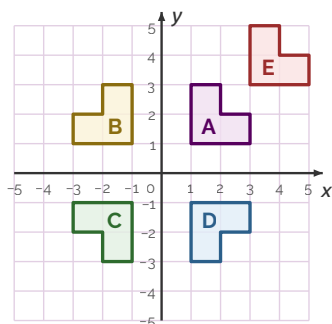
Common mistakes

- Describing a rotation without its centre or direction.
- Writing "flip" or "turn" instead of reflection or rotation — use the correct words.

Exam tip: Use tracing paper for rotations: pin it at the centre with your pencil and turn it.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Describe fully the single transformation that maps shape A onto shape B.



1

2

Using the diagram in question 1, describe fully the transformation that maps A onto C.

3

Describe fully the transformation that maps A onto D.

4

Describe fully the transformation that maps A onto E.

5

Describe fully the transformation that maps B onto C.

6

Reflect the point (3, -2) in the x-axis.

7

Reflect (3, -2) in the line $y = x$.

8

Reflect (3, -2) in the line $y = -x$.

9

Reflect (3, -2) in the line $x = 1$.

10

Rotate (3, -2) through 90° anticlockwise about (0, 0).

11

Rotate (3, -2) through 180° about (1, 1).

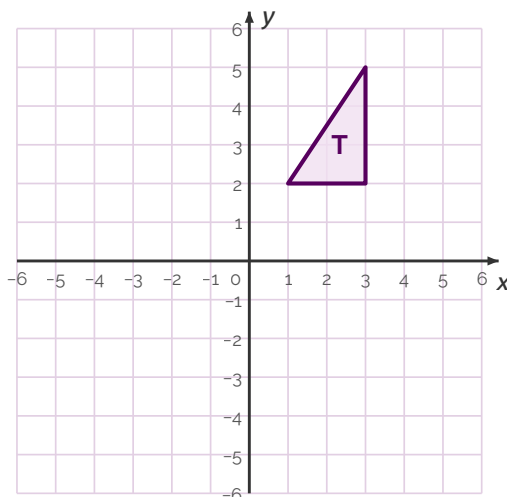
12

Name the three transformations that always give a congruent image.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 The diagram shows triangle T on a coordinate grid.

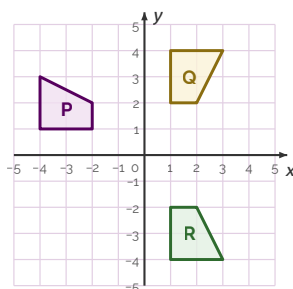


- (a) Reflect triangle T in the line $y = 1$. Label the image A. [2]
- (b) Rotate triangle T through 90° anticlockwise about $(-1, 0)$. Label the image B. [2]
- (c) Translate triangle T by the vector shown. Label the image C. [2]

$$\mathbf{v} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$$

- (d) Reflect triangle T in the line $y = -x$. Label the image D. [2]
- (e) Describe fully the single transformation that maps triangle A onto triangle D. [3]

- 2 The diagram shows three shapes P, Q and R.



(a) Describe fully the single transformation that maps P onto Q. [3]

(b) Describe fully the single transformation that maps P onto R. [2]

(c) Describe fully the single transformation that maps Q onto R. [2]

- 3 The point M(4, 1) is reflected in the x-axis. Its image is then rotated through 90° anticlockwise about the origin.

(a) Find the coordinates of the final image of M. [2]

Answer (.....,)

(b) Describe fully the single transformation equivalent to these two transformations. [2]

- 4 A rotation of 90° clockwise maps the point (1, 4) onto the point (5, 2).

(a) Find the coordinates of the centre of rotation. [3]

Answer (.....,)

(b) Find the image of (5, 2) under the same rotation. [2]

Answer (.....,)

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Enlargement

I can... enlarge shapes by positive, fractional and negative scale factors and describe enlargements fully.

Syllabus ref: C7.1, E7.1 · Core & Extended

An **enlargement** multiplies every length by the **scale factor**, measured from a **centre**. Rays through corresponding vertices meet at the centre.

Key points

- Describe fully: enlargement, scale factor k , centre (a, b) .
- $0 < k < 1$ makes it smaller; a **negative** k puts the image on the other side of the centre, upside down (Extended).
- Areas $\times k^2$.

Key words

enlargement

scale factor

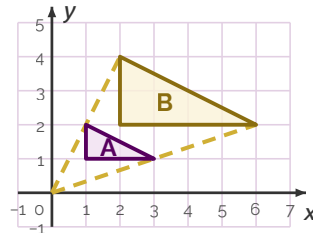
centre

ray

similar

Worked example 1

Describe fully the single transformation that maps A onto B.



Lengths double and the rays meet at $(0, 0)$: **enlargement, scale factor 2, centre $(0, 0)$**

Worked example 2

Enlarge the point $(5, 3)$ by scale factor $-1/2$, centre $(1, 1)$.

From the centre: 4 right, 2 up. $\times (-1/2)$: 2 left, 1 down. Image = **$(-1, 0)$**

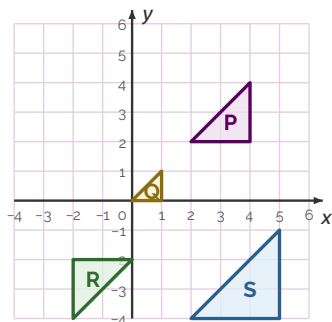
Common mistakes

- Missing the centre or the scale factor in a description.
- Measuring from the origin when the centre is somewhere else.

Exam tip: Count squares from the centre to each vertex, multiply by the scale factor, then count again from the centre.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

Describe fully the single transformation that maps P onto Q.



1

2

Using the diagram in question 1, describe fully the transformation that maps Q onto S.

3

Describe fully the transformation that maps P onto R. (Extended)

4

Describe fully the transformation that maps Q onto P.

5

Enlarge the point (2, 3) by scale factor 3, centre (0, 0).

6

Enlarge (6, 4) by scale factor $\frac{1}{2}$, centre (0, 0).

7

Enlarge (4, 1) by scale factor 2, centre (1, 1).

8

Enlarge (3, 5) by scale factor -1, centre (1, 2).

9

Enlarge (4, 4) by scale factor $-\frac{1}{2}$, centre (0, 0).

10

A shape of area 5 cm^2 is enlarged by scale factor 3. Find the area of the image.

11

A shape with perimeter 18 cm is enlarged by scale factor $\frac{1}{3}$. Find the new perimeter.

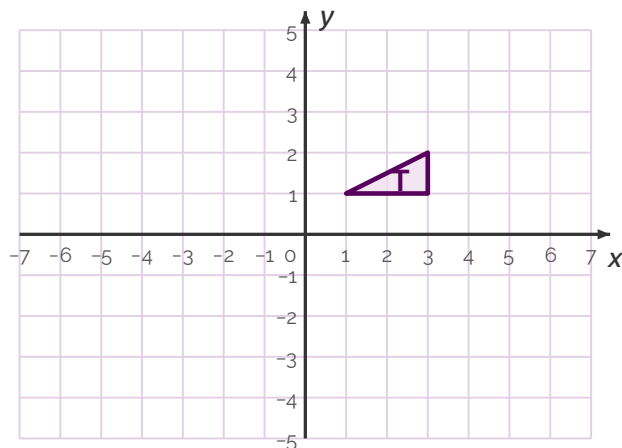
12

An enlargement with scale factor -1 is the same as which single transformation?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

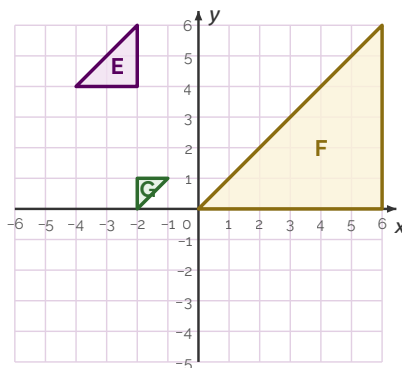
Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 The diagram shows triangle T.



- (a) Enlarge triangle T by scale factor 2, centre $(-1, 0)$. Label the image A. [3]
- (b) (Extended) Enlarge triangle T by scale factor -2 , centre $(0, 0)$. Label the image B. [3]

- 2 The diagram shows triangles E, F and G.



- (a) Describe fully the single transformation that maps E onto F. [3]

- (b) Describe fully the single transformation that maps E onto G. [3]

3 A rectangle measures 3 cm by 2 cm. It is enlarged by scale factor 4.

(a) Find the perimeter of the image.

[2]

Answer cm

(b) Find the area of the image.

[2]

Answer cm^2

(c) A different enlargement maps the 3 cm by 2 cm rectangle onto a rectangle with area 1.5 cm^2 . Find the positive scale factor.

[2]

Answer

4 The point W(7, 5) is enlarged with centre (1, 2).

(a) Find the image of W when the scale factor is $\frac{1}{3}$.

[2]

Answer (..... ,)

(b) Find the image of W when the scale factor is -2.

[2]

Answer (..... ,)

5 Triangle V has vertices (2, 4), (6, 4) and (6, 10). It is enlarged by scale factor $\frac{1}{2}$, centre (0, 0).

(a) Find the coordinates of the vertices of the image.

[2]

(b) The area of V is 12 square units. Find the area of the image.

[2]

Answer units^2

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Vectors and column vectors

I can... add, subtract and multiply column vectors and find the magnitude of a vector.

Syllabus ref: C7.2, E7.2, E7.3 · Core & Extended

A **vector** has size (magnitude) and direction. A column vector shows the movement: the top number is across (right is positive), the bottom number is up (up is positive). Vectors are written in bold, **a**, or with an arrow over the letters, e.g. $\overrightarrow{AB}$.

Key points

- Add or subtract the top numbers and the bottom numbers separately.
- $k\mathbf{a}$ multiplies both numbers by k ; it is parallel to **a**.
- Magnitude of a vector with top x and bottom y : $\sqrt{x^2 + y^2}$ (Extended).

Key words

vector

column vector

magnitude

scalar

parallel

Worked example 1

Find $2\mathbf{a} - \mathbf{b}$ and $|2\mathbf{a} - \mathbf{b}|$.

$$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$2\mathbf{a} - \mathbf{b}$: top $6 - (-2) = 8$, bottom $-2 - 4 = -6$

$$|2\mathbf{a} - \mathbf{b}| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$$

Worked example 2

A is (1, 5) and B is (4, -2). Write $\overrightarrow{AB}$ as a column vector.

Across: $4 - 1 = 3$. Up: $-2 - 5 = -7$. $\overrightarrow{AB}$ has top number **3** and bottom number **-7**.

Common mistakes

- Writing a column vector like a coordinate, (3, -1).
- Forgetting that subtracting a negative number adds.

Exam tip: Write the steps for each row of the column vector separately, then check the signs.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

a, **b** and **c** are shown below. Find **a + b**.

1 $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

2 Using **a**, **b**, **c** from question 1, find **a - b**.

3 Find **3c**.

4 Find **2a + b**.

5 Find **a - 2b + c**.

6 Find **|a|**.

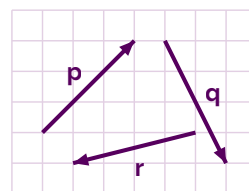
7 Find **|b|** to 3 s.f.

8 Find **|a + b|** to 3 s.f.

Write the vectors **p**, **q** and **r** as column vectors.

9 Find the vector **d** such that **a + d = c**.

10



11 A is (2, -1) and B is (-3, 5). Write AB as a column vector.

12 Is **e** parallel to **a** from question 1? Explain.

$$\mathbf{e} = \begin{pmatrix} 6 \\ -4.5 \end{pmatrix}$$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 Vectors $\mathbf{p}$ and $\mathbf{q}$ are shown below.

$$\mathbf{p} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

- (a) Find $\mathbf{p} + \mathbf{q}$. [1]

- (b) Find $2\mathbf{p} - 3\mathbf{q}$. [2]

- (c) Find $|\mathbf{q}|$. [2]

Answer

- (d) $m\mathbf{p} + n\mathbf{q}$ is equal to the vector shown. Find the values of m and n . [3]

$$m\mathbf{p} + n\mathbf{q} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

Answer $m = \dots\dots$, $n = \dots\dots$

- 2 A is the point $(-2, 1)$, B is $(3, 3)$ and C is $(5, -1)$.

- (a) Write $\overrightarrow{AB}$ as a column vector. [1]

- (b) Calculate $|\overrightarrow{AB}|$. [2]

Answer

- (c) ABCD is a parallelogram. Find the coordinates of D. [2]

Answer $(\dots\dots, \dots\dots)$

- (d) Calculate the length of the diagonal AC. [2]

Answer

- 3 The vector $\mathbf{v}$ is shown below, where k is positive. $|\mathbf{v}| = 13$.

$$\mathbf{v} = \begin{pmatrix} k \\ 12 \end{pmatrix}$$

- (a) Find the value of k .

[3]

Answer $k = \dots\dots$

- (b) Find the vector in the same direction as $\mathbf{v}$ with magnitude 39.

[1]

- 4 Vectors $\mathbf{u}$, $\mathbf{w}$ and $\mathbf{s}$ are shown below.

$$\mathbf{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} t \\ 3 \end{pmatrix} \quad \mathbf{s} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

- (a) Write $\mathbf{u} + \mathbf{w}$ as a column vector in terms of t .

[1]

- (b) $\mathbf{u} + \mathbf{w}$ is parallel to $\mathbf{s}$. Find the value of t .

[2]

Answer $t = \dots\dots$

- 5 A translation by the vector $\mathbf{t}$ shown below maps shape S onto shape S'.

$$\mathbf{t} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

- (a) One vertex of S' is at (1, 7). Find the coordinates of the corresponding vertex of S.

[2]

Answer (.....,

- (b) Write down the vector of the translation that maps S' back onto S.

[1]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Vector geometry

I can... use vectors to find routes, midpoints and ratios, and prove that lines are parallel or collinear.

Syllabus ref: E7.4 · Extended

In vector geometry you write a vector as a **route** along known vectors. Going against an arrow changes the sign. Two vectors are **parallel** if one is a multiple of the other.

Key points

- $AB = AO + OB = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$ when $OA = \mathbf{a}$, $OB = \mathbf{b}$.
- If P divides AB in the ratio 1 : 3, then $AP = \frac{1}{4} AB$.
- Points X, Y, Z are **collinear** if XY is a multiple of YZ (they share point Y).

Key words

position vector

route

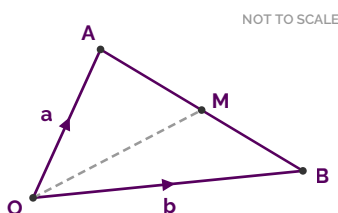
midpoint

parallel

collinear

Worked example 1

$OA = \mathbf{a}$ and $OB = \mathbf{b}$. M is the midpoint of AB. Find OM in terms of $\mathbf{a}$ and $\mathbf{b}$.



$$AB = -\mathbf{a} + \mathbf{b}, \text{ so } AM = \frac{1}{2}(\mathbf{b} - \mathbf{a}).$$

$$OM = OA + AM = \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

Worked example 2

Show that $PQ = 2\mathbf{a} - 6\mathbf{b}$ is parallel to $RS = -\mathbf{a} + 3\mathbf{b}$.

$$PQ = -2(-\mathbf{a} + 3\mathbf{b}) = -2 RS. \text{ PQ is a multiple of RS, so they are parallel (and PQ is twice as long).}$$

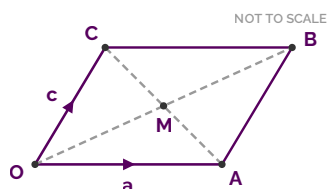
Common mistakes

- Forgetting to change the sign when going backwards along a vector.
- Using the ratio 1 : 3 as the fraction $\frac{1}{3}$ instead of $\frac{1}{4}$.

Exam tip: Write the route in letters first (e.g. $OP = OA + AP$) before you substitute any vectors.

Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

OABC is a parallelogram. $OA = \mathbf{a}$ and $OC = \mathbf{c}$. M is the point where the diagonals cross. Find AB in terms of $\mathbf{a}$ and/or $\mathbf{c}$.



2 Using the parallelogram in question 1, find CB.

3 Find OB.

4 Find AC.

5 Find BO.

6 Find OM.

7 Find AM.

8 $OA = \mathbf{a}$, $OB = \mathbf{b}$. P is on AB with AP : PB = 1 : 3. Find AP.

9 For the same triangle, find OP. Simplify your answer.

10 Simplify $3(\mathbf{a} - \mathbf{b}) + 2(2\mathbf{b} - \mathbf{a})$.

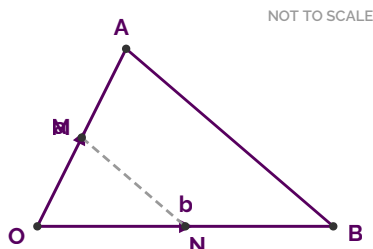
11 Is $2\mathbf{a} + 6\mathbf{b}$ parallel to $\mathbf{a} + 3\mathbf{b}$? Explain.

12 $XY = 4\mathbf{a} - \mathbf{b}$ and $YZ = 8\mathbf{a} - 2\mathbf{b}$. What can you say about X, Y and Z?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Give lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.

- 1 OAB is a triangle. $OA = \mathbf{a}$ and $OB = \mathbf{b}$. M is the midpoint of OA and N is the midpoint of OB.



- (a) Find AB in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

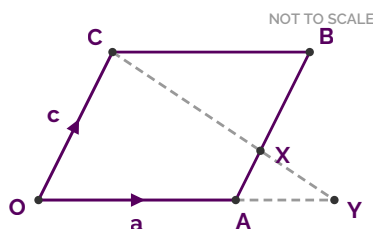
Answer AB =

- (b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

Answer MN =

- (c) Write down two facts about the lines MN and AB. [2]

- 2 OABC is a parallelogram. $OA = \mathbf{a}$ and $OC = \mathbf{c}$. X is the point on AB such that $AX : XB = 1 : 2$. Y is the point on OA extended such that $OY = \frac{3}{2}\mathbf{a}$.



- (a) Find CX in terms of $\mathbf{a}$ and $\mathbf{c}$. Simplify your answer. [2]

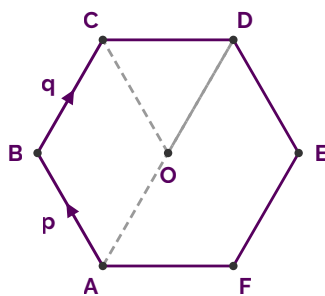
Answer CX =

- (b) Find XY in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]

Answer XY =

- (c) Show that C, X and Y lie on a straight line. [2]

- 3 ABCDEF is a regular hexagon with centre O. $AB = \mathbf{p}$ and $BC = \mathbf{q}$.



- (a) Find CD in terms of $\mathbf{p}$ and $\mathbf{q}$. [2]

Answer $CD = \dots\dots$

- (b) Find AD in terms of $\mathbf{q}$. [2]

Answer $AD = \dots\dots$

- (c) Find FC . [1]

Answer $FC = \dots\dots$

- 4 $OP = 3\mathbf{a} + \mathbf{b}$ and $OQ = \mathbf{a} + k\mathbf{b}$, where k is a constant.

- (a) Find PQ in terms of $\mathbf{a}$, $\mathbf{b}$ and k . [2]

Answer $PQ = \dots\dots$

- (b) PQ is parallel to $\mathbf{a} - 2\mathbf{b}$. Find the value of k . [3]

Answer $k = \dots\dots$

- 5 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. P is the point on OA such that $OP : PA = 2 : 1$. Q is the midpoint of AB. R is the point on OB extended such that $OR = 2\mathbf{b}$.

- (a) Find OQ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

Answer $OQ = \dots\dots$

- (b) Find PQ and PR in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

- (c) Prove that P, Q and R are collinear. [2]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

End-of-book check

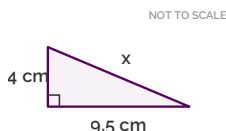
End-of-book check

Name _____ Date _____ Score _____ / 12

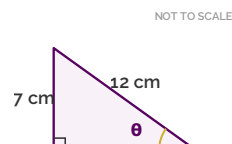
Answer every question. Give lengths to 3 significant figures and angles to 1 decimal place.

Find x .

1

Answer $x = \dots\dots\dots$ cmFind θ .

2

Answer $\theta = \dots\dots\dots^\circ$

3

From 30 m away, the angle of elevation of the top of a tower is 40° . Find the height of the tower.

4

The bearing of B from A is 250° . Find the bearing of A from B.

5

Find the exact value of $\cos 30^\circ \times \tan 60^\circ$.

6

Solve $\cos x = -0.5$ for $0^\circ \leq x \leq 360^\circ$.

7

In triangle ABC, $a = 10$ cm, $A = 50^\circ$ and $B = 65^\circ$. Find b .

8

In triangle ABC, $b = 6$ cm, $c = 9$ cm and $A = 75^\circ$. Find a .

9

Find the area of a triangle with sides 5 cm and 7 cm and an included angle of 120° .

10

Find the length of the longest diagonal of a 3 cm by 4 cm by 12 cm cuboid.

11

An enlargement, centre (0, 0), maps (2, 1) onto (-6, -3). Find the scale factor.

12

A vector has top number -5 and bottom number 12. Find its magnitude.



Practice paper

Name _____ Date _____ Marks _____ / 60

Time: 1 hour

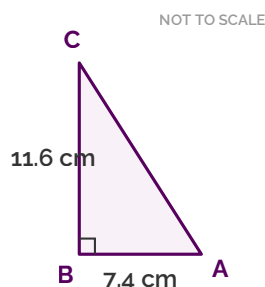
Calculator allowed unless a question says otherwise

Total: 60 marks

Give non-exact answers to 3 significant figures (angles to 1 decimal place).

Answer all questions. Show your working.

- 1 Triangle ABC has a right angle at B. $AB = 7.4$ cm and $BC = 11.6$ cm.



- (a) Calculate AC.

[2]

Answer AC = cm

- (b) Calculate angle BAC.

[2]

Answer°

- 2 A helicopter H hovers 260 m vertically above a point P on the sea. A boat B is due east of P. The angle of depression of B from H is 8° .

- (a) Calculate the distance PB.

[2]

Answer m

- (b) A second boat C is 1500 m due south of P. Calculate the bearing of B from C.

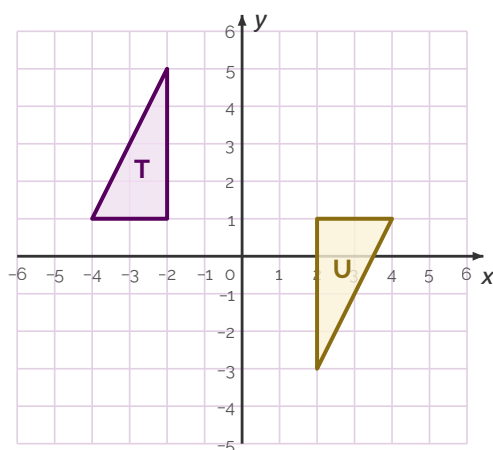
[3]

Answer°

- (c) Calculate the distance from the helicopter to boat B.

[2]

- 3 The diagram shows triangles T and U.



- (a) Describe fully the single transformation that maps T onto U. [3]

- (b) Reflect triangle T in the x-axis. [2]

- (c) Enlarge triangle T by scale factor $\frac{1}{2}$, centre $(-6, -3)$. [2]

- 4 Vectors **a** and **b** are shown below.

$$\mathbf{a} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

- (a) Find $3\mathbf{a} + 2\mathbf{b}$. [2]

- (b) Find $|\mathbf{a} - \mathbf{b}|$. [3]

Answer

- 5 Do not use a calculator in part (a).

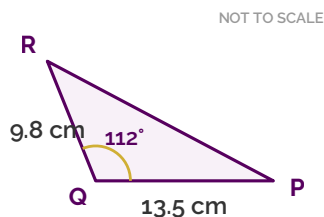
- (a) Find the exact value of $\sin 45^\circ \times \cos 45^\circ + \tan^2 60^\circ$. [2]

Answer

- (b) Solve $\sin x = -0.35$ for $0^\circ \leq x \leq 360^\circ$. [3]

Answer $x =$

- 6 In triangle PQR, PQ = 13.5 cm, QR = 9.8 cm and angle PQR = 112° .



- (a) Calculate PR.

[4]

Answer PR = cm

- (b) Calculate angle QPR.

[3]

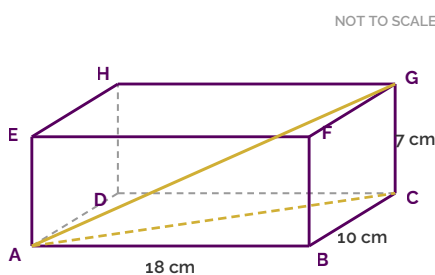
Answer $^\circ$

- (c) Calculate the area of triangle PQR.

[2]

Answer cm^2

- 7 The diagram shows a cuboid ABCDEFGH. AB = 18 cm, BC = 10 cm and CG = 7 cm.



- (a) Calculate the length AG.

[3]

Answer AG = cm

- (b) Calculate the angle between AG and the base ABCD.

[3]

Answer $^\circ$

- 8 The area of triangle XYZ is 54 cm^2 . $XY = 15 \text{ cm}$ and $YZ = 9 \text{ cm}$. Angle XYZ is obtuse.

(a) Calculate angle XYZ.

[3]

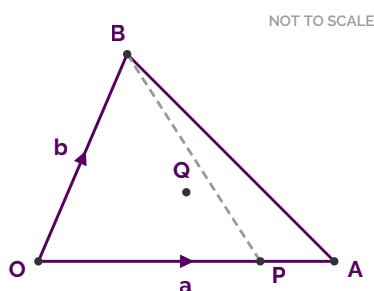
Answer°

(b) Calculate XZ.

[2]

Answer XZ = cm

- 9 OAB is a triangle. $OA = \mathbf{a}$ and $OB = \mathbf{b}$. P is the point on OA such that $OP : PA = 3 : 1$. Q is the point on BP such that $BQ : QP = 2 : 1$.



(a) Find BP in terms of $\mathbf{a}$ and $\mathbf{b}$.

[2]

Answer BP =

(b) Show that $OQ = \frac{1}{2} \mathbf{a} + \frac{1}{3} \mathbf{b}$.

[2]

(c) M is the point on AB such that $OM = \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{b}$. Explain why O, Q and M are **not** on a straight line.

[2]

- 10 Triangle J has vertices (1, 1), (3, 1) and (1, 4).

(a) J is enlarged by scale factor -2 , centre (0, 0). Find the coordinates of the vertices of the image.

[2]

(b) The area of J is 3 square units. Find the area of the image.

[2]

Answer units²

(c) Describe fully the single transformation that maps the image back onto J.

[2]





FOR TEACHERS & PARENTS

Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

Starting point check

- 1 10 cm
- 2 23.5
- 3 0.5
- 4 48°
- 5 270°
- 6 $x = 3$
- 7 $(-2, 5)$
- 8 $(5, 1)$
- 9 18 cm^2
- 10 7.00 cm
- 11 10 cm by 6 cm
- 12 $5\sqrt{2}$

Unit 1 · Sheet A — Pythagoras' theorem

- 1 13 cm
- 2 10.5 cm
- 3 11.9 cm
- 4 15 cm
- 5 12.6 cm
- 6 6.58 cm
- 7 26 cm
- 8 7.07 cm
- 9 Yes: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$
- 10 10 units
- 11 12 cm
- 12 7.74 km

Unit 1 · Sheet B — Pythagoras' theorem

- 1 (a) $AC^2 = 6.4^2 + 4.8^2$ M1; AC = 8 cm A1 [2] (b) $CD^2 = 13.5^2 - 8^2$ M1; = 118.25 M1 (dep); CD = 10.9 cm A1 [3] (c) $6.4 + 4.8 + 10.9 + 13.5 = 35.6$ cm B1ft [1]
- 2 (a) $6.5^2 - 1.8^2$ M1; $\sqrt{42.25 - 3.24} = \sqrt{39.01}$ M1; 6.25 m A1 [3] (b) new height 5.7458... B1ft; $6.5^2 - 5.746^2$ M1; 3.04 m A1 [3]
- 3 (a) horizontal 8, vertical 6 B1; $\sqrt{8^2 + 6^2}$ M1; 10 units A1 [3] (b) AC horizontal, BC vertical so angle C = 90° B1; $\frac{1}{2} \times 8 \times 6$ M1; 24 square units A1 [3]
- 4 (a) half-diagonals 8 and 15 B1; $\sqrt{8^2 + 15^2}$ M1; 17 cm A1 [3] (b) $4 \times 17 = 68$ cm B1ft [1]
- 5 (a) half base 7 B1; $\sqrt{11^2 - 7^2} = \sqrt{72}$ M1; 8.49 cm A1 [3] (b) $\frac{1}{2} \times 14 \times 8.49$ M1; 59.4 cm^2 A1 [2]

Unit 2 · Sheet A — Trigonometry in right-angled triangles

- 1 6.88 cm
- 2 6.68 cm
- 3 9.82 cm
- 4 19.8 cm
- 5 51.3°
- 6 66.0°
- 7 38.7°
- 8 6.68
- 9 69.5°
- 10 19.2
- 11 12.6
- 12 26.4°

Unit 2 · Sheet B — Trigonometry in right-angled triangles

- 1 (a) $13 \sin 68^\circ$ M1; 12.1 cm A1 [2] (b) $QM = 13 \cos 68^\circ$ M1; = 4.870... A1; QR = 9.74 cm A1 [3] (c) $\frac{1}{2} \times 9.74 \times 12.1$ M1; 58.7 cm^2 A1 [2]
- 2 (a) $\sin \theta = 0.9/7.5$ M1; 6.9° A1 [2] (b) $\sin 4.5^\circ = 0.9/L$ M1; $L = 0.9 \div \sin 4.5^\circ$ M1; 11.5 m A1 [3] (c) $7.5 \cos 6.9^\circ$ or $\sqrt{7.5^2 - 0.9^2}$ M1; 7.45 m A1 [2]
- 3 (a) $\tan 40^\circ = BD/8$ M1; BD = 6.71 cm A1 [2] (b) $\tan 25^\circ = 6.71/DC$ M1; DC = 14.4 cm A1ft [2] (c) $BD/\sin 25^\circ$ or Pythagoras M1; 15.9 cm A1ft [2]
- 4 (a) $\tan^{-1}(6.5/14.2)$ M1; 24.6° A1 [2] (b) $180 - 2 \times 24.6^\circ$ M1; 130.8° A1ft [2]
- 5 (a) shortest distance is perpendicular B1; $50 \sin 32^\circ = 26.5$ m B1 [2] (b) The perpendicular from a point to a line is the shortest distance B1 [1]

Unit 3 · Sheet A — Angles of elevation and depression

- 1 25.2 m
- 2 16.7°
- 3 180 m
- 4 21.8°
- 5 33.9 m
- 6 410 m
- 7 11.2 m
- 8 14.8 m
- 9 4230 m
- 10 53.1°
- 11 28°
- 12 61.0°

Unit 3 · Sheet B — Angles of elevation and depression

- 1** (a) $50 \div \tan 22^\circ$ M1; 124 m A1 **[2]** (b) $BF = 50 \div \tan 37^\circ = 66.4$ M1A1; $AB = 57.4$ m A1 **[3]**
- 2** (a) P, because its angle of depression is larger B1 **[1]** (b) $95/\tan 14^\circ = 381$ M1A1; $95/\tan 26^\circ = 195$ A1; $PQ = 186$ m A1 **[4]**
- 3** (a) $28 \tan 18^\circ$ M1; 9.10 m A1 **[2]** (b) $28 \tan 40^\circ = 23.5$ M1A1; total 32.6 m A1ft **[3]**
- 4** (a) $150 \sin 63^\circ$ M1; 134 m A1 **[2]** (b) horizontal distance from R = $150 \cos 63^\circ = 68.1$ M1; from Lina = 28.1 A1; $\tan^{-1}(134 \div 28.1) = 78.1^\circ$ A1 **[3]**
- 5** (a) $h = x \tan 40^\circ$ B1; $h = (x + 20) \tan 25^\circ$ B1 **[2]** (b) $x \tan 40^\circ = (x + 20) \tan 25^\circ$ M1; $x = 20 \tan 25^\circ \div (\tan 40^\circ - \tan 25^\circ) = 25.0$ m A1; $h = 21.0$ m A1 **[3]**

Unit 4 · Sheet A — Bearings and trigonometry

- 1** 225°
- 2** 252°
- 3** 125°
- 4** 15.3 km
- 5** 303 km
- 6** 3.03 km
- 7** 029.1°
- 8** 233.1°
- 9** 284.9°
- 10** 15.5 km
- 11** 19.2 km
- 12** 16.1 km

Unit 4 · Sheet B — Bearings and trigonometry

- 1** (a) angle at Q between QP (bearing 228°) and QR (bearing 138°): $228 - 138 = 90^\circ$ M1A1 (or co-interior: $180 - 48 = 132$, $132 - 42 = 90$) **[2]** (b) $\sqrt{18^2 + 11^2}$ M1; 21.1 km A1 **[2]** (c) $\tan^{-1}(11/18) = 31.4^\circ$ M1A1; $048 + 31.4 = 079.4^\circ$ A1 **[3]**
- 2** (a) $64 \cos 20^\circ$ or $64 \sin 110^\circ$ M1; 60.1 km A1 **[2]** (b) $64 \sin 20^\circ$ M1; 21.9 km A1 **[2]** (c) 290° B1 **[1]**
- 3** (a) $6.2 \tan 34^\circ$ M1; 4.18 km A1 **[2]** (b) $6.2 \div \cos 34^\circ$ M1; 7.48 km A1 **[2]** (c) $034 + 180$ M1; 214° A1 **[2]**
- 4** (a) $3.1 \sin 65^\circ$ M1; 2.81 km A1 **[2]** (b) $2.4 - 3.1 \cos 65^\circ$ M1; 1.09 km A1 **[2]** (c) $\sqrt{(2.81^2 + 1.09^2)} = 3.01$ km M1A1; bearing $\tan^{-1}(\text{east/north}) = 068.8^\circ$ A1 **[3]**

Unit 5 · Sheet A — Exact trigonometric values

- 1** $1/2$
- 2** $\sqrt{2}/2$ (or $1/\sqrt{2}$)
- 3** $\sqrt{3}$
- 4** $\sqrt{3}/2$
- 5** $\sqrt{3}/3$ (or $1/\sqrt{3}$)
- 6** 1
- 7** $3/2$
- 8** 1
- 9** $4\sqrt{3}$ cm
- 10** $5\sqrt{2}$ cm
- 11** 30°
- 12** 12 cm

Unit 5 · Sheet B — Exact trigonometric values

- 1** $\sin 60^\circ = \sqrt{3}/2$; $\cos 30^\circ = \sqrt{3}/2$; $\cos 90^\circ = 0$; $\tan 30^\circ = \sqrt{3}/3$ (or $1/\sqrt{3}$); $\tan 45^\circ = 1$ — B4 for all five, B3 for 4, B2 for 3, B1 for 2 **[4]**
- 2** (a) $BC = 20 \sin 30^\circ$ M1; 10 cm A1 **[2]** (b) $AB = 20 \cos 30^\circ$ M1; $= 10\sqrt{3}$ cm A1 **[2]** (c) triangle BDC is isosceles so $BD = BC = 10$ (or $10/\tan 45^\circ$) M1A1; $AD = 10\sqrt{3} - 10 = 10(\sqrt{3} - 1)$ A1 **[3]**
- 3** (a) $\sqrt{3}/2 \times 1/\sqrt{3}$ M1; $= 1/2$ A1 **[2]** (b) $3/4 - 1/4$ M1; $= 1/2$ A1 **[2]** (c) $\sqrt{3} \div \sqrt{3}/2$ M1; $= 2$ A1 **[2]**
- 4** (a) $8 \sin 60^\circ$ or $\sqrt{(8^2 - 4^2)}$ M1; $= 8 \times \sqrt{3}/2 = 4\sqrt{3}$ A1 **[2]** (b) $\frac{1}{2} \times 8 \times 4\sqrt{3}$ M1; $16\sqrt{3}$ cm² A1 **[2]**
- 5** (a) $\tan RPS = 6\sqrt{3}/6 = \sqrt{3}$ M1; 60° A1 **[2]** (b) $6 \div \cos 60^\circ$ or $\sqrt{(36 + 108)}$ M1; 12 cm A1 **[2]**

Unit 6 · Sheet A — Trigonometric graphs and equations

- 1** 1
- 2** 180°
- 3** 90° , 270°
- 4** 40
- 5** 60 (also accept -60)
- 6** 30° , 150°
- 7** 72.5° , 287.5°
- 8** 63.4° , 243.4°
- 9** 203.6° , 336.4°
- 10** 143.1° , 216.9°
- 11** 41.8° , 138.2°
- 12** $\cos x$

Unit 6 · Sheet B — Trigonometric graphs and equations

- 1** (a) starts at (0, 1), through (90, 0), min (180, -1), (270, 0), ends at (360, 1) B2 (B1 for correct shape with one error) **[2]** (b) 2 B1 **[1]** (c) $\sin^{-1}(-0.5) = -30^\circ$ M1; $180 + 30 = 210^\circ$ A1; $360 - 30 = 330^\circ$ A1 **[3]**
- 2** (a) $\sin x = 0.75$ M1; 48.6° A1; 131.4° A1 **[3]** (b) $\tan x = -3$, $\tan^{-1}(-3) = -71.6^\circ$ M1; 108.4° A1; 288.4° A1 **[3]** (c) $\cos x = 0.4$ M1; 66.4° , 293.6° A1A1 **[3]**
- 3** (a) 290° B1 **[1]** (b) 110° B1; 250° B1 **[2]**
- 4** (a) $x = 90$ B1; $x = 270$ B1 **[2]** (b) 35.0° B1; 215.0° B1ft **[2]**
- 5** (a) 1 B1; -1 B1 **[2]** (b) $k = 1$; $x = 90^\circ$ B1; $k = -1$; $x = 270^\circ$ B1 **[2]**

Unit 7 · Sheet A — The sine rule

- 1** 9.05 cm
- 2** 20.3 cm
- 3** 13.7 cm
- 4** 40.8°
- 5** 39.3°
- 6** 50
- 7** 126.9°
- 8** 9.93 cm
- 9** 48.6° or 131.4°
- 10** 8.16 cm
- 11** 9.04 cm
- 12** No angle is known, so no side-opposite angle pair

Unit 7 · Sheet B — The sine rule

- 1** (a) $BC/\sin 38^\circ = 14.5/\sin 57^\circ$ M1; $BC = 14.5 \sin 38^\circ \div \sin 57^\circ$ M1; 10.6 cm A1 **[3]** (b) angle C = 85° B1; $AB = 14.5 \sin 85^\circ \div \sin 57^\circ$ M1; 17.2 cm A1 **[3]**
- 2** (a) $60 - 20 = 40$ B1; bearing of A from B is 240° , $310 - 240 = 70$ B1 **[2]** (b) angle ALB = 70° B1; $BL = 40 \sin 40^\circ \div \sin 70^\circ$ M1; 27.4 km A1 **[3]** (c) $BL \sin 70^\circ$ M1; 25.7 km A1ft **[2]**
- 3** (a) $\sin Q/8.4 = \sin 35^\circ/5.6$ M1; $\sin Q = 0.8604$ A1; $Q = 59.4^\circ$ A1; or 120.6° A1 **[4]** (b) $180 - 35 - 120.6 = 24.4^\circ$ B1ft **[1]**
- 4** (a) $BD = 9 \sin 80^\circ \div \sin 55^\circ$ M1; 10.8 cm A1 **[2]** (b) $CD/\sin 65^\circ = BD/\sin 70^\circ$ M1; $CD = 10.8 \sin 65^\circ \div \sin 70^\circ$ M1; 10.4 cm A1ft **[3]** (c) $AD = 9 \sin 45^\circ \div \sin 55^\circ = 7.77$, $BC = BD \sin 45^\circ \div \sin 70^\circ = 8.14$ M1; $\frac{1}{2} \times 9 \times AD \sin 80^\circ + \frac{1}{2} \times BD \times BC \sin 65^\circ$ M1; 74.4 cm² A1 **[3]**

Unit 8 · Sheet A — The cosine rule and area of a triangle

- 1** 5.87 cm
2 18.5 cm
3 3.86 cm
4 95.7°
5 126.6°
6 71.4 cm²
7 30.0 cm²
8 48.6°
9 12.3 cm
10 obtuse ($4^2 + 5^2 = 41 < 64$)
11 43.3 cm²
12 51.4 cm²

Unit 8 · Sheet B — The cosine rule and area of a triangle

- 1** (a) $85^2 + 120^2 - 2 \times 85 \times 120 \cos 63^\circ$ M1; = 12363.6 A1; $BC = 111$ m A1 **[3]** (b) $\frac{1}{2} \times 85 \times 120 \times \sin 63^\circ$ M1; 4540 m² A1 **[2]** (c) area = $\frac{1}{2} \times BC \times h$ M1; $h = 81.7$ m A1ft **[2]**
- 2**
(a) $\cos R = (12^2 + 9^2 - 7^2) \div (2 \times 12 \times 9)$ M1; = 0.8148 A1; 35.4° A1 **[3]**
(b) $\frac{1}{2} \times 12 \times 9 \times \sin 35.4^\circ$ M1; 31.3 cm² A1ft **[2]**
- 3** (a) angle between BH (bearing 220°) and BI (110°): $220 - 110 = 110$ M1A1 **[2]** (b) $30^2 + 45^2 - 2 \times 30 \times 45 \cos 110^\circ$ M1A1; 62.0 km A1 **[3]**
- 4** (a) $\frac{1}{2} \times 9 \times BC \times \sin 48^\circ = 40$ M1; $BC = 80 \div (9 \sin 48^\circ)$ M1; 12.0 cm A1 **[3]** (b) $AC^2 = 9^2 + 12.0^2 - 2 \times 9 \times 12.0 \cos 48^\circ$ M1A1; 8.94 cm A1ft **[3]**
- 5** (a) $11^2 + 7^2 - 2 \times 11 \times 7 \cos 58^\circ$ M1A1; 9.40 cm A1 **[3]** (b) $11 \times 7 \times \sin 58^\circ$ (or $2 \times \frac{1}{2} \times 11 \times 7 \sin 58^\circ$) M1; 65.3 cm² A1 **[2]**

Unit 9 · Sheet A — Pythagoras and trigonometry in 3D

- 1** 11.2 cm
2 12.8 cm
3 34.6 cm
4 9.43 cm
5 9.85 cm
6 11.5 cm
7 31.4°
8 33.7°
9 7.07 cm
10 13.9 cm
11 59.5°
12 19.8°

Unit 9 · Sheet B — Pythagoras and trigonometry in 3D

- 1** (a) $\sqrt{15^2 + 8^2}$ M1; 17 cm A1 **[2]**
(b) $\sqrt{17^2 + 6^2}$ M1; 18.0 cm A1 **[2]** (c) angle GAC identified M1; $\tan^{-1}(6/17)$ M1; 19.4° A1 **[3]** (d) angle AGB (B is the projection of A) M1; $\sin^{-1}(15/18.0)$ or $\tan^{-1}(15/\sqrt{8^2 + 6^2})$ M1; 56.3° A1 **[3]**
- 2** (a) $AM = \frac{1}{2}\sqrt{16^2 + 16^2} = 11.3$ M1A1; $VA = \sqrt{11.3^2 + 15^2} = 18.8$ cm A1 **[3]** (b) $\tan^{-1}(15/11.3)$ M1; 53.0° A1 **[2]** (c) angle VNM M1; $MN = 8$ B1; $\tan^{-1}(15/8) = 61.9^\circ$ A1 **[3]**
- 3** (a) $PR = \sqrt{40^2 + 30^2} = 50$ M1A1; $\tan^{-1}(24/50) = 25.6^\circ$ A1 **[3]** (b) $\sqrt{50^2 + 24^2}$ M1; 55.5 m A1 **[2]**
- 4** (a) $\sqrt{40^2 + 30^2 + 20^2}$ M1; = 53.9 cm A1; $55 > 53.9$ so it will not fit A1 **[3]**

Unit 10 · Sheet A — Reflections, rotations and translations

- 1** reflection in the y-axis ($x = 0$)
2 rotation 180° about (0, 0)
3 rotation 90° clockwise about (0, 0)
4 translation by (2 over 2)
5 reflection in the x-axis ($y = 0$)
6 (3, 2)
7 (-2, 3)
8 (2, -3)
9 (-1, -2)
10 (2, 3)
11 (-1, 4)
12 reflection, rotation, translation

Unit 10 · Sheet B — Reflections, rotations and translations

- 1** (a) vertices (1, 0), (3, 0), (3, -3) B2 (B1 for reflection in another horizontal line or 2 correct vertices) **[2]** (b) vertices (-3, 2), (-3, 4), (-6, 4) B2 (B1 for correct orientation, wrong centre) **[2]** (c) vertices (4, -6), (6, -6), (6, -3) B2 (B1 for one component correct) **[2]** (d) vertices (-2, -1), (-2, -3), (-5, -3) B2 (B1 for 2 correct vertices) **[2]**
(e) rotation B1; 90° clockwise B1; centre (-1, 1) B1 **[3]**
- 2** (a) rotation B1; 90° clockwise (or -90°) B1; about (0, 0) B1 **[3]**
(b) reflection B1; in the line $y = x$ B1 **[2]** (c) reflection in $y = 0$ B2 (B1 for reflection with wrong line) **[2]**
- 3** (a) (4, -1) B1; (1, 4) B1ft **[2]** (b) reflection B1; in $y = x$ B1 **[2]**
- 4** (a) perpendicular bisector of the two points, or trial with tracing paper M1; check 90° M1; centre (2, 1) A1 **[3]** (b) (3, -2) B2 **[2]**

Unit 11 · Sheet A — Enlargement

- 1** enlargement, scale factor $1/2$, centre (-2, -2)
2 enlargement, scale factor 3, centre (-1, 2)
3 enlargement, scale factor -1, centre (1, 0) (or rotation 180° about (1, 0))
4 enlargement, scale factor 2, centre (-2, -2)
5 (6, 9)
6 (3, 2)
7 (7, 1)
8 (-1, -1)
9 (-2, -2)
10 45 cm²
11 6 cm
12 rotation 180° about the same centre

Unit 11 · Sheet B — Enlargement

- 1** (a) vertices (3, 2), (7, 2), (7, 4) B3 (B2 for correct size, wrong position; B1 for 2 correct vertices) [3] (b) vertices (-2, -2), (-6, -2), (-6, -4) B3 (B2 for scale factor 2 or -2 with wrong centre; B1 for correct size and orientation) [3]
- 2** (a) enlargement B1; scale factor 3 B1; centre (-6, 6) B1 [3] (b) enlargement B1; scale factor $-1/2$ B1; centre (-2, 2) B1 [3]
- 3** (a) 12×4 or $2(12 \times 8)$ M1; 40 cm A1 [2] (b) 6×4^2 or 12×8 M1; 96 cm² A1 [2] (c) $1.5 \div 6 = 1/4$ M1; scale factor $1/2$ A1 [2]
- 4** (a) (3, 3) B2 (B1 for one coordinate) [2] (b) (-11, -4) B2 (B1 for one coordinate) [2]
- 5** (a) (1, 2), (3, 2), (3, 5) B2 (B1 for two correct) [2] (b) $12 \times (1/2)^2$ M1; 3 square units A1 [2]

Unit 12 · Sheet A — Vectors and column vectors

- 1** (3 over -1)
- 2** (5 over -5)
- 3** (0 over 15)
- 4** (7 over -4)
- 5** (6 over -2)
- 6** 5
- 7** 2.24
- 8** 3.16
- 9** (-4 over 8)
- 10** $p = (3 \text{ over } 3)$, $q = (2 \text{ over } -4)$, $r = (-4 \text{ over } -1)$
- 11** (-5 over 6)
- 12** Yes: $e = 1.5a$

Unit 12 · Sheet B — Vectors and column vectors

- 1** (a) (2 over 6) B1 [1] (b) (19 over -8) B2 (B1 for one component) [2] (c) $\sqrt{3^2 + 4^2}$ M1; 5 A1 [2] (d) $5m - 3n = 7$ and $2m + 4n = 8$ M1; eliminate one variable M1; $m = 2$, $n = 1$ A1 [3]
- 2** (a) (5 over 2) B1 [1] (b) $\sqrt{5^2 + 2^2}$ M1; 5.39 A1 [2] (c) $DC = AB$ or $AD = BC = (2 \text{ over } -4)$ M1; $D = (0, -3)$ A1 [2] (d) $AC = (7 \text{ over } -2)$ M1; $\sqrt{53} = 7.28$ A1 [2]
- 3** (a) $k^2 + 12^2 = 13^2$ M1; $k^2 = 25$ M1; $k = 5$ A1 [3] (b) $3v = (15 \text{ over } 36)$ B1ft [1]
- 4** (a) $(2 + t \text{ over } 2)$ B1 [1] (b) $2 + t = 3 \times 2$ M1; $t = 4$ A1 [2]
- 5** (a) $(1 + 4, 7 - 3)$ M1; (5, 4) A1 [2] (b) (4 over -3) B1 [1]

Unit 13 · Sheet A — Vector geometry

- 1** c
- 2** a
- 3** $a + c$
- 4** $-a + c$
- 5** $-a - c$
- 6** $1/2a + 1/2c$
- 7** $-1/2a + 1/2c$
- 8** $-1/4a + 1/4b$
- 9** $3/4a + 1/4b$
- 10** $a + b$
- 11** Yes: $2a + 6b = 2(a + 3b)$
- 12** $YZ = 2XY$, so X, Y, Z are collinear (YZ is twice XY)

Unit 13 · Sheet B — Vector geometry

- 1** (a) $b - a$ B1 [1] (b) $MN = MO + ON = -\frac{1}{2}a + \frac{1}{2}b$ M1; $\frac{1}{2}b - \frac{1}{2}a$ A1 [2]
- (c) MN is parallel to AB B1; MN is half the length of AB B1 [2]
- 2** (a) $CX = CO + OA + AX = -c + a + 1/3 c$ M1; $a - 2/3 c$ A1 [2] (b) $XY = XA + AY = -1/3 c + 1/2 a$ M1; $\frac{1}{2}a - 1/3 c$ A1 [2] (c) $CX = 2XY$ so CX is parallel to XY M1; and they share point X, so collinear A1 [2]
- 3** (a) $CD = q - p$ B2 (M1 for a correct route) [2] (b) $AD = AO + OD = q + q$ M1; $2q$ A1 [2] (c) $2p$ B1 [1]
- 4** (a) $OQ = OP$ M1; $-2a + (k - 1)b$ A1 [2] (b) $-2a + (k - 1)b = -2(a - 2b)$ M1; $k - 1 = 4$ M1; $k = 5$ A1 [3]
- 5** (a) $\frac{1}{2}a + \frac{1}{2}b$ B1 [1] (b) $PQ = -1/6 a + \frac{1}{2}b$ B1; $PR = -2/3 a + 2b$ B1 [2] (c) $PR = 4PQ$ M1; parallel with common point P, so collinear A1 [2]

End-of-book check

- 1** 10.3 cm
- 2** 35.7°
- 3** 25.2 m
- 4** 070°
- 5** $3/2$
- 6** 120°, 240°
- 7** 11.8 cm
- 8** 9.44 cm
- 9** 15.2 cm²
- 10** 13 cm
- 11** -3
- 12** 13

Practice paper

- 1** (a) $\sqrt{7.4^2 + 11.6^2}$ M1; 13.8 cm A1 [2] (b) $\tan^{-1}(11.6/7.4)$ M1; 57.5° A1 [2]
- 2** (a) $260 \div \tan 8^\circ$ M1; 1850 m A1 [2] (b) $\tan^{-1}(1850 \div 1500)$ M1; = 51.0 A1; bearing 051.0° A1 [3] (c) $260 \div \sin 8^\circ$ or $\sqrt{260^2 + PB^2}$ M1; 1870 m A1 [2]
- 3** (a) rotation B1; 180° B1; about (0, 1) B1 (or enlargement, scale factor -1, centre (0, 1)) [3] (b) vertices (-4, -1), (-2, -1), (-2, -5) B2 (B1 for reflection in another horizontal line) [2] (c) vertices (-5, -1), (-4, -1), (-4, 1) B2 (B1 for correct size) [2]
- 4** (a) $(-1 \text{ over } 19)$ B2 (B1 for one component) [2] (b) $a - b = (-7 \text{ over } 3)$ B1; $\sqrt{49 + 9}$ M1; 7.62 A1 [3]
- 5** (a) $1/2 + 3$ M1; $7/2$ A1 [2] (b) $\sin^{-1}(-0.35) = -20.5$ M1; 200.5° A1; 339.5° A1 [3]
- 6** (a) $13.5^2 + 9.8^2 - 2 \times 13.5 \times 9.8 \cos 112^\circ$ M2 (M1 for correct substitution with one error); = 377.41 A1; $PR = 19.4$ cm A1 [4] (b) $\sin P / 9.8 = \sin 112^\circ / 19.4$ M1; $\sin P = 0.4677$ A1; 27.9° A1 [3] (c) $\frac{1}{2} \times 13.5 \times 9.8 \times \sin 112^\circ$ M1; 61.3 cm² A1 [2]
- 7** (a) $AC^2 = 18^2 + 10^2$ M1; $AG = \sqrt{424 + 49}$ M1; 21.7 cm A1 [3] (b) angle GAC M1; $\tan^{-1}(7/\sqrt{424})$ M1; 18.8° A1 [3]
- 8** (a) $\frac{1}{2} \times 15 \times 9 \times \sin Y = 54$ M1; $\sin Y = 0.8$ A1; $Y = 180 - 53.1 = 126.9^\circ$ A1 [3] (b) $15^2 + 9^2 - 2 \times 15 \times 9 \cos 126.9^\circ$ M1; 21.6 cm A1 [2]
- 9** (a) $BO + OP$ M1; $3/4 a - b$ A1 [2] (b) $OQ = b + 2/3(3/4 a - b)$ M1; = $1/2 a + 1/3 b$ A1 [2] (c) OM is not a multiple of OQ (ratio of coefficients 1 : 1 $\neq$ 3 : 2) B2 [2]
- 10** (a) (-2, -2), (-6, -2), (-2, -8) B2 (B1 for two correct) [2] (b) $3 \times (-2)^2$ M1; 12 square units A1 [2] (c) enlargement, scale factor $-1/2$ B1, centre (0, 0) B1 [2]