



BRITISH LEARNING CENTRE

CAMBRIDGE IGCSE · 0580

# Mathematics

## Geometry & Mensuration

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Fourteen units covering Topic 4 Geometry and Topic 5 Mensuration of the Cambridge IGCSE Mathematics 0580 syllabus, for Core and Extended candidates. Each unit has clear notes with worked examples, practice questions and exam-style questions with full mark schemes.

Name: .....

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Key knowledge · Practice · Exam-style questions · Mark schemes

# How to use this workbook

1

## Key knowledge

Read the notes, worked examples and common mistakes.

2

## Practice

Build the skill on Sheet A.

3

## Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [ ].

4

## Check & reflect

Mark your work with the mark scheme, then colour the tracker.

## My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

| #  | Syllabus ref                         | Topic · I can...                                                                                                                                            | R | A | G |
|----|--------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|---|
| 1  | <b>C4.1, E4.1</b><br>Core & Extended | <b>Geometrical terms and congruence</b><br>I can use the correct words for angles, lines, polygons, circles and solids, and spot congruent shapes.          |   |   |   |
| 2  | <b>C4.2, E4.2</b><br>Core & Extended | <b>Measuring, constructions and nets</b><br>I can measure and draw lines and angles, construct triangles with compasses and use nets.                       |   |   |   |
| 3  | <b>C4.3, E4.3</b><br>Core & Extended | <b>Scale drawings and bearings</b><br>I can use map scales, make scale drawings and use three-figure bearings.                                              |   |   |   |
| 4  | <b>C4.6, E4.6</b><br>Core & Extended | <b>Angles and parallel lines</b><br>I can find missing angles using angle facts and parallel lines, giving geometrical reasons.                             |   |   |   |
| 5  | <b>C4.6, E4.6</b><br>Core & Extended | <b>Angles in polygons</b><br>I can use the angle sums of triangles, quadrilaterals and polygons, including regular polygons.                                |   |   |   |
| 6  | <b>C4.5, E4.5</b><br>Core & Extended | <b>Symmetry in 2D and 3D</b><br>I can find lines of symmetry and rotational symmetry, and planes of symmetry of solids (Extended).                          |   |   |   |
| 7  | <b>C4.4, E4.4</b><br>Core & Extended | <b>Similar shapes</b><br>I can use scale factors to find lengths in similar shapes, and areas and volumes (Extended).                                       |   |   |   |
| 8  | <b>C4.7, E4.7</b><br>Core & Extended | <b>Circle theorems</b><br>I can use the angle in a semicircle, tangent-radius and (Extended) centre and same-segment theorems.                              |   |   |   |
| 9  | <b>E4.7, E4.8</b><br>Extended        | <b>Cyclic quadrilaterals, tangents and chords</b><br>I can use cyclic quadrilaterals, the alternate segment theorem and the symmetry properties of circles. |   |   |   |
| 10 | <b>C5.1, E5.1</b><br>Core & Extended | <b>Units of measure</b><br>I can convert between metric units of length, mass, capacity, area and volume.                                                   |   |   |   |
| 11 | <b>C5.2, E5.2</b><br>Core & Extended | <b>Perimeter and area</b><br>I can find perimeters and areas of rectangles, triangles, parallelograms, trapezia and compound shapes.                        |   |   |   |
| 12 | <b>C5.3, E5.3</b><br>Core & Extended | <b>Circles, arcs and sectors</b><br>I can find circumferences and areas of circles, and arc lengths and areas of sectors.                                   |   |   |   |
| 13 | <b>C5.4, E5.4</b><br>Core & Extended | <b>Prisms and cylinders</b><br>I can find the volume and surface area of cuboids, prisms and cylinders.                                                     |   |   |   |
| 14 | <b>C5.4, E5.5</b><br>Core & Extended | <b>Pyramids, cones and spheres</b><br>I can find volumes and surface areas of pyramids, cones, spheres and compound solids.                                 |   |   |   |



CHECK

## Starting point check

### Starting point check

Name \_\_\_\_\_ Date \_\_\_\_\_ Score \_\_\_\_\_ / 12

Answer as many questions as you can. This shows what you already know.

- 1 What type of angle is  $156^\circ$ ?

Answer .....

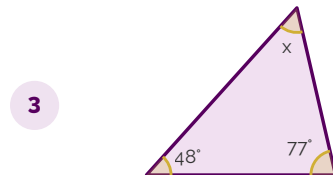
Find x.



Not to scale

Answer  $x = \dots\dots$

Find x.



Not to scale

Answer  $x = \dots\dots$

- 4 Work out the sum of the interior angles of a pentagon.

Answer ..... $^\circ$

- 5 Work out the exterior angle of a regular hexagon.

Answer ..... $^\circ$

- 6 The bearing of B from A is  $070^\circ$ . Find the bearing of A from B.

Answer ..... $^\circ$

- 7 A map has scale 1 : 50 000. Two points are 4 cm apart on the map. Find the real distance in km.

Answer ..... km

- 8 Find the area of a triangle with base 10 cm and height 6 cm.

Answer .....  $\text{cm}^2$

- 9 Find the circumference of a circle of radius 10 cm (3 s.f.).

Answer ..... cm

- 10 Find the volume of a cuboid 6 cm by 5 cm by 2 cm.

Answer .....  $\text{cm}^3$

- 11 Change  $2.5 \text{ m}^2$  to  $\text{cm}^2$ .

Answer .....  $\text{cm}^2$

- 12 How many lines of symmetry does a regular pentagon have?

Answer .....

# Geometrical terms and congruence

**I can...** use the correct words for angles, lines, polygons, circles and solids, and spot congruent shapes.

Syllabus ref: C4.1, E4.1 · Core & Extended

Lines can be **parallel** (never meet) or **perpendicular** (meet at  $90^\circ$ ). Angles are **acute** ( $< 90^\circ$ ), **obtuse** ( $90^\circ - 180^\circ$ ) or **reflex** ( $180^\circ - 360^\circ$ ). **Congruent** shapes are identical in shape and size. **Similar** shapes have equal angles and all lengths multiplied by the same **scale factor**.

## Key points

- Quadrilaterals: square, rectangle, rhombus, parallelogram, trapezium, kite.
- Polygons: pentagon (5), hexagon (6), heptagon (7), octagon (8), nonagon (9), decagon (10).
- Circles: radius, diameter, chord, tangent, arc, sector, segment.
- Solids have faces, edges and vertices. A **net** folds to make a solid.

## Key words

**perpendicular**

**reflex**

**congruent**

**similar**

**chord**

**net**

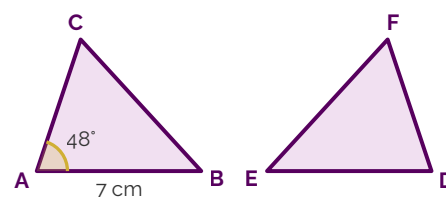
## Worked example

Triangles ABC and DEF are congruent.  $AB = 7 \text{ cm}$  and angle  $BAC = 48^\circ$ . Find DE and angle EDF.

Match the vertices:  $A \leftrightarrow D$ ,  $B \leftrightarrow E$ ,  $C \leftrightarrow F$ .

DE matches AB, so  $DE = 7 \text{ cm}$ .

Angle EDF matches angle BAC, so angle  $EDF = 48^\circ$ .

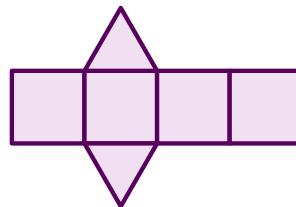


Not to scale

## Common mistakes

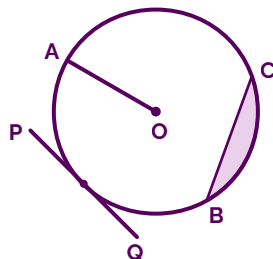
- Calling a  $200^\circ$  angle obtuse. It is reflex.
- Thinking a trapezium has two pairs of parallel sides. It has exactly one pair.

**Exam tip:** Write the vertices of congruent shapes in matching order, then corresponding sides and angles are easy to read off.

**Answer each question. Show your working where it helps.****1** An angle of  $137^\circ$  is \_\_\_\_\_.**2** An angle of  $215^\circ$  is \_\_\_\_\_.**3** A polygon with 8 sides is an \_\_\_\_\_.**4** A polygon with 7 sides is a \_\_\_\_\_.**5** How many faces does a triangular prism have? **6** How many edges does a square-based pyramid have? **7** How many vertices does a cuboid have? **8** Four equal sides, no right angles: a \_\_\_\_\_.**9** Two pairs of equal adjacent sides: a \_\_\_\_\_.**10** A triangle with angles  $40^\circ$ ,  $70^\circ$ ,  $70^\circ$  is \_\_\_\_\_.**11** A chord through the centre is a \_\_\_\_\_.**12** A line touching a circle at one point is a \_\_\_\_\_.**13** The region between a chord and an arc is a \_\_\_\_\_.**14** Shapes with the same shape and size are \_\_\_\_\_.**15** A line cutting AB in half at  $90^\circ$  is the \_\_\_\_\_ of AB.**16** This net folds to make a solid. Name the solid.**Answer** .....**Reflect:** ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 The diagram shows a circle, centre O. The line PQ touches the circle at one point.



Not to scale

| Line or region          | Name |
|-------------------------|------|
| OA                      |      |
| Straight line BC        |      |
| Curved line from B to C |      |
| Shaded region           |      |
| PQ                      |      |

- (a) Complete the table with the mathematical name of each line or region.

[5]

- 2 Complete the table with the mathematical name of each quadrilateral.

| Description                                                       | Name |
|-------------------------------------------------------------------|------|
| All sides equal, diagonals cross at $90^\circ$ , no right angles  |      |
| Exactly one pair of parallel sides                                |      |
| Two pairs of equal adjacent sides, one line of symmetry           |      |
| Four right angles, but not all sides equal                        |      |
| Two pairs of parallel sides, no right angles, sides not all equal |      |

- (a) Complete the table.

[5]

- 3 Complete the table.

| Solid                | Faces | Edges | Vertices |
|----------------------|-------|-------|----------|
| Cuboid               | 6     |       |          |
| Triangular prism     | 5     |       |          |
| Square-based pyramid |       | 8     | 5        |

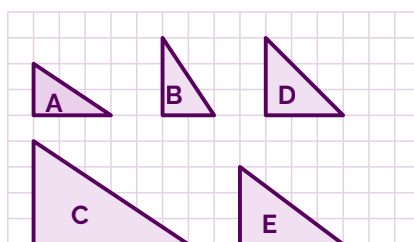
- (a) Fill in the five missing numbers.

[5]

- (b) For each solid, work out faces + vertices – edges. What do you notice?

[2]

- 4 The diagram shows five triangles, A, B, C, D and E, on a grid.



- (a) Write down the letter of the triangle that is congruent to triangle A. [1]

Answer .....

- (b) Write down the letter of the triangle that is similar to triangle A but **not** congruent to it. [1]

Answer .....

- (c) Write down the scale factor of the enlargement that maps triangle A onto this triangle. [1]

Answer .....

- (d) Explain why triangle E is **not** similar to triangle A. [1]

- 5 Triangle ABC is congruent to triangle PQR, with  $A \leftrightarrow P$ ,  $B \leftrightarrow Q$  and  $C \leftrightarrow R$ .  
 $AB = 8$  cm,  $BC = 11$  cm, angle  $BAC = 64^\circ$  and angle  $ABC = 47^\circ$ .

- (a) Write down the length of QR. [1]

Answer QR = ..... cm

- (b) Work out the size of angle PRQ. [2]

Answer angle PRQ = .....°

- (c) Write down the size of angle PQR. [1]

Answer angle PQR = .....°

- (d)  $AC = 9.5$  cm. Work out the perimeter of triangle PQR. [1]

Answer ..... cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Measuring, constructions and nets

**I can...** measure and draw lines and angles, construct triangles with compasses and use nets.

Syllabus ref: C4.2, E4.2 · Core & Extended

Measure lengths to the nearest millimetre and angles to the nearest degree. To **construct** a triangle from three sides, use only a ruler and compasses, and leave your construction arcs showing.

## Key points

- Accuracy expected: lengths  $\pm 2$  mm, angles  $\pm 2^\circ$ .
- Perpendicular bisector of AB: equal arcs from A and from B, above and below AB; join the two crossing points.
- A **net** is a 2D shape that folds into a solid. A cube has 11 different nets.

## Key words

protractor

construct

compasses

arc

net

## Worked example

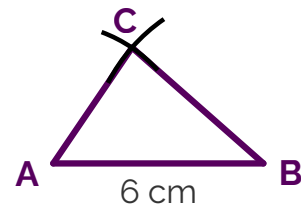
**Construct triangle ABC with AB = 6 cm, AC = 4 cm and BC = 5 cm.**

Draw AB = 6 cm.

Compasses at 4 cm, point on A: draw an arc above AB.

Compasses at 5 cm, point on B: draw an arc to cross the first arc.

The crossing point is C. Join AC and BC. Keep the arcs.



## Common mistakes

- Reading the wrong protractor scale ( $140^\circ$  instead of  $40^\circ$ ). Ask: is the angle acute or obtuse?
- Rubbing out construction arcs: you lose the marks.

**Exam tip:** Use a sharp pencil: a thick line can be 1–2 mm out, which is the whole tolerance.

Answer each question. Show your working where it helps.

Measure the length of this line in millimetres.

1



Answer ..... mm

Measure the length of this line in centimetres.

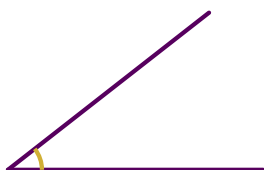
2



Answer ..... cm

Measure this angle.

3



Answer .....°

Measure this angle.

4



Answer .....°

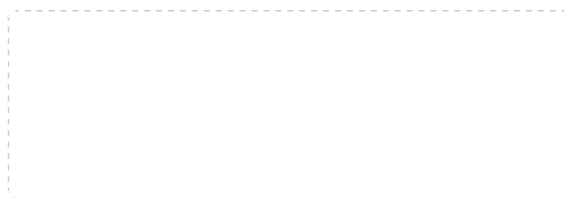
Draw a line 6.4 cm long.

5



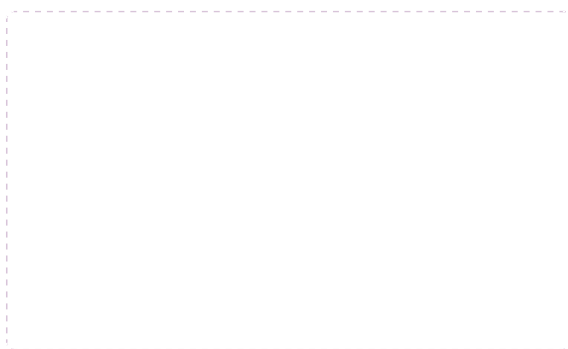
Draw an angle of 72°.

6



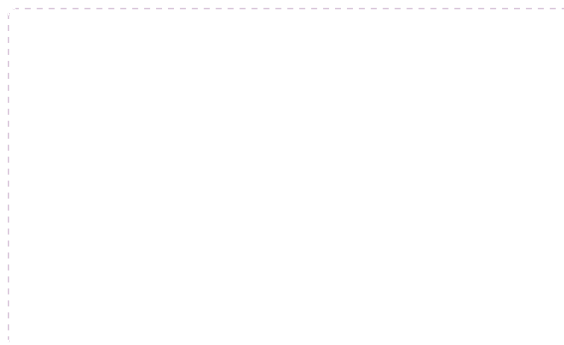
Using a ruler and compasses only, construct an equilateral triangle with sides 5 cm.

7



Draw a line AB 8 cm long. Construct the perpendicular bisector of AB.

8



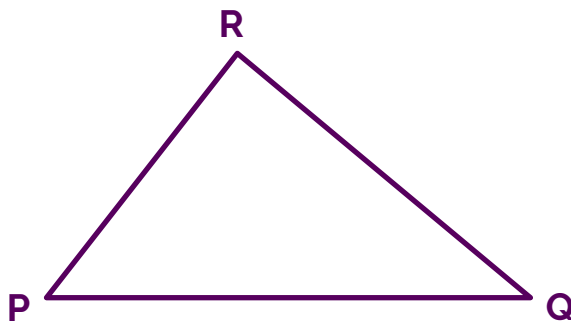
Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 Using a ruler and compasses only, construct triangle XYZ with  $XY = 9$  cm,  $XZ = 7$  cm and  $YZ = 5.5$  cm. XY has been drawn for you. Leave in your construction arcs. [3]



- 2 The triangle PQR is drawn accurately.



Drawn accurately

- (a) Measure the lengths of QR and PR. [2]

Answer QR = ..... cm PR = ..... cm

- (b) Work out the perimeter of the triangle. [1]

Answer ..... cm

- (c) Measure angle PQR and angle PRQ. [2]

Answer PQR = .....° PRQ = .....°

- 3 The net of a square-based pyramid is a square of side 5 cm with four congruent triangles. Each triangle has base 5 cm and height 7 cm.

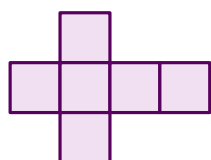
- (a) How many faces does the pyramid have? [1]

Answer .....

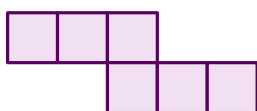
- (b) Work out the total area of the net. [3]

Answer ..... cm<sup>2</sup>

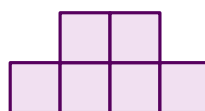
- 4 Here are three arrangements of six squares, A, B and C.



A



B



C

- (a) Which arrangement is **not** the net of a cube? Give a reason. [2]

- (b) Net A is folded to make a cube. How many edges does the cube have? [1]

Answer .....

- (c) How many vertices does the cube have? [1]

Answer .....

- 5 A cuboid measures 4 cm by 3 cm by 2 cm.

- (a) Draw an accurate net of the cuboid. [3]

- (b) Work out the total area of the net. [2]

Answer ..... cm<sup>2</sup>

- 6 Draw a line AB 7 cm long.

- (a) Using a ruler and compasses only, construct the perpendicular bisector of AB. [2]

- (b) The bisector crosses AB at M. Write down the length AM. [1]

Answer AM = ..... cm

- (c) T is any point on the bisector. Explain why TA = TB. [1]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Scale drawings and bearings

**I can...** use map scales, make scale drawings and use three-figure bearings.

Syllabus ref: C4.3, E4.3 · Core & Extended

A map scale of 1 : 50 000 means 1 cm on the map is 50 000 cm (500 m) in real life. A **bearing** is measured from North, clockwise, and is written with three figures, e.g. 064° or 305°.

## Key points

- Real length = map length  $\times$  scale number; then convert (100 cm = 1 m, 1000 m = 1 km).
- Bearing of A from B: stand at B, face North, turn clockwise to face A.
- Back bearing: add 180° if the bearing is less than 180°, otherwise subtract 180°.

## Key words

scale

ratio

bearing

clockwise

North line

## Worked example 1

**Scale 1 : 25 000.** Two towns are 6.4 cm apart on the map. Find the real distance in km.

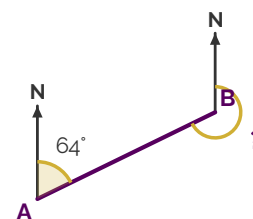
$$6.4 \times 25\,000 = 160\,000 \text{ cm} = 1600 \text{ m} = \mathbf{1.6 \text{ km}}$$

## Worked example 2

**The bearing of B from A is 064°. Find the bearing of A from B.**

The North lines are parallel, so add 180°.

$$64 + 180 = \mathbf{244^\circ}$$



Not to scale

## Common mistakes

- Measuring anticlockwise, or from the wrong point. The word after "from" tells you where to stand.
- Writing 64° instead of 064°.

**Exam tip:** Draw a North line at the point you are measuring **from** before you measure or calculate a bearing.

Answer each question. Show your working where it helps.

- 1 A map scale is 1 : 50 000. Write 1 cm on the map as a real distance in metres.

- 2 Scale 1 : 20 000. Map length 7 cm. Real length in km?

- 3 Scale 1 : 200. A wall is 4.6 cm long on the plan. Real length in m?

- 4 Scale 1 : 50. A room is 6 m long. Length on the plan in cm?

- 5 Scale 1 : 25 000. Real distance 3 km. Map distance in cm?

- 6 Scale 1 cm to 5 km. Map distance 8.4 cm. Real distance?

- 7 Write the bearing for the direction due East.

- 8 Write the bearing for the direction South-West.

- 9 Bearing of B from A is  $035^\circ$ . Bearing of A from B?

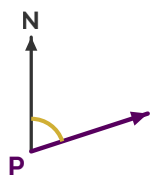
- 10 Bearing of B from A is  $290^\circ$ . Bearing of A from B?

- 11 Bearing of D from C is  $180^\circ$ . Bearing of C from D?

- 12 Bearing of F from E is  $147^\circ$ . Bearing of E from F?

Measure the bearing of the arrow from P.

13

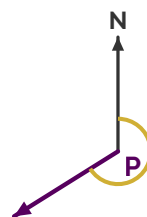


Drawn accurately

Answer .....°

Measure the bearing of the arrow from P.

14



Drawn accurately

Answer .....°

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 A plan of a school is drawn to a scale of 1 : 500.

(a) The library is 3.8 cm long on the plan. Work out the real length of the library in metres. [2]

Answer ..... m

(b) The playground is 42 m long. Work out its length on the plan in centimetres. [2]

Answer ..... cm

(c) The hall is 12.5 m wide. Work out its width on the plan. [2]

Answer ..... cm

- 2 A map has a scale of 1 : 40 000.

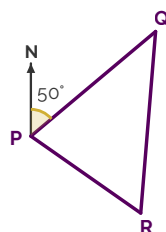
(a) Two villages are 9.5 cm apart on the map. Work out the real distance between them in kilometres. [2]

Answer ..... km

(b) A straight road is 2.6 km long. How long is the road on the map? [2]

Answer ..... cm

- 3 The bearing of Q from P is 050°. The bearing of R from P is 125°.



Not to scale

(a) Work out the bearing of P from Q. [1]

Answer .....°

(b) Work out the bearing of P from R. [1]

Answer .....°

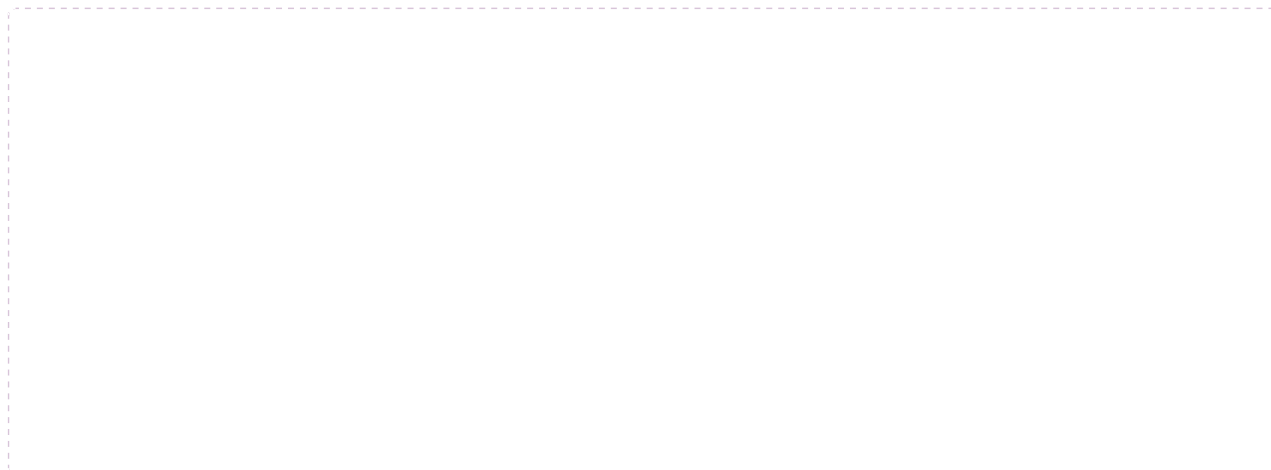
(c) Work out the size of angle QPR. [1]

Answer .....°

- 4 A ship leaves harbour H and sails 12 km on a bearing of  $060^\circ$  to a buoy B. It then sails 9 km on a bearing of  $150^\circ$  to a lighthouse L.

(a) Using a scale of 1 cm to 2 km, make an accurate scale drawing of the journey.

[3]



(b) Use your drawing to find the distance HL in km.

[2]

Answer ..... km

(c) Use your drawing to find the bearing of L from H.

[1]

Answer ..... $^\circ$

- 5 A model of a building is made to a scale of 1 : 120.

(a) The model is 45 cm tall. How tall is the real building in metres?

[2]



Answer ..... m

(b) A door on the real building is 2.4 m tall. How tall is the door on the model in centimetres?

[2]



Answer ..... cm

- 6 A plane flies from airport A to airport B on a bearing of  $300^\circ$ .

(a) Work out the bearing of A from B.

[1]

Answer ..... $^\circ$

(b) At B the plane turns and flies on a bearing of  $040^\circ$  to airport C. Work out the size of angle ABC.

[2]



Answer ..... $^\circ$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Angles and parallel lines

**I can...** find missing angles using angle facts and parallel lines, giving geometrical reasons.

Syllabus ref: C4.6, E4.6 · Core & Extended

Angle facts let you find missing angles. In an exam you must give a **reason** for each step, using the correct words. Arrows on lines show that they are parallel.

## Key points

- Angles on a straight line add up to  $180^\circ$ ; angles at a point add up to  $360^\circ$ .
- Vertically opposite angles are equal.
- Parallel lines: **corresponding** angles are equal (F shape), **alternate** angles are equal (Z shape), **co-interior** angles add up to  $180^\circ$  (C shape).
- Angles in a triangle add up to  $180^\circ$ . An isosceles triangle has two equal base angles.

## Key words

vertically opposite

corresponding

alternate

co-interior

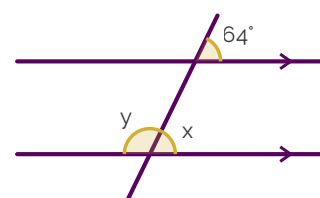
isosceles

## Worked example

Find  $x$  and  $y$ . Give reasons.

$$x = 64^\circ \text{ (corresponding angles are equal)}$$

$$y = 180 - 64 = 116^\circ \text{ (angles on a straight line add up to } 180^\circ \text{)}$$



Not to scale

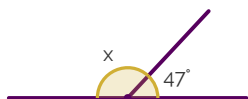
## Common mistakes

- Writing "F angles" or "Z angles" as a reason. Use the words corresponding and alternate.
- Assuming lines are parallel when there are no arrows.

**Exam tip:** Write each reason next to the angle you find, in full words. Reasons often carry a mark of their own.

Find the value of  $x$  in each diagram. Write down the angle fact you used.

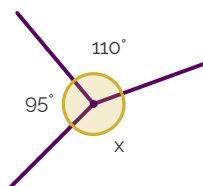
1



Not to scale

Answer  $x = \dots\dots\dots$ 

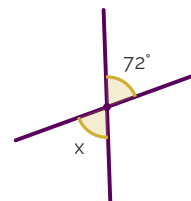
2



Not to scale

Answer  $x = \dots\dots\dots$ 

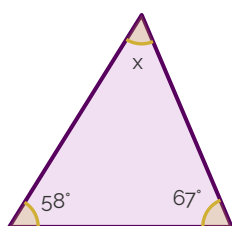
3



Not to scale

Answer  $x = \dots\dots\dots$ 

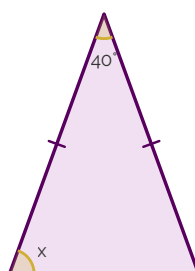
4



Not to scale

Answer  $x = \dots\dots\dots$ 

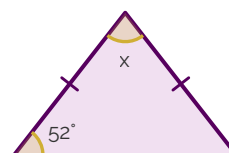
5



Not to scale

Answer  $x = \dots\dots\dots$ 

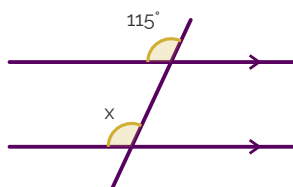
6



Not to scale

Answer  $x = \dots\dots\dots$ 

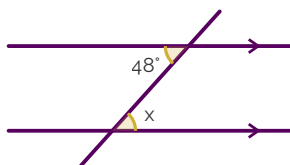
7



Not to scale

Answer  $x = \dots\dots\dots$ 

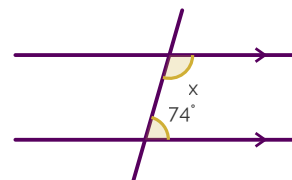
8



Not to scale

Answer  $x = \dots\dots\dots$ 

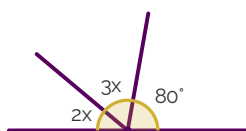
9



Not to scale

Answer  $x = \dots\dots\dots$ 

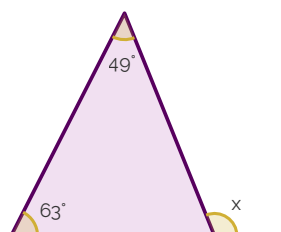
10



Not to scale

Answer  $x = \dots\dots\dots$ 

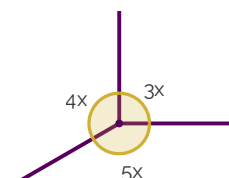
11



Not to scale

Answer  $x = \dots\dots\dots$ 

12



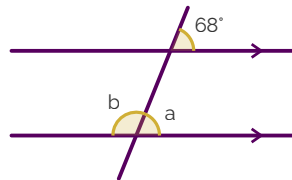
Not to scale

Answer  $x = \dots\dots\dots$ 

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 The diagram shows two parallel lines crossed by a straight line.



Not to scale

- (a) Find the size of angle  $a$ . Give a reason for your answer.

[2]

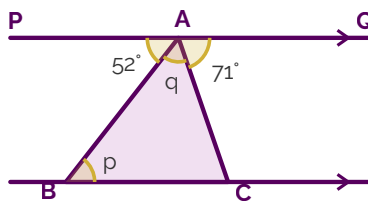
Answer  $a = \dots\dots$

- (b) Find the size of angle  $b$ . Give a reason for your answer.

[2]

Answer  $b = \dots\dots$

- 2 PQ is parallel to BC. A lies on PQ.



Not to scale

- (a) Find the size of angle  $p$ . Give a reason.

[2]

Answer  $p = \dots\dots$

- (b) Find the size of angle  $q$ .

[2]

Answer  $q = \dots\dots$

- 3 The angles of a triangle are  $x^\circ$ ,  $2x^\circ$  and  $(3x - 30)^\circ$ .

- (a) Find the value of  $x$ .

[3]

Answer  $x = \dots\dots$

- (b) Is the triangle isosceles? Explain your answer.

[1]

4 Angles and algebra.

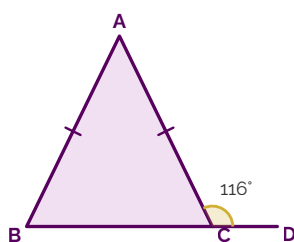
- (a) Three angles on a straight line are  $75^\circ$ ,  $(2x - 5)^\circ$  and  $(3x + 10)^\circ$ . Find the value of  $x$ . [3]

Answer  $x = \dots\dots$

- (b) Two straight lines AB and CD cross at O. Angle AOC =  $3y^\circ$  and angle BOD =  $(2y + 24)^\circ$ . Find the size of angle AOD. [3]

Answer angle AOD =  $\dots\dots$

- 5 In triangle ABC,  $AB = AC$ . BC is extended to D. Angle ACD =  $116^\circ$ .



Not to scale

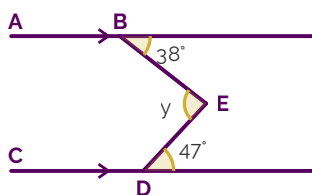
- (a) Work out the size of angle ACB. Give a reason. [2]

Answer  $\dots\dots^\circ$

- (b) Work out the size of angle BAC. Give a reason. [2]

Answer  $\dots\dots^\circ$

- 6 AB is parallel to CD. Find the size of angle  $y$ . Show your reasoning. [3]



Not to scale

Answer  $y = \dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Angles in polygons

**I can...** use the angle sums of triangles, quadrilaterals and polygons, including regular polygons.

Syllabus ref: C4.6, E4.6 · Core & Extended

A polygon with  $n$  sides can be split into  $(n - 2)$  triangles, so its interior angles add up to  $(n - 2) \times 180^\circ$ . The exterior angles of any polygon add up to  $360^\circ$ . In a **regular** polygon all sides and all angles are equal.

## Key points

- Sum of interior angles =  $(n - 2) \times 180^\circ$ .
- Regular polygon: exterior angle =  $360^\circ \div n$ ; interior angle =  $180^\circ - \text{exterior angle}$ .
- Number of sides  $n = 360^\circ \div \text{exterior angle}$ .
- Quadrilateral angles add up to  $360^\circ$ . A parallelogram has equal opposite angles.

## Key words

**polygon**

**regular**

**interior angle**

**exterior angle**

**tessellate**

## Worked example 1

**Find the interior angle of a regular octagon.**

$$\text{Exterior angle} = 360 \div 8 = 45^\circ$$

$$\text{Interior angle} = 180 - 45 = \mathbf{135^\circ}$$

## Worked example 2

**Each interior angle of a regular polygon is  $156^\circ$ . How many sides does it have?**

$$\text{Exterior angle} = 180 - 156 = 24^\circ$$

$$n = 360 \div 24 = \mathbf{15 \text{ sides}}$$

## Common mistakes

- Using  $n \times 180^\circ$  instead of  $(n - 2) \times 180^\circ$ .
- Dividing  $360^\circ$  by the interior angle instead of the exterior angle.

**Exam tip:** If you are asked for the number of sides, work with the exterior angle first: it is much quicker.

Answer each question. Show your working where it helps.

1 Sum of the interior angles of a hexagon

2 Sum of the interior angles of a pentagon

3 Sum of the interior angles of a decagon

4 Sum of the interior angles of a 20-sided polygon

5 Exterior angle of a regular octagon

6 Interior angle of a regular hexagon

7 Interior angle of a regular decagon

8 Interior angle of a regular 12-sided polygon

9 A regular polygon has exterior angle  $30^\circ$ . Number of sides?

10 A regular polygon has interior angle  $160^\circ$ . Number of sides?

11 A quadrilateral has angles  $85^\circ$ ,  $110^\circ$ ,  $72^\circ$  and  $x$ . Find  $x$ .

12 A pentagon has angles  $100^\circ$ ,  $120^\circ$ ,  $95^\circ$ ,  $130^\circ$  and  $x$ . Find  $x$ .

13 A kite has angles  $115^\circ$ ,  $115^\circ$ ,  $70^\circ$  and  $x$ . Find  $x$ .

14 A parallelogram has one angle of  $64^\circ$ . Find the largest angle.

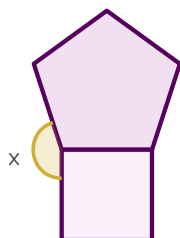
15 The interior angles of a polygon add up to  $1980^\circ$ . How many sides?

16 Can a regular polygon have an exterior angle of  $25^\circ$ ? (yes/no)

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

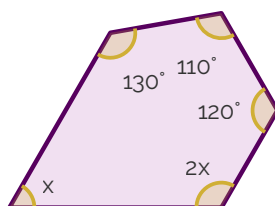
- 1 The diagram shows a regular pentagon and a square that share a side. Work out the size of angle  $x$ . [3]



Not to scale

Answer  $x = \dots\dots$

- 2 The diagram shows a pentagon.



Not to scale

- (a) Find the value of  $x$ . [3]

Answer  $x = \dots\dots$

- (b) Write down the size of the largest angle of the pentagon. [1]

Answer  $\dots\dots^\circ$

- 3 Shapes **tessellate** if they fit together around a point with no gaps.

- (a) Explain why regular hexagons tessellate. [1]

- (b) Show that regular pentagons do not tessellate. [2]

- (c) Show that one square and two regular octagons fit together exactly around a point. [1]

4 Regular polygons.

(a) Each exterior angle of a regular polygon is  $40^\circ$ . How many sides does it have? [2]

Answer .....

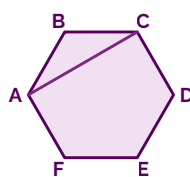
(b) Work out the sum of the interior angles of this polygon. [2]

Answer .....°

(c) Each interior angle of a different regular polygon is 5 times the size of each exterior angle. How many sides does this polygon have? [3]

Answer ..... sides

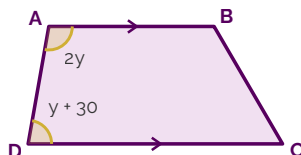
5 ABCDEF is a regular hexagon. Work out the size of angle FAC. Give reasons. [3]



Not to scale

Answer angle FAC = .....

6 ABCD is a trapezium. AB is parallel to DC.



Not to scale

(a) Find the value of  $y$ . Give a reason. [3]

Answer  $y =$  .....

(b) Find the size of angle DAB. [1]

Answer .....°

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Symmetry in 2D and 3D

**I can...** find lines of symmetry and rotational symmetry, and planes of symmetry of solids (Extended).

Syllabus ref: C4.5, E4.5 · Core & Extended

A shape has **line symmetry** if a mirror line splits it into two halves that are reflections of each other. It has **rotational symmetry of order  $n$**  if it fits onto itself  $n$  times in one full turn. Extended: a solid has a **plane of symmetry** if a flat cut splits it into two mirror-image halves.

## Key points

- A regular polygon with  $n$  sides has  $n$  lines of symmetry and rotational symmetry of order  $n$ .
- Order 1 means no rotational symmetry.
- Extended: a cuboid (all edges different) has 3 planes of symmetry; a cube has 9.
- Extended: cylinders and cones have an infinite number of planes of symmetry.

## Key words

line symmetry

rotational symmetry

order

plane of symmetry

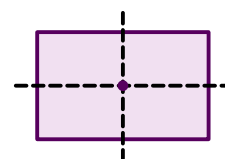
axis

## Worked example

**Describe the symmetry of a rectangle (not a square).**

2 lines of symmetry (through the midpoints of opposite sides, not the diagonals).

Rotational symmetry of **order 2** about the centre.



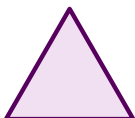
## Common mistakes

- Thinking the diagonals of a rectangle are lines of symmetry. Fold one to check: they are not.
- Saying "order 0". A shape with no rotational symmetry has order 1.

**Exam tip:** Turn the page (or use tracing paper) to check rotational symmetry: count how many times the shape looks the same in one full turn.

Questions 1–6: write the number of lines of symmetry and the order of rotational symmetry of each shape.

1



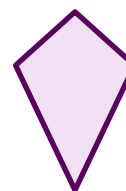
**Answer** lines .... order ....

2



**Answer** lines .... order ....

3



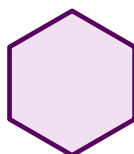
**Answer** lines .... order ....

4



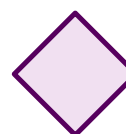
**Answer** lines .... order ....

5



**Answer** lines .... order ....

6



**Answer** lines .... order ....

7

Number of lines of symmetry of a regular octagon

8

Order of rotational symmetry of a regular pentagon

9

Lines of symmetry of a square

10

Order of rotational symmetry of a scalene triangle

11

*Extended:* planes of symmetry of a cube

12

*Extended:* planes of symmetry of a cuboid with all edges different

13

*Extended:* planes of symmetry of a square-based pyramid (regular)

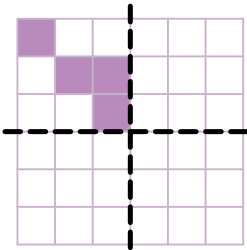
14

*Extended:* order of rotational symmetry of a square-based pyramid about its vertical axis

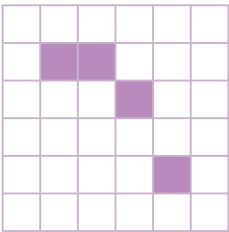
Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 Shade squares so that the pattern has both dashed lines as lines of symmetry. [2]



- 2 Shade **two** more squares so that the pattern has rotational symmetry of order 2 about the centre of the grid. [2]

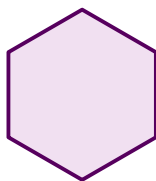


- 3 Complete the table.

| Shape                | Lines of symmetry | Order of rotational symmetry |
|----------------------|-------------------|------------------------------|
| Square               |                   |                              |
| Rectangle            |                   |                              |
| Parallelogram        |                   |                              |
| Regular pentagon     |                   |                              |
| Equilateral triangle |                   |                              |

- (a) Fill in the table. [5]

- 4 The diagram shows a regular hexagon.



(a) Draw all the lines of symmetry on the hexagon. [2]

(b) Write down the order of rotational symmetry of the hexagon. [1]

Answer .....

(c) How many lines of symmetry does a regular polygon with 12 sides have? [1]

Answer .....

- 5 Write down the mathematical name of each shape described.

| Description                                                                      | Name |
|----------------------------------------------------------------------------------|------|
| A quadrilateral with rotational symmetry of order 2 and no lines of symmetry     |      |
| A triangle with exactly one line of symmetry                                     |      |
| A quadrilateral with one line of symmetry and diagonals that cross at $90^\circ$ |      |
| A quadrilateral with four lines of symmetry                                      |      |

(a) Complete the table. [4]

- 6 *Extended.* Complete the table for these solids.

| Solid                                            | Number of planes of symmetry |
|--------------------------------------------------|------------------------------|
| Cylinder                                         |                              |
| Cone                                             |                              |
| Regular square-based pyramid                     |                              |
| Cube                                             |                              |
| Prism with an equilateral triangle cross-section |                              |
| Cuboid with a square cross-section (not a cube)  |                              |

(a) Write down the number of planes of symmetry of each solid. [6]

(b) The triangular prism has an axis of rotational symmetry through the centres of its two triangular faces. Write down the order of rotational symmetry about this axis. [1]

Answer .....

(c) Write down the order of rotational symmetry of the regular square-based pyramid about its vertical axis. [1]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

## 7

## UNIT 7 · KEY KNOWLEDGE

## Similar shapes

**I can...** use scale factors to find lengths in similar shapes, and areas and volumes (Extended).

Syllabus ref: C4.4, E4.4 · Core & Extended

Similar shapes have equal angles and corresponding lengths in the same ratio. The **scale factor**  $k$  = new length  $\div$  original length. When a line is drawn parallel to one side of a triangle, it cuts off a smaller similar triangle.

**Key points**

- Lengths  $\times k$ . *Extended:* areas  $\times k^2$ ; volumes  $\times k^3$ .
- *Extended:* from an area ratio,  $k = \sqrt{\text{area ratio}}$ ; from a volume ratio,  $k = \sqrt[3]{\text{volume ratio}}$ .

**Key words**

similar

scale factor

corresponding

enlargement

area factor

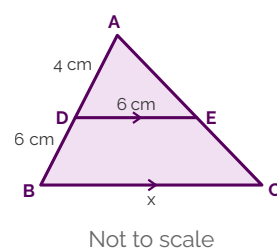
volume factor

**Worked example 1**

DE is parallel to BC. Find  $x$ .

$$AB = 4 + 6 = 10 \text{ cm, so } k = 10 \div 4 = 2.5$$

$$x = 6 \times 2.5 = \mathbf{15 \text{ cm}}$$

**Worked example 2 (Extended)**

Two similar jugs are 12 cm and 18 cm tall. The small jug holds  $640 \text{ cm}^3$ . Find the volume of the large jug.

$$k = 18 \div 12 = 1.5$$

$$\text{Volume} = 640 \times 1.5^3 = \mathbf{2160 \text{ cm}^3}$$

**Common mistakes**

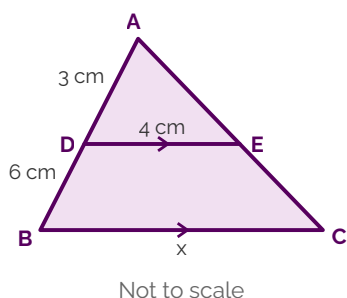
- Using AD and DB instead of AD and AB to find  $k$ .
- Multiplying an area by  $k$  instead of  $k^2$ .

**Exam tip:** Redraw the two similar triangles separately, the same way round, before matching sides.

Answer each question. Show your working where it helps.

DE is parallel to BC. Find  $x$ .

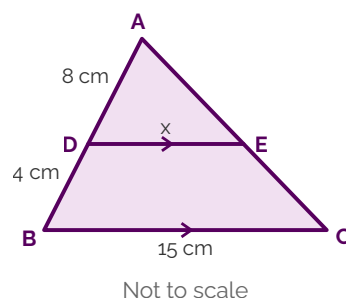
1



Answer  $x = \dots\dots$

DE is parallel to BC. Find  $x$ .

2



Answer  $x = \dots\dots$

3

A shape is enlarged with scale factor 3. A side of 4.5 cm becomes

4

Scale factor from a 8 cm side to a 20 cm side

5

Scale factor from a 12 cm side to a 3 cm side

6

Rectangles 4 cm  $\times$  6 cm and 10 cm  $\times$  15 cm. Similar? (yes/no)

7

Triangle sides 6, 8, 10 cm. A similar triangle has shortest side 9 cm. Longest side?

8

Similar shapes: 5 cm  $\leftrightarrow$  7.5 cm. Then 12 cm  $\leftrightarrow$  ?

9

Extended: length scale factor 3. Area scale factor?

10

Extended: length scale factor 4. Volume scale factor?

11

Extended:  $k = 2$ . Small area 15 cm<sup>2</sup>. Large area?

12

Extended:  $k = 1.5$ . Small volume 40 cm<sup>3</sup>. Large volume?

13

Extended: areas 20 cm<sup>2</sup> and 45 cm<sup>2</sup>. Length scale factor?

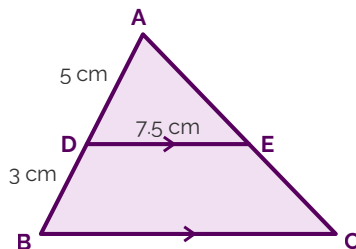
14

Extended: volumes 54 cm<sup>3</sup> and 250 cm<sup>3</sup>. Length scale factor?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 In the diagram, DE is parallel to BC. AD = 5 cm, DB = 3 cm and DE = 7.5 cm.



Not to scale

- (a) Explain why triangle ADE is similar to triangle ABC. [1]

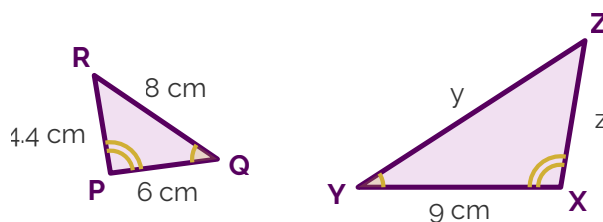
- (b) Work out the length of BC. [2]

Answer BC = ..... cm

- (c) AC = 9.6 cm. Work out the length of AE. [2]

Answer AE = ..... cm

- 2 Triangles PQR and XYZ are similar. Angle P = angle X and angle Q = angle Y.



Not to scale

- (a) Write down the scale factor of the enlargement from PQR to XYZ. [1]

Answer .....

- (b) Work out the lengths y and z. [2]

Answer y = ..... cm    z = ..... cm

- (c) *Extended.* The area of triangle PQR is  $12.8 \text{ cm}^2$ . Work out the area of triangle XYZ. [2]

Answer .....  $\text{cm}^2$

- 3 *Extended.* Two bottles are mathematically similar. The small bottle is 15 cm tall and the large bottle is 24 cm tall.

(a) The surface area of the small bottle is  $210 \text{ cm}^2$ . Work out the surface area of the large bottle. [2]

Answer .....  $\text{cm}^2$

(b) The large bottle holds 1.2 litres. Work out the volume of the small bottle in  $\text{cm}^3$ . Give your answer correct to 3 significant figures. [3]

Answer .....  $\text{cm}^3$

- 4 A photograph is 15 cm long and 10 cm high. It is enlarged so that it is 24 cm long.

(a) Work out the height of the enlarged photograph. [2]

Answer ..... cm

(b) Is a frame measuring 30 cm by 21 cm similar to the original photograph? Show how you decide. [2]

- 5 *Extended.* Two solid cylinders are mathematically similar. Their volumes are  $128 \text{ cm}^3$  and  $432 \text{ cm}^3$ .

(a) The radius of the small cylinder is 4 cm. Work out the radius of the large cylinder. [3]

Answer ..... cm

(b) The total surface area of the large cylinder is  $360 \text{ cm}^2$ . Work out the total surface area of the small cylinder. [2]

Answer .....  $\text{cm}^2$

- 6 *Extended.* A map has a scale of 1 : 25 000. A lake has an area of  $12 \text{ cm}^2$  on the map. Work out the real area of the lake in  $\text{km}^2$ . [3]

Answer .....  $\text{km}^2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Circle theorems

**I can...** use the angle in a semicircle, tangent–radius and (Extended) centre and same-segment theorems.

Syllabus ref: C4.7, E4.7 · Core & Extended

Circle theorems describe angles made by chords, radii and tangents. Look for radii: two radii make an **isosceles** triangle. Always quote the theorem as your reason.

## Key points

- **Core:** the angle in a semicircle is  $90^\circ$ .
- **Core:** a tangent is perpendicular to the radius at the point of contact.
- **Extended:** the angle at the centre is twice the angle at the circumference (from the same arc).
- **Extended:** angles in the same segment are equal.

## Key words

semicircle

tangent

radius

circumference

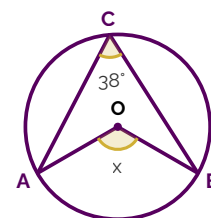
segment

## Worked example (Extended)

**O is the centre. Angle  $ACB = 38^\circ$ . Find angle  $AOB$ .**

Angle  $AOB$  and angle  $ACB$  stand on the same arc  $AB$ .

$x = 2 \times 38 = \mathbf{76^\circ}$  (angle at the centre is twice the angle at the circumference)



Not to scale

## Common mistakes

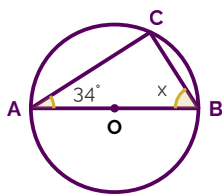
- Using "angle at centre =  $2 \times$  angle at circumference" when the two angles stand on different arcs.
- Forgetting that the two radii  $OA$  and  $OB$  are equal.

**Exam tip:** Mark every right angle and every pair of equal radii on the diagram before you start calculating.

**O is the centre of each circle. Find  $x$  and give a reason. Questions 3–6 and 8 are Extended.**

AB is a diameter.

1

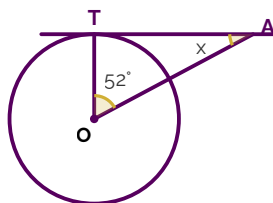


Not to scale

**Answer**  $x = \dots\dots$

AT is a tangent.

2

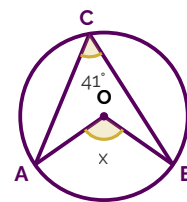


Not to scale

**Answer**  $x = \dots\dots$

*Extended*

3

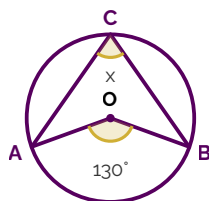


Not to scale

**Answer**  $x = \dots\dots$

*Extended*

4

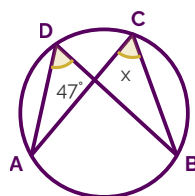


Not to scale

**Answer**  $x = \dots\dots$

*Extended*

5

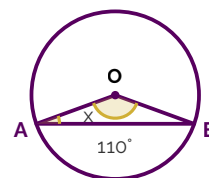


Not to scale

**Answer**  $x = \dots\dots$

*Extended*

6

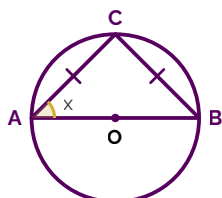


Not to scale

**Answer**  $x = \dots\dots$

AB is a diameter.

7

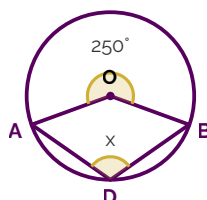


Not to scale

**Answer**  $x = \dots\dots$

*Extended*

8



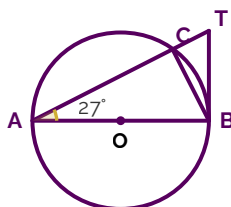
Not to scale

**Answer**  $x = \dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 A, B and C lie on a circle with centre O. AB is a diameter. The tangent to the circle at B meets AC extended at T. Angle BAC =  $27^\circ$ .



Not to scale

- (a) Write down the size of angle ACB. Give a reason. [2]

Answer .....°

- (b) Work out the size of angle ABC. [1]

Answer .....°

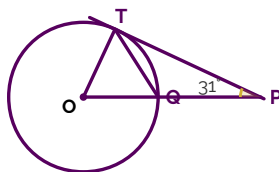
- (c) Write down the size of angle ABT. Give a reason. [1]

Answer .....°

- (d) Work out the size of angle CBT. [1]

Answer .....°

- 2 PT is a tangent to the circle, centre O, at T. OP meets the circle at Q. Angle TPO =  $31^\circ$ .



Not to scale

- (a) Write down the size of angle OTP. Give a reason. [2]

Answer .....°

- (b) Work out the size of angle TOP. [1]

Answer .....°

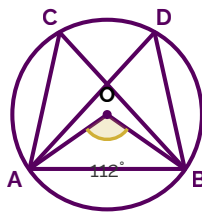
- (c) Work out the size of angle OTQ. [2]

Answer .....°

- 3 AB is a diameter of a circle and C lies on the circle. Angle BAC =  $2x^\circ$  and angle ABC =  $(x + 15)^\circ$ . Find the value of x. [3]

Answer  $x =$  .....

- 4 *Extended.* A, B, C and D lie on the circle, centre O. Angle AOB =  $112^\circ$ .



Not to scale

- (a) Find angle ACB. Give a reason.

[2]

Answer .....

- (b) Find angle ADB. Give a reason.

[2]

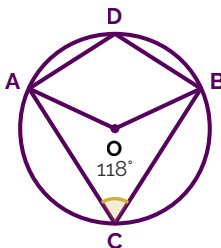
Answer .....

- (c) Work out the size of angle OAB.

[2]

Answer .....

- 5 *Extended.* A, B, C and D lie on the circle, centre O. C is on the minor arc AB and D is on the major arc. Angle ACB =  $118^\circ$ .



Not to scale

- (a) Work out the size of the obtuse angle AOB.

[3]

Answer .....

- (b) Work out the size of angle ADB.

[2]

Answer .....

- 6 *Extended.* A, B and C lie on a circle with centre O. C is on the major arc. Angle AOB =  $(5y - 10)^\circ$  and angle ACB =  $(2y + 5)^\circ$ . Find the size of angle AOB.

[3]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Cyclic quadrilaterals, tangents and chords

**I can...** use cyclic quadrilaterals, the alternate segment theorem and the symmetry properties of circles.

Syllabus ref: E4.7, E4.8 · Extended

These theorems are for the Extended tier. A **cyclic quadrilateral** has all four vertices on a circle. The **alternate segment theorem** links the angle between a tangent and a chord to an angle in the other segment.

## Key points

- Opposite angles of a cyclic quadrilateral add up to  $180^\circ$ .
- Alternate segment: angle between tangent and chord = angle in the alternate segment.
- Two tangents from a point outside a circle are equal in length.
- The perpendicular from the centre to a chord bisects the chord. Equal chords are equidistant from the centre.

## Key words

cyclic quadrilateral

alternate segment

tangent

chord

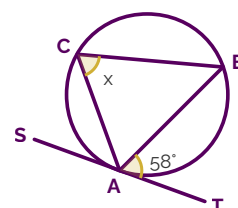
equidistant

## Worked example

**ST is a tangent at A. Angle TAB =  $58^\circ$ . Find angle ACB.**

Angle TAB is between the tangent and chord AB.

$x = 58^\circ$  (alternate segment theorem)



Not to scale

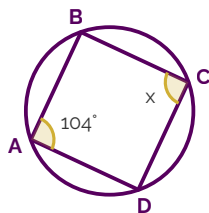
## Common mistakes

- Using the cyclic quadrilateral rule on a quadrilateral that has a vertex at the centre O.
- Choosing the angle in the wrong segment for the alternate segment theorem.

**Exam tip:** For chord lengths, draw the radius to the end of the chord: you get a right-angled triangle, then use Pythagoras.

These questions are Extended. O is the centre where shown. Find each unknown and give a reason.

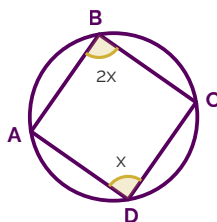
1



Not to scale

Answer  $x = \dots\dots$ 

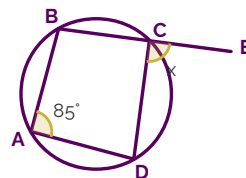
2



Not to scale

Answer  $x = \dots\dots$ 

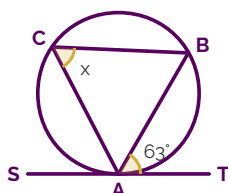
3



Not to scale

Answer  $x = \dots\dots$ 

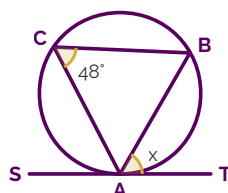
4



Not to scale

Answer  $x = \dots\dots$ 

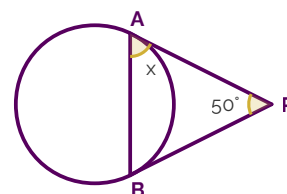
5



Not to scale

Answer  $x = \dots\dots$ 

6

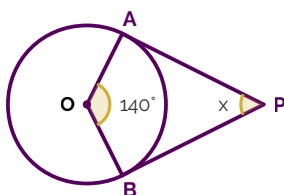


Not to scale

Answer  $x = \dots\dots$ 

PA and PB are tangents.

7

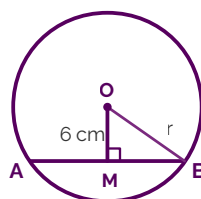


Not to scale

Answer  $x = \dots\dots$ 

OM  $\perp$  AB and AB = 16 cm. Find r.

8

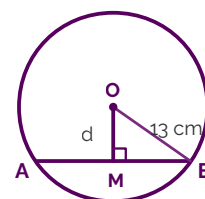


Not to scale

Answer  $r = \dots\dots$ 

OM  $\perp$  AB and AB = 24 cm. Find d.

9



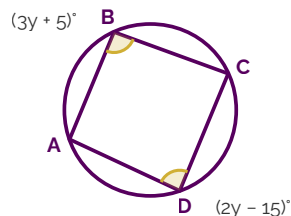
Not to scale

Answer  $d = \dots\dots$ 

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 ABCD is a cyclic quadrilateral. Angle  $ABC = (3y + 5)^\circ$  and angle  $ADC = (2y - 15)^\circ$ .



Not to scale

- (a) Find the value of  $y$ . Give a reason.

[3]

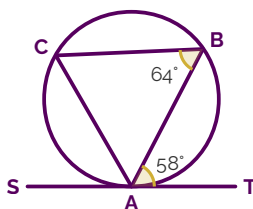
Answer  $y = \dots\dots$

- (b) Work out the size of angle  $ABC$ .

[1]

Answer  $\dots\dots^\circ$

- 2 ST is a tangent to the circle at A. Angle  $TAB = 58^\circ$  and angle  $ABC = 64^\circ$ .



Not to scale

- (a) Find angle  $ACB$ . Give a reason.

[2]

Answer  $\dots\dots^\circ$

- (b) Find angle  $BAC$ .

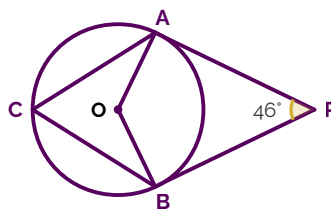
[1]

Answer  $\dots\dots^\circ$

- (c) Explain why  $AB = BC$ .

[1]

- 3 PA and PB are tangents to the circle, centre O. Angle APB =  $46^\circ$ . C lies on the major arc AB.



Not to scale

- (a) Write down the size of angle OAP. Give a reason. [1]

Answer .....

- (b) Work out the size of angle AOB. [2]

Answer .....

- (c) Work out the size of angle ACB. Give a reason. [2]

Answer .....

- 4 A circle, centre O, has radius 8.5 cm. A chord AB has length 15 cm.

- (a) Work out the distance from O to the chord AB. [3]

Answer ..... cm

- (b) Another chord CD is also 15 cm long. Write down the distance from O to CD. Give a reason. [2]

Answer ..... cm

- 5 PA and PB are tangents from P to a circle with centre O. Angle APB =  $72^\circ$ .

- (a) Work out the size of angle PAB. Give a reason. [3]

Answer .....

- (b) Work out the size of angle OAB. [1]

Answer .....

- 6 ABCD is a cyclic quadrilateral. BC is extended to E. Angle DCE =  $84^\circ$  and angle ABD =  $31^\circ$ . Work out the size of angle ADB. [3]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

## Units of measure

**I can...** convert between metric units of length, mass, capacity, area and volume.

Syllabus ref: C5.1, E5.1 · Core & Extended

Metric units are linked by powers of 10. For area, the conversion factor is **squared**; for volume it is **cubed**. So  $1\text{ m} = 100\text{ cm}$ , but  $1\text{ m}^2 = 100^2 = 10\,000\text{ cm}^2$  and  $1\text{ m}^3 = 100^3 = 1\,000\,000\text{ cm}^3$ .

## Key points

- Length:  $10\text{ mm} = 1\text{ cm}$ ,  $100\text{ cm} = 1\text{ m}$ ,  $1000\text{ m} = 1\text{ km}$ .
- Mass:  $1000\text{ g} = 1\text{ kg}$ ,  $1000\text{ kg} = 1\text{ tonne}$ . Capacity:  $1000\text{ ml} = 1\text{ litre}$ .
- $1\text{ cm}^3 = 1\text{ ml}$ ,  $1000\text{ cm}^3 = 1\text{ litre}$ ,  $1\text{ m}^3 = 1000\text{ litres}$ .
- Big unit  $\rightarrow$  small unit: multiply. Small unit  $\rightarrow$  big unit: divide.

## Key words

metric

capacity

litre

 $\text{cm}^2$  $\text{cm}^3$ 

## Worked example 1

Change  $3.5\text{ m}^2$  to  $\text{cm}^2$ .

$$1\text{ m}^2 = 100 \times 100 = 10\,000\text{ cm}^2$$

$$3.5 \times 10\,000 = \mathbf{35\,000\text{ cm}^2}$$

## Worked example 2

A tank holds  $2\,400\,000\text{ cm}^3$ . Write this in  $\text{m}^3$  and in litres.

$$2\,400\,000 \div 1\,000\,000 = \mathbf{2.4\text{ m}^3}$$

$$2\,400\,000 \div 1000 = \mathbf{2400\text{ litres}}$$

## Common mistakes

- Using 100 instead of 10 000 to convert  $\text{m}^2$  to  $\text{cm}^2$ .
- Multiplying when you should divide. Check: the number of smaller units must be bigger.

**Exam tip:** Convert all lengths to the same unit **before** you calculate an area or a volume.

**Fill in each box.**

1  $4.7 \text{ m} = \boxed{\phantom{000}} \text{ cm}$

2  $365 \text{ mm} = \boxed{\phantom{000}} \text{ cm}$

3  $2.3 \text{ km} = \boxed{\phantom{000}} \text{ m}$

4  $850 \text{ m} = \boxed{\phantom{000}} \text{ km}$

5  $3.2 \text{ kg} = \boxed{\phantom{000}} \text{ g}$

6  $450 \text{ g} = \boxed{\phantom{000}} \text{ kg}$

7  $2.5 \text{ tonnes} = \boxed{\phantom{000}} \text{ kg}$

8  $1.75 \text{ litres} = \boxed{\phantom{000}} \text{ ml}$

9  $600 \text{ ml} = \boxed{\phantom{000}} \text{ litres}$

10  $3 \text{ m}^2 = \boxed{\phantom{000}} \text{ cm}^2$

11  $45\,000 \text{ cm}^2 = \boxed{\phantom{000}} \text{ m}^2$

12  $5 \text{ cm}^2 = \boxed{\phantom{000}} \text{ mm}^2$

13  $2 \text{ m}^3 = \boxed{\phantom{000}} \text{ cm}^3$

14  $750\,000 \text{ cm}^3 = \boxed{\phantom{000}} \text{ m}^3$

15  $4 \text{ cm}^3 = \boxed{\phantom{000}} \text{ mm}^3$

16  $2500 \text{ cm}^3 = \boxed{\phantom{000}} \text{ litres}$

17  $1.8 \text{ m}^3 = \boxed{\phantom{000}} \text{ litres}$

18  $0.35 \text{ km}^2 = \boxed{\phantom{000}} \text{ m}^2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 A water tank is a cuboid measuring 1.2 m by 80 cm by 50 cm.

(a) Work out the volume of the tank in  $\text{cm}^3$ . [2]

Answer .....  $\text{cm}^3$

(b) Write the volume in litres. [1]

Answer ..... litres

(c) Water flows into the empty tank at 12 litres per minute. How long does it take to fill the tank? [2]

Answer ..... minutes

- 2 A rectangular floor measures 4.5 m by 3 m. It is covered with square tiles of side 30 cm.

(a) Work out the area of the floor in  $\text{m}^2$ . [1]

Answer .....  $\text{m}^2$

(b) Work out the area of one tile in  $\text{m}^2$ . [2]

Answer .....  $\text{m}^2$

(c) How many tiles are needed? [2]

Answer .....

- 3 A bottle contains 0.25 litres of medicine.

(a) One dose is 5 ml. How many doses are in the bottle? [2]

Answer .....

(b) A tablet has a mass of 250 mg. Work out the total mass of 40 tablets in grams. [2]

Answer ..... g

4 Areas and volumes.

(a) A field has an area of  $0.042 \text{ km}^2$ . Write this area in  $\text{m}^2$ . [1]

Answer .....  $\text{m}^2$

(b) A cube has a volume of  $0.008 \text{ m}^3$ . Work out the length of one edge in centimetres. [3]

Answer ..... cm

5 A swimming pool holds  $375 \text{ m}^3$  of water.

(a) Write  $375 \text{ m}^3$  in litres. [1]

Answer ..... litres

(b) The pool is emptied at a rate of 2.5 litres per second. How many hours does it take to empty the pool? Give your answer correct to 3 significant figures. [3]

Answer ..... hours

6 Lengths.

(a) A ribbon 3.6 m long is cut into pieces 45 cm long. How many pieces are there? [2]

Answer .....

(b) Write these lengths in order, smallest first: 0.5 km, 480 m, 52 000 cm, 0.49 km [2]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Perimeter and area

**I can...** find perimeters and areas of rectangles, triangles, parallelograms, trapezia and compound shapes.

Syllabus ref: C5.2, E5.2 · Core & Extended

**Perimeter** is the distance all the way round a shape. **Area** is the space inside it, measured in square units. Always use the **perpendicular** height, not a sloping side. For a compound shape, split it into rectangles and triangles, or subtract a part.

## Key points

- Rectangle:  $A = lw$ . Triangle:  $A = \frac{1}{2}bh$ . Parallelogram:  $A = bh$ .
- Trapezium:  $A = \frac{1}{2}(a + b)h$ , where  $a$  and  $b$  are the parallel sides.
- Perimeter: add all the sides; work out any missing sides first.

## Key words

perimeter

area

perpendicular height

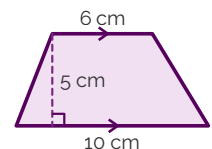
trapezium

compound shape

## Worked example

Find the area of the trapezium.

$$\begin{aligned} A &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2} \times (6 + 10) \times 5 \\ &= 40 \text{ cm}^2 \end{aligned}$$



Not to scale

## Common mistakes

- Using the sloping side instead of the perpendicular height.
- Forgetting the  $\frac{1}{2}$  in the triangle and trapezium formulas.

**Exam tip:** Write the formula first, then substitute. A correct substitution often earns the method mark even if you slip later.

Find the area or perimeter asked for. All lengths are in cm.

Area

1

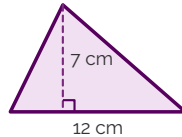


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area

2

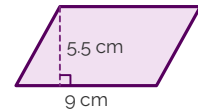


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area

3

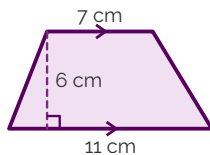


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area

4

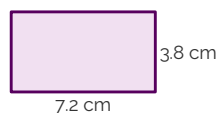


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Perimeter

5

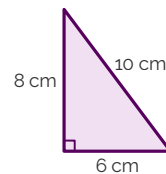


Not to scale

**Answer** P = ..... cm

Perimeter

6

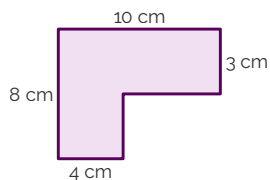


Not to scale

**Answer** P = ..... cm

Area

7

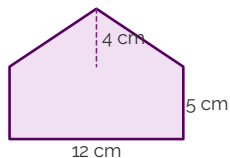


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area

8



Not to scale

**Answer** A = ..... cm<sup>2</sup>

A square has area 49 cm<sup>2</sup>. Find its side.

9

**Answer** ..... cm

A rectangle has area 54 cm<sup>2</sup> and length 9 cm. Find its width.

10

**Answer** ..... cm

A triangle has area 30 cm<sup>2</sup> and base 12 cm. Find its height.

11

**Answer** ..... cm

A trapezium has area 52 cm<sup>2</sup> and parallel sides 5 cm and 8 cm. Find its height.

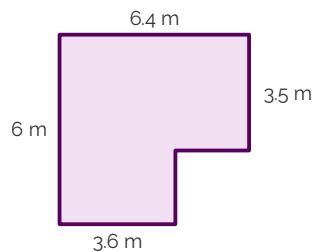
12

**Answer** ..... cm

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 The diagram shows the plan of an L-shaped room. All corners are right angles.



Not to scale

- (a) Work out the perimeter of the room. [2]

Answer ..... m

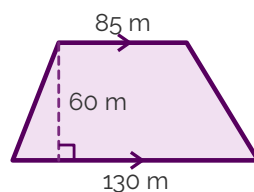
- (b) Work out the area of the room. [2]

Answer .....  $\text{m}^2$

- (c) Carpet costs \$18.50 per  $\text{m}^2$ . Work out the cost of carpeting the room. [2]

Answer \$ .....

- 2 A field is in the shape of a trapezium.



Not to scale

- (a) Work out the area of the field. [2]

Answer .....  $\text{m}^2$

- (b) Fertiliser is spread at 35 g per  $\text{m}^2$ . How many kilograms of fertiliser are needed? [3]

Answer ..... kg

- 3 A rectangle measures 15 cm by 9 cm. A triangle with base 6 cm and height 5 cm is cut out of it.

(a) Work out the area that is left.

[3]

Answer ..... cm<sup>2</sup>

- 4 A triangle has base 8 cm and height 9 cm. A parallelogram has base 12 cm and the same area as the triangle. Work out the perpendicular height of the parallelogram.

[3]

Answer ..... cm

- 5 *Extended.* A rectangle has length  $(x + 5)$  cm and width  $x$  cm. Its area is 84 cm<sup>2</sup>.

(a) Show that  $x^2 + 5x - 84 = 0$ .

[1]

(b) Solve the equation to find the dimensions of the rectangle.

[3]

Answer ..... cm by ..... cm

(c) Work out the perimeter of the rectangle.

[1]

Answer ..... cm

- 6 A square has the same perimeter as a rectangle measuring 11 cm by 7 cm. Work out the area of the square.

[3]

Answer ..... cm<sup>2</sup>

Reflect:    ☐ Red — not yet    ☐ Amber — nearly    ☐ Green — secure

## Circles, arcs and sectors

**I can...** find circumferences and areas of circles, and arc lengths and areas of sectors.

Syllabus ref: C5.3, E5.3 · Core & Extended

For a circle of radius  $r$ : circumference  $C = 2\pi r$  (or  $\pi d$ ) and area  $A = \pi r^2$ . A **sector** is a fraction of a circle: the fraction is  $\theta/360$ , where  $\theta$  is the angle at the centre. Use the  $\pi$  button and round only at the end.

## Key points

- Arc length =  $(\theta/360) \times 2\pi r$ . Sector area =  $(\theta/360) \times \pi r^2$ .
- Perimeter of a sector = arc length + 2 radii.
- Core questions use angles that divide into 360 (e.g.  $90^\circ$ ,  $60^\circ$ ); Extended questions use any angle and may ask you to find  $r$  or  $\theta$ .

## Key words

circumference

$\pi$

arc

sector

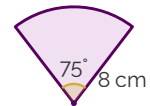
segment

## Worked example (Extended)

Find the arc length and area of the sector. Give your answers to 3 s.f.

$$\text{Arc} = 75/360 \times 2 \times \pi \times 8 = \mathbf{10.5 \text{ cm}}$$

$$\text{Area} = 75/360 \times \pi \times 8^2 = \mathbf{41.9 \text{ cm}^2}$$



Not to scale

## Common mistakes

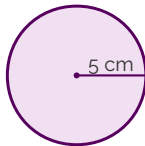
- Using the diameter in  $A = \pi r^2$ . Halve it first.
- Forgetting the two radii when finding the perimeter of a sector.

**Exam tip:** Keep full calculator values in your working and round to 3 significant figures only in the final answer.

Give your answers correct to 3 significant figures.

Circumference

1

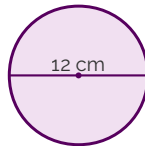


Not to scale

**Answer** C = ..... cm

Circumference

2

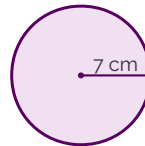


Not to scale

**Answer** C = ..... cm

Area

3

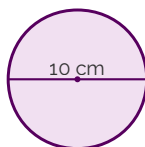


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area

4

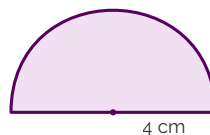


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Area of the semicircle

5

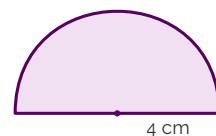


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Perimeter of the semicircle

6

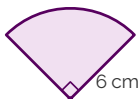


Not to scale

**Answer** P = ..... cm

Area of the quarter circle

7

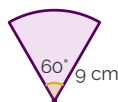


Not to scale

**Answer** A = ..... cm<sup>2</sup>

Arc length

8

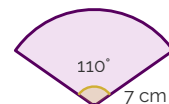


Not to scale

**Answer** ..... cm

Extended: sector area

9



Not to scale

**Answer** ..... cm<sup>2</sup>

Extended: a circle has circumference 50 cm. Find its radius.

10

**Answer** r = ..... cm

Extended: a circle has area 200 cm<sup>2</sup>. Find its radius.

11

**Answer** r = ..... cm

Extended: a sector of radius 8 cm has arc length 12 cm. Find the angle.

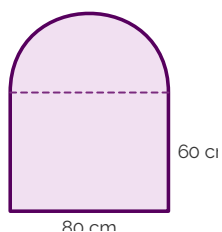
12

**Answer**  $\theta$  = ..... °

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 A window is a rectangle 80 cm wide and 60 cm high with a semicircle on top.



Not to scale

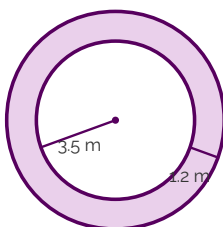
- (a) Work out the area of the window. Give your answer correct to 3 significant figures. [3]

Answer .....  $\text{cm}^2$

- (b) Work out the perimeter of the window. [3]

Answer ..... cm

- 2 A circular pond has radius 3.5 m. A path 1.2 m wide goes all the way round the pond.



Not to scale

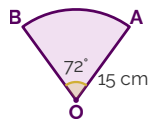
- (a) Work out the area of the path. [3]

Answer .....  $\text{m}^2$

- (b) A fence is put round the outside edge of the path. Work out the length of the fence. [2]

Answer ..... m

- 3 *Extended.* OAB is a sector of a circle, centre O, radius 15 cm. Angle AOB =  $72^\circ$ .



Not to scale

- (a) Work out the arc length AB. [2]

Answer ..... cm

- (b) Work out the perimeter of the sector. [1]

Answer ..... cm

- (c) Work out the area of the sector. [2]

Answer .....  $\text{cm}^2$

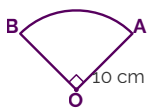
- 4 *Extended.* A sector of a circle of radius 9 cm has area  $50 \text{ cm}^2$ . Work out the angle of the sector. [3]

Answer .....  $^\circ$

- 5 A bicycle wheel has diameter 65 cm. How many complete turns does the wheel make when the bicycle travels 1 km? [3]

Answer .....

- 6 *Extended.* OAB is a quarter circle, radius 10 cm. Work out the area of the segment between the chord AB and the arc AB. [3]



Not to scale

Answer .....  $\text{cm}^2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Prisms and cylinders

**I can...** find the volume and surface area of cuboids, prisms and cylinders.

Syllabus ref: C5.4, E5.4 · Core & Extended

A **prism** has the same cross-section all the way through. Its volume is the area of the cross-section multiplied by the length. A cylinder is a prism with a circular cross-section. **Surface area** is the total area of all the faces: think of the net.

## Key points

- Volume of a prism = area of cross-section  $\times$  length.  
Cuboid:  $V = lwh$ .
- Cylinder:  $V = \pi r^2 h$ ; curved surface area =  $2\pi rh$ ; total surface area =  $2\pi rh + 2\pi r^2$ .
- Units: volume in  $\text{cm}^3$  or  $\text{m}^3$ ; area in  $\text{cm}^2$  or  $\text{m}^2$ .

## Key words

prism

cross-section

cylinder

volume

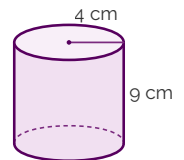
surface area

## Worked example

A cylinder has radius 4 cm and height 9 cm. Find its volume and total surface area (3 s.f.).

$$V = \pi \times 4^2 \times 9 = 144\pi = \mathbf{452 \text{ cm}^3}$$

$$SA = 2\pi \times 4 \times 9 + 2\pi \times 4^2 = 104\pi = \mathbf{327 \text{ cm}^2}$$



Not to scale

## Common mistakes

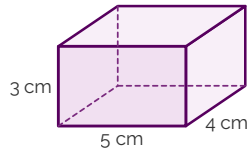
- Forgetting the two circular ends in the total surface area.
- Mixing units, e.g. a length in m with others in cm.

**Exam tip:** For a prism, first find the area of the cross-section and write it down. Then multiply by the length.

Give answers to 3 significant figures where necessary.

Volume

1

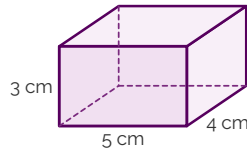


Not to scale

Answer .....  $\text{cm}^3$ 

Total surface area

2

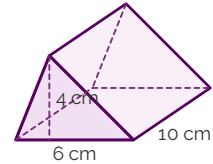


Not to scale

Answer .....  $\text{cm}^2$ 

Volume

3

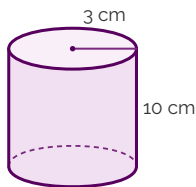


Not to scale

Answer .....  $\text{cm}^3$ 

Volume

4

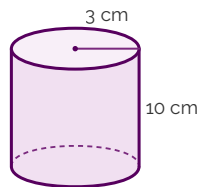


Not to scale

Answer .....  $\text{cm}^3$ 

Curved surface area

5

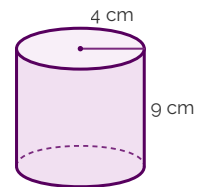


Not to scale

Answer .....  $\text{cm}^2$ 

Total surface area

6

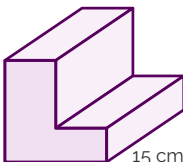


Not to scale

Answer .....  $\text{cm}^2$ 

The cross-section has area  $22 \text{ cm}^2$ . Find the volume.

7

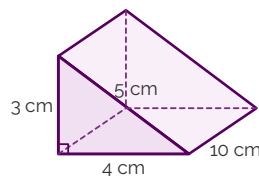


Not to scale

Answer .....  $\text{cm}^3$ 

Total surface area

8



Not to scale

Answer .....  $\text{cm}^2$ 

A cuboid has volume  $240 \text{ cm}^3$  and a base  $8 \text{ cm}$  by  $6 \text{ cm}$ . Find its height.

9

Answer .....  $\text{cm}$ 

A cube has total surface area  $150 \text{ cm}^2$ . Find its volume.

10

Answer .....  $\text{cm}^3$ 

*Extended:* a cylinder has volume  $500 \text{ cm}^3$  and radius  $5 \text{ cm}$ . Find its height.

11

Answer .....  $\text{cm}$ 

*Extended:* a cylinder has height  $10 \text{ cm}$  and volume  $1000 \text{ cm}^3$ . Find its radius.

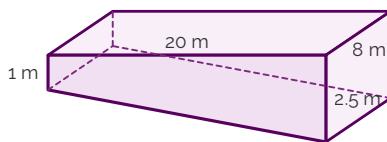
12

Answer .....  $\text{cm}$ 

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

- 1 A swimming pool is a prism. Its cross-section is a trapezium. The pool is 20 m long and 8 m wide. It is 1 m deep at one end and 2.5 m deep at the other.



Not to scale

- (a) Work out the area of the cross-section. [2]

Answer .....  $\text{m}^2$

- (b) Work out the volume of the pool. [1]

Answer .....  $\text{m}^3$

- (c) Water is pumped into the empty pool at 400 litres per minute. How long does it take to fill? Give your answer in hours and minutes. [3]

Answer ..... h ..... min

- 2 A tin of beans is a cylinder with radius 4 cm and height 11 cm.

- (a) Work out the volume of the tin. [2]

Answer .....  $\text{cm}^3$

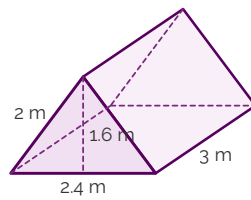
- (b) A paper label covers the curved surface. Work out the area of the label. [2]

Answer .....  $\text{cm}^2$

- (c) The tin is full of water. The water is poured into an empty cuboid with base 10 cm by 8 cm. Work out the depth of the water. [2]

Answer ..... cm

- 3 A tent is a triangular prism. The triangular ends are isosceles with base 2.4 m, sloping sides 2 m and height 1.6 m. The tent is 3 m long.



Not to scale

- (a) Work out the volume of the tent. [2]

Answer .....  $\text{m}^3$

- (b) The tent has a groundsheet. Work out the total surface area of the tent, including the groundsheet. [3]

Answer .....  $\text{m}^2$

- 4 *Extended.* A cylindrical jug has radius 5 cm and holds exactly 1 litre. Work out its height. [3]

Answer ..... cm

- 5 A cuboid has volume  $360 \text{ cm}^3$ . It is 12 cm long and 6 cm wide.

- (a) Work out its height. [1]

Answer ..... cm

- (b) Work out its total surface area. [2]

Answer .....  $\text{cm}^2$

- (c) How many cubes of edge 2 cm can be packed inside the cuboid? [2]

Answer .....

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

# Pyramids, cones and spheres

**I can...** find volumes and surface areas of pyramids, cones, spheres and compound solids.

Syllabus ref: C5.4, E5.5 · Core & Extended

These formulas are given in the exam, but you must know how to use them. A pyramid or cone has one third of the volume of the prism or cylinder with the same base and height. For a compound solid, work out each part, then add or subtract.

## Key points

- Pyramid:  $V = \frac{1}{3} \times \text{base area} \times h$ . Cone:  $V = \frac{1}{3}\pi r^2 h$ , curved surface area =  $\pi r l$  ( $l$  = slant height).
- Sphere:  $V = \frac{4}{3}\pi r^3$ , surface area =  $4\pi r^2$ . A hemisphere is half a sphere.
- Extended:* rearrange to find  $r$  or  $h$ ; a frustum = large cone – small cone.

## Key words

pyramid

cone

slant height

sphere

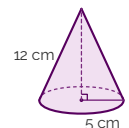
hemisphere

frustum

## Worked example

A cone has radius 5 cm and height 12 cm. Find its volume (3 s.f.).

$$\begin{aligned} V &= \frac{1}{3} \times \pi \times 5^2 \times 12 \\ &= 100\pi = \mathbf{314 \text{ cm}^3} \end{aligned}$$



Not to scale

## Common mistakes

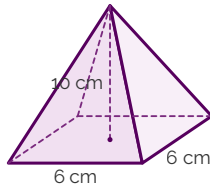
- Using the slant height  $l$  instead of the vertical height  $h$  in the volume formula.
- Forgetting the flat circular face when finding the total surface area of a hemisphere.

**Exam tip:** Write the formula from the formula list, substitute, and keep  $\pi$  until the last step.

Give answers to 3 significant figures where necessary.

Volume (square base)

1

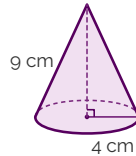


Not to scale

**Answer** .....  $\text{cm}^3$

Volume

2

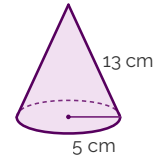


Not to scale

**Answer** .....  $\text{cm}^3$

Curved surface area

3

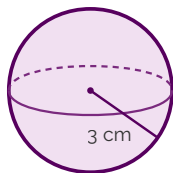


Not to scale

**Answer** .....  $\text{cm}^2$

Volume

4

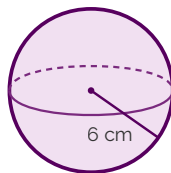


Not to scale

**Answer** .....  $\text{cm}^3$

Surface area

5



Not to scale

**Answer** .....  $\text{cm}^2$

Volume

6



Not to scale

**Answer** .....  $\text{cm}^3$

Total surface area

7

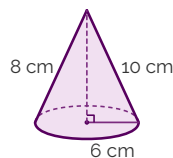


Not to scale

**Answer** .....  $\text{cm}^2$

Total surface area

8

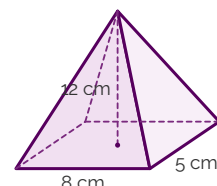


Not to scale

**Answer** .....  $\text{cm}^2$

Volume (rectangular base)

9



Not to scale

**Answer** .....  $\text{cm}^3$

*Extended:* a sphere has volume  $1000 \text{ cm}^3$ . Find its radius.

10

**Answer**  $r = \text{..... cm}$

*Extended:* a cone has volume  $300 \text{ cm}^3$  and radius  $6 \text{ cm}$ . Find its height.

11

**Answer**  $h = \text{..... cm}$

*Extended:* a sphere has surface area  $200 \text{ cm}^2$ . Find its radius.

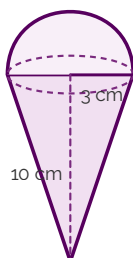
12

**Answer**  $r = \text{..... cm}$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working. Marks are shown in brackets.

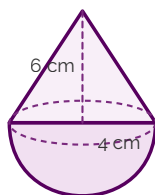
- 1 An ice cream is a cone of radius 3 cm and height 10 cm with a hemisphere of radius 3 cm on top. Work out the total volume. Give your answer correct to 3 significant figures. [4]



Not to scale

Answer ..... cm<sup>3</sup>

- 2 A toy is made from a hemisphere of radius 4 cm and a cone of radius 4 cm and height 6 cm. cm.



Not to scale

- (a) Work out the volume of the toy. [3]

Answer ..... cm<sup>3</sup>

- (b) Work out the total surface area of the toy. [4]

Answer ..... cm<sup>2</sup>

- 3 *Extended.* A solid metal sphere of radius 6 cm is melted down and made into a cylinder of radius 4 cm. Work out the height of the cylinder. [3]

Answer ..... cm

- 4 A pyramid has a square base of side 10 m and a vertical height of 12 m. The apex is directly above the centre of the base.

(a) Work out the volume of the pyramid.

[2]

Answer .....  $\text{m}^3$

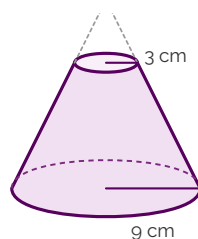
(b) The height of each triangular face is 13 m. Work out the total surface area of the pyramid.

[3]

Answer .....  $\text{m}^2$

- 5 *Extended.* A frustum is made by cutting a small cone of height 8 cm and radius 3 cm from the top of a large cone of height 24 cm and radius 9 cm. Work out the volume of the frustum.

[4]



Not to scale

Answer .....  $\text{cm}^3$

- 6 A football is a sphere with diameter 22 cm. Work out its volume.

[2]

Answer .....  $\text{cm}^3$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

## End-of-book check

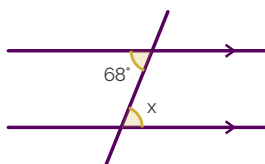
### End-of-book check

Name \_\_\_\_\_ Date \_\_\_\_\_ Score \_\_\_\_\_ / 12

Answer every question. Compare your score with the starting point check.

Find  $x$ .

1

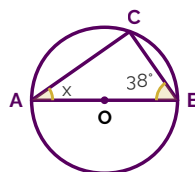


Not to scale

Answer  $x = \dots\dots$

AB is a diameter. Find  $x$ .

2



Not to scale

Answer  $x = \dots\dots$

Find the interior angle of a regular 15-sided polygon.

3

Answer  $\dots\dots^\circ$

Two triangles are similar with scale factor 2.5. A side of the small triangle is 6 cm. Find the matching side of the large triangle.

4

Answer  $\dots\dots$  cm

Find the area of a trapezium with parallel sides 5 cm and 9 cm and height 4 cm.

5

Answer  $\dots\dots$  cm<sup>2</sup>

Find the area of a circle with diameter 14 cm (3 s.f.).

6

Answer  $\dots\dots$  cm<sup>2</sup>

Find the arc length of a sector with angle  $45^\circ$  and radius 8 cm (3 s.f.).

7

Answer  $\dots\dots$  cm

Find the volume of a cylinder with radius 2 cm and height 7 cm (3 s.f.).

8

Answer  $\dots\dots$  cm<sup>3</sup>

Find the volume of a sphere with radius 4 cm (3 s.f.).

9

Answer  $\dots\dots$  cm<sup>3</sup>

Find the volume of a cone with radius 3 cm and height 4 cm (3 s.f.).

10

Answer  $\dots\dots$  cm<sup>3</sup>

Change 3 600 000 cm<sup>3</sup> to m<sup>3</sup>.

11

Answer  $\dots\dots$  m<sup>3</sup>

Extended: ABCD is a cyclic quadrilateral with angle  $A = 72^\circ$ . Find angle C.

12

Answer  $\dots\dots^\circ$



## Practice paper

Name \_\_\_\_\_ Date \_\_\_\_\_ Marks \_\_\_\_\_ / 60

Time: 1 hour 15 minutes

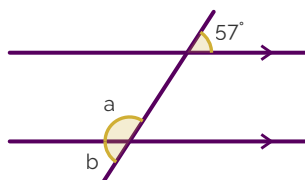
Calculator allowed. Use the  $\pi$  button on your calculator.

Give answers to 3 significant figures unless told otherwise.

Total: 60 marks

Answer all questions. Show your working. Questions marked *Extended* are for Extended candidates.

- 1 The diagram shows two parallel lines crossed by a straight line.



Not to scale

- (a) Find angle a. Give a reason.

[2]

Answer a = .....°

- (b) Find angle b. Give a reason.

[2]

Answer b = .....°

- 2 The bearing of a lighthouse L from a port P is  $128^\circ$ .

- (a) Work out the bearing of P from L.

[1]

Answer .....°

- (b) On a map of scale 1 : 25 000, PL is 14.4 cm. Work out the real distance PL in km.

[2]

Answer ..... km

- 3 Each exterior angle of a regular polygon is  $24^\circ$ .

- (a) Work out the number of sides.

[2]

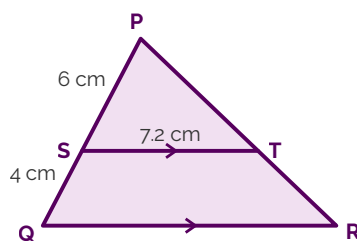
Answer .....

- (b) Work out the sum of the interior angles of the polygon.

[2]

Answer .....°

- 4 ST is parallel to QR. PS = 6 cm, SQ = 4 cm and ST = 7.2 cm.



Not to scale

- (a) Work out the length of QR.

[2]

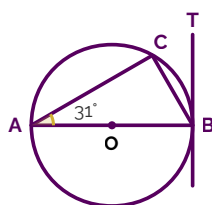
Answer ..... cm

- (b) PR = 13.5 cm. Work out the length of PT.

[2]

Answer ..... cm

- 5 AB is a diameter of the circle, centre O. TB is a tangent at B. Angle BAC =  $31^\circ$ .



Not to scale

- (a) Find angle ACB. Give a reason.

[2]

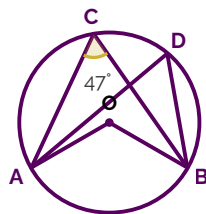
Answer ..... $^\circ$

- (b) Find angle CBT. Give reasons.

[2]

Answer ..... $^\circ$

- 6 *Extended.* A, B, C and D lie on a circle, centre O. Angle ACB =  $47^\circ$ .



Not to scale

- (a) Find angle AOB. Give a reason.

[2]

Answer ..... $^\circ$

- (b) Find angle ADB. Give a reason.

[2]

Answer ..... $^\circ$

- (c) Work out angle OAB.

[1]

Answer ..... $^\circ$

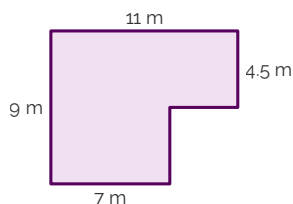
- 7 A fish tank is a cuboid 60 cm long, 35 cm wide and 40 cm high.

- (a) Work out the capacity of the tank in litres.

[3]

Answer ..... litres

- 8 The diagram shows an L-shaped garden. All corners are right angles.



Not to scale

- (a) Work out the perimeter of the garden.

[2]

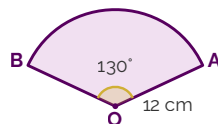
Answer ..... m

- (b) Work out the area of the garden.

[2]

Answer .....  $\text{m}^2$

- 9 *Extended.* OAB is a sector of a circle, centre O, radius 12 cm. Angle AOB =  $130^\circ$ .



Not to scale

- (a) Work out the arc length AB.

[2]

Answer ..... cm

- (b) Work out the area of the sector.

[2]

Answer .....  $\text{cm}^2$

- 10 A solid cylinder has radius 6 cm and height 15 cm.

- (a) Work out the volume of the cylinder.

[2]

Answer .....  $\text{cm}^3$

- (b) Work out the total surface area of the cylinder.

[3]

Answer .....  $\text{cm}^2$

- 11 A solid is made from a cone of radius 5 cm and height 12 cm on top of a hemisphere of radius 5 cm. Work out the total volume of the solid.

[4]

Answer .....  $\text{cm}^3$

- 12 *Extended.* Two vases are mathematically similar. The heights are 12 cm and 18 cm.

- (a) The small vase holds  $400 \text{ cm}^3$ . Work out the volume of the large vase.

[2]

Answer .....  $\text{cm}^3$

- (b) The surface area of the large vase is  $594 \text{ cm}^2$ . Work out the surface area of the small vase.

[2]

Answer .....  $\text{cm}^2$

**13** Symmetry.

**(a)** Write down the order of rotational symmetry of a regular octagon.

[1]

**Answer** .....

**(b)** *Extended.* How many planes of symmetry does a cuboid measuring 2 cm by 3 cm by 4 cm have?

[1]

**Answer** .....

**(c)** *Extended.* How many planes of symmetry does a cube have?

[1]

**Answer** .....

**(d)** How many lines of symmetry does an isosceles trapezium have?

[1]

**Answer** .....

**14** AB = 8 cm has been drawn for you.



**(a)** Using a ruler and compasses only, construct triangle ABC with AC = 6 cm and BC = 5 cm. Leave in your construction arcs.

[3]

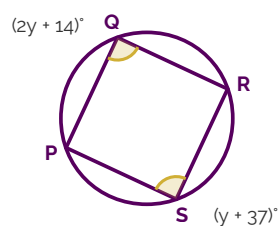
**(b)** Measure angle BAC.

[1]

**Answer** .....°

**15** *Extended.* PQRS is a cyclic quadrilateral. Angle PQR =  $(2y + 14)^\circ$  and angle PSR =  $(y + 37)^\circ$ . Find the value of  $y$  and the size of angle PSR.

[4]



Not to scale

**Answer**  $y = \dots\dots\dots$  angle PSR = .....°



Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

**Starting point check**

- 1 obtuse
- 2  $116^\circ$  (angles on a straight line)
- 3  $55^\circ$
- 4  $540^\circ$
- 5  $60^\circ$
- 6  $250^\circ$
- 7 2 km
- 8  $30 \text{ cm}^2$
- 9 62.8 cm
- 10  $60 \text{ cm}^3$
- 11  $25\,000 \text{ cm}^2$
- 12 5

**Unit 1 · Sheet A — Geometrical terms and congruence**

- 1 obtuse
- 2 reflex
- 3 octagon
- 4 heptagon
- 5 5
- 6 8
- 7 8
- 8 rhombus
- 9 kite
- 10 isosceles
- 11 diameter
- 12 tangent
- 13 segment
- 14 congruent
- 15 perpendicular bisector
- 16 triangular prism

**Unit 1 · Sheet B — Geometrical terms and congruence**

- 1 (a) OA radius; BC chord; curve BC arc; shaded region (minor) segment; PQ tangent. B1 each [5]
- 2 (a) rhombus; trapezium; kite; rectangle; parallelogram. B1 each [5]
- 3 (a) cuboid 12 edges, 8 vertices; triangular prism 9 edges, 6 vertices; square-based pyramid 5 faces. B1 each [5] (b)  $6 + 8 - 12 = 2$ ,  $5 + 6 - 9 = 2$ ,  $5 + 5 - 8 = 2$  M1; always 2 A1 [2]
- 4 (a) B B1 [1] (b) C B1 (E is not similar: its sides are not in the same ratio) [1] (c) 2 B1 [1] (d) A is 3 by 2 (ratio 1.5) but E is 4 by 3; sides not in the same ratio / not all multiplied by the same scale factor B1 [1]
- 5 (a) 11 cm B1 [1] (b)  $180 - 64 - 47$  M1 (angles in a triangle add up to  $180^\circ$ ; angle PRQ = angle ACB);  $69^\circ$  A1 [2] (c)  $47^\circ$  B1 [1] (d)  $8 + 11 + 9.5 = 28.5$  cm B1 [1]

**Unit 2 · Sheet A — Measuring, constructions and nets**

- 1 73 mm (accept 71–75)
- 2 5.8 cm (accept 5.6–6.0)
- 3  $38^\circ$  (accept  $36^\circ$ – $40^\circ$ )
- 4  $124^\circ$  (accept  $122^\circ$ – $126^\circ$ )
- 5 line 6.4 cm  $\pm$  2 mm
- 6 angle  $72^\circ \pm 2^\circ$
- 7 three sides 5 cm  $\pm$  2 mm with arcs from both ends of the base
- 8 AB = 8 cm; two pairs of equal arcs; bisector crosses AB at 4 cm at  $90^\circ$

**Unit 2 · Sheet B — Measuring, constructions and nets**

- 1 arc centre X radius 7 cm B1; arc centre Y radius 5.5 cm B1; triangle completed with Z at the crossing point, sides  $\pm$  2 mm B1 [3]
- 2 (a) QR = 5.0 cm B1; PR = 4.1 cm B1 ( $\pm$  0.2 cm) [2] (b)  $6.4 + 4.1 + 5.0 = 15.5$  cm (ft their measurements) B1 [1] (c) PQR =  $40^\circ$  B1; PRQ =  $88^\circ$  B1 ( $\pm 2^\circ$ ) [2]
- 3 (a) 5 B1 [1]  
(b)  $5 \times 5 = 25$  B1;  $4 \times \frac{1}{2} \times 5 \times 7 = 70$  M1;  $95 \text{ cm}^2$  A1 [3]
- 4 (a) C B1; the two squares on top would fold onto the same face (overlap), leaving one face open B1 [2] (b) 12 B1 [1]  
(c) 8 B1 [1]
- 5 (a) six rectangles: two  $4 \times 3$ , two  $4 \times 2$ , two  $3 \times 2$  B1; arranged so that they fold into a cuboid B1; accurate to  $\pm$  2 mm B1 [3] (b)  $2(4 \times 3 + 4 \times 2 + 3 \times 2)$  M1;  $52 \text{ cm}^2$  A1 [2]
- 6 (a) two pairs of arcs, centres A and B, same radius (more than 3.5 cm) B1; correct line through both crossings B1 [2] (b) 3.5 cm B1 [1] (c) every point on the perpendicular bisector is the same distance from A and B (oe) B1 [1]

**Unit 3 · Sheet A — Scale drawings and bearings**

- 1 500 m
- 2 1.4 km
- 3 9.2 m
- 4 12 cm
- 5 12 cm
- 6 42 km
- 7  $090^\circ$
- 8  $225^\circ$
- 9  $215^\circ$
- 10  $110^\circ$
- 11  $000^\circ$
- 12  $327^\circ$
- 13  $072^\circ$  (accept  $070^\circ$ – $074^\circ$ )
- 14  $238^\circ$  (accept  $236^\circ$ – $240^\circ$ )

### Unit 3 · Sheet B — Scale drawings and bearings

- 1** (a)  $3.8 \times 500 = 1900$  cm M1; 19 m A1 **[2]** (b)  $4200 \div 500$  M1; 8.4 cm A1 **[2]** (c)  $1250 \div 500$  M1; 2.5 cm A1 **[2]**
- 2** (a)  $9.5 \times 40\,000 = 380\,000$  cm M1; 3.8 km A1 **[2]** (b)  $260\,000 \div 40\,000$  M1; 6.5 cm A1 **[2]**
- 3** (a)  $230^\circ$  B1 **[1]** (b)  $305^\circ$  B1 **[1]** (c)  $125 - 50 = 75^\circ$  B1 **[1]**
- 4** (a) HB = 6 cm on bearing  $060^\circ$  B1; BL = 4.5 cm B1; on bearing  $150^\circ$  ( $\pm 2^\circ$ ) B1 **[3]** (b) measures HL  $\approx 7.5$  cm M1; 15 km (accept 14.6–15.4) A1 **[2]** (c)  $097^\circ$  (accept  $\pm 2^\circ$ ) B1 **[1]**
- 5** (a)  $45 \times 120 = 5400$  cm M1; 54 m A1 **[2]** (b)  $240 \div 120$  M1; 2 cm A1 **[2]**
- 6** (a)  $120^\circ$  B1 **[1]** (b) bearing of A from B =  $120^\circ$  M1;  $120 - 40 = 80^\circ$  A1 **[2]**

### Unit 4 · Sheet A — Angles and parallel lines

- 1**  $133^\circ$  (angles on a straight line add up to  $180^\circ$ )
- 2**  $155^\circ$  (angles at a point add up to  $360^\circ$ )
- 3**  $72^\circ$  (vertically opposite angles are equal)
- 4**  $55^\circ$  (angles in a triangle add up to  $180^\circ$ )
- 5**  $70^\circ$  (isosceles triangle:  $(180 - 40) \div 2$ )
- 6**  $76^\circ$  ( $180 - 2 \times 52$ )
- 7**  $115^\circ$  (corresponding angles are equal)
- 8**  $48^\circ$  (alternate angles are equal)
- 9**  $106^\circ$  (co-interior angles add up to  $180^\circ$ )
- 10**  $x = 20$  ( $2x + 3x + 80 = 180$ )
- 11**  $112^\circ$  (exterior angle = sum of the two opposite interior angles:  $63 + 49$ )
- 12**  $x = 30$  ( $3x + 4x + 5x = 360$ )

### Unit 4 · Sheet B — Angles and parallel lines

- 1** (a)  $a = 68^\circ$  B1; corresponding angles are equal B1 **[2]** (b)  $b = 112^\circ$  B1; angles on a straight line add up to  $180^\circ$  (or co-interior angles add up to  $180^\circ$ ) B1 **[2]**
- 2** (a)  $p = 52^\circ$  B1; alternate angles are equal B1 **[2]** (b)  $180 - 52 - 71$  M1 (angles on a straight line);  $q = 57^\circ$  A1 **[2]**
- 3** (a)  $x + 2x + 3x - 30 = 180$  M1;  $6x = 210$  M1;  $x = 35$  A1 **[3]** (b) No: the angles are  $35^\circ$ ,  $70^\circ$  and  $75^\circ$ , all different B1 (ft their  $x$ ) **[1]**
- 4** (a)  $(2x - 5) + (3x + 10) + 75 = 180$  M1;  $5x = 100$  M1;  $x = 20$  A1 **[3]** (b)  $3y = 2y + 24$  (vertically opposite angles) M1;  $y = 24$  so angle AOC =  $72^\circ$  A1; angle AOD =  $180 - 72 = 108^\circ$  B1 **[3]**
- 5** (a)  $64^\circ$  B1; angles on a straight line add up to  $180^\circ$  B1 **[2]** (b) angle ABC =  $64^\circ$  (base angles of an isosceles triangle are equal) B1; angle BAC =  $180 - 128 = 52^\circ$  B1 **[2]**
- 6** draw a line through E parallel to AB M1; alternate angles give  $38^\circ$  and  $47^\circ$  M1;  $y = 85^\circ$  A1 **[3]**

### Unit 5 · Sheet A — Angles in polygons

- 1**  $720^\circ$
- 2**  $540^\circ$
- 3**  $1440^\circ$
- 4**  $3240^\circ$
- 5**  $45^\circ$
- 6**  $120^\circ$
- 7**  $144^\circ$
- 8**  $150^\circ$
- 9** 12
- 10** 18
- 11**  $93^\circ$
- 12**  $95^\circ$
- 13**  $60^\circ$
- 14**  $116^\circ$
- 15** 13
- 16** no:  $360 \div 25 = 14.4$  is not a whole number

### Unit 5 · Sheet B — Angles in polygons

- 1** interior angle of pentagon =  $540 \div 5 = 108^\circ$  M1;  $360 - 108 - 90$  M1;  $x = 162^\circ$  A1 **[3]**
- 2** (a) sum of angles =  $540^\circ$  B1;  $x + 2x + 110 + 130 + 120 = 540$  M1;  $x = 60$  A1 **[3]** (b)  $130^\circ$  B1 **[1]**
- 3** (a) interior angle  $120^\circ$  and  $3 \times 120 = 360$  B1 **[1]** (b) interior angle =  $108^\circ$  M1;  $360 \div 108 = 3.33\dots$  not a whole number ( $3 \times 108 = 324$ ,  $4 \times 108 = 432$ ) A1 **[2]** (c)  $90 + 135 + 135 = 360$  B1 **[1]**
- 4** (a)  $360 \div 40$  M1; 9 A1 **[2]** (b)  $(9 - 2) \times 180$  M1;  $1260^\circ$  A1 **[2]** (c)  $e + 5e = 180$  M1;  $e = 30^\circ$  A1;  $360 \div 30 = 12$  sides A1 **[3]**
- 5** interior angle  $120^\circ$  B1; triangle ABC is isosceles ( $AB = BC$ ), so angle BAC =  $(180 - 120) \div 2 = 30^\circ$  M1; angle FAC =  $120 - 30 = 90^\circ$  A1 **[3]**
- 6** (a)  $2y + y + 30 = 180$  M1 (co-interior angles add up to  $180^\circ$ ) B1;  $y = 50$  A1 **[3]** (b)  $100^\circ$  B1 ft **[1]**

### Unit 6 · Sheet A — Symmetry in 2D and 3D

- 1** 3 lines; order 3
- 2** 0 lines; order 2
- 3** 1 line; order 1
- 4** 1 line; order 1
- 5** 6 lines; order 6
- 6** 2 lines; order 2
- 7** 8
- 8** 5
- 9** 4
- 10** 1
- 11** 9
- 12** 3
- 13** 4
- 14** 4

## Unit 6 · Sheet B — Symmetry in 2D and 3D

- 1 reflect each shaded square in the vertical line and in the horizontal line (and both): 16 squares shaded in total, symmetric about both lines B2 (B1 for symmetry in one line only) [2]
- 2 shades the square in column 4, row 5 and the square in column 3, row 4 (counting from the top left) B1 B1 [2]
- 3 (a) square 4, 4; rectangle 2, 2; parallelogram 0, 2; regular pentagon 5, 5; equilateral triangle 3, 3. B1 for each correct row [5]
- 4 (a) 6 lines: 3 joining opposite vertices and 3 joining midpoints of opposite sides B2 (B1 for 3 to 5 correct lines and none wrong) [2] (b) 6 B1 [1] (c) 12 B1 [1]
- 5 (a) parallelogram; isosceles triangle; kite; square. B1 each [4]
- 6 (a) infinitely many; infinitely many; 4; 9; 4; 5 B1 each [6] (b) 3 B1 [1] (c) 4 B1 [1]

## Unit 7 · Sheet A — Similar shapes

- 1  $k = 9 \div 3 = 3$ ;  $x = 12$  cm
- 2  $k = 12 \div 8 = 1.5$ ;  $x = 15 \div 1.5 = 10$  cm
- 3 13.5 cm
- 4 2.5
- 5  $1/4$  (0.25)
- 6 yes ( $k = 2.5$ )
- 7 15 cm
- 8 18 cm
- 9 9
- 10 64
- 11  $60 \text{ cm}^2$
- 12  $135 \text{ cm}^3$
- 13 1.5
- 14  $5/3$  ( $\sqrt[3]{(125/27)}$ )

## Unit 7 · Sheet B — Similar shapes

- 1 (a) angle A is common and angle ADE = angle ABC (corresponding angles), so all angles are equal B1 [1] (b)  $k = 8 \div 5 = 1.6$  M1;  $BC = 12$  cm A1 [2] (c)  $9.6 \div 1.6$  M1;  $AE = 6$  cm A1 [2]
- 2 (a) 1.5 B1 [1] (b)  $y = 8 \times 1.5 = 12$  cm B1;  $z = 4.4 \times 1.5 = 6.6$  cm B1 [2] (c)  $12.8 \times 1.5^2$  M1;  $28.8 \text{ cm}^2$  A1 [2]
- 3 (a)  $(24 \div 15)^2 = 2.56$  M1;  $537.6 \text{ cm}^2$  A1 [2] (b) 1.2 litres =  $1200 \text{ cm}^3$  B1;  $1200 \div 1.6^3$  M1;  $293 \text{ cm}^3$  A1 [3]
- 4 (a)  $k = 24 \div 15 = 1.6$  M1; 16 cm A1 [2] (b)  $30 \div 15 = 2$  but  $21 \div 10 = 2.1$  (or ratios 1.5 and 1.43) M1; not similar A1 [2]
- 5 (a)  $432 \div 128 = 3.375$  M1;  $k = \sqrt[3]{3.375} = 1.5$  M1; 6 cm A1 [3] (b)  $360 \div 1.5^2$  M1;  $160 \text{ cm}^2$  A1 [2]
- 6 area factor  $25\,000^2$  M1;  $12 \times 25\,000^2 = 7\,500\,000\,000 \text{ cm}^2$  M1;  $\div 10^{10} = 0.75 \text{ km}^2$  A1 [3]

## Unit 8 · Sheet A — Circle theorems

- 1  $56^\circ$  (angle in a semicircle =  $90^\circ$ ;  $180 - 90 - 34$ )
- 2  $38^\circ$  (tangent  $\perp$  radius so angle OTA =  $90^\circ$ ;  $180 - 90 - 52$ )
- 3  $82^\circ$  (angle at centre =  $2 \times$  angle at circumference)
- 4  $65^\circ$  (angle at circumference = half the angle at the centre)
- 5  $47^\circ$  (angles in the same segment are equal)
- 6  $35^\circ$  (OA = OB, isosceles:  $(180 - 110) \div 2$ )
- 7  $45^\circ$  (angle ACB =  $90^\circ$  in a semicircle; isosceles so  $(180 - 90) \div 2$ )
- 8  $125^\circ$  (angle at circumference = half the reflex angle at the centre:  $250 \div 2$ )

## Unit 8 · Sheet B — Circle theorems

- 1 (a)  $90^\circ$  B1; angle in a semicircle B1 [2] (b)  $63^\circ$  B1 [1] (c)  $90^\circ$ , tangent  $\perp$  radius (diameter) B1 [1] (d)  $90 - 63 = 27^\circ$  B1 ft [1]
- 2 (a)  $90^\circ$  B1; tangent is perpendicular to the radius B1 [2] (b)  $59^\circ$  B1 [1] (c) OT = OQ (radii) so triangle OTQ is isosceles M1;  $(180 - 59) \div 2 = 60.5^\circ$  A1 [2]
- 3 angle ACB =  $90^\circ$  (angle in a semicircle) B1;  $2x + x + 15 + 90 = 180$  M1;  $x = 25$  A1 [3]
- 4 (a)  $56^\circ$  B1; angle at the centre is twice the angle at the circumference B1 [2] (b)  $56^\circ$  B1; angles in the same segment are equal B1 [2] (c) OA = OB so  $(180 - 112) \div 2$  M1;  $34^\circ$  A1 [2]
- 5 (a) reflex angle AOB =  $2 \times 118 = 236^\circ$  M1 (angle at centre); obtuse angle AOB =  $360 - 236$  M1;  $124^\circ$  A1 [3] (b)  $124 \div 2$  (or  $180 - 118$ , cyclic quadrilateral) M1;  $62^\circ$  A1 [2]
- 6  $5y - 10 = 2(2y + 5)$  M1;  $y = 20$  A1; angle AOB =  $90^\circ$  A1 [3]

## Unit 9 · Sheet A — Cyclic quadrilaterals, tangents and chords

- 1  $76^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )
- 2  $x = 60$  ( $2x + x = 180$ )
- 3  $85^\circ$  (exterior angle of a cyclic quadrilateral = interior opposite angle)
- 4  $63^\circ$  (alternate segment theorem)
- 5  $48^\circ$  (alternate segment theorem)
- 6  $65^\circ$  (tangents from P are equal so triangle PAB is isosceles:  $(180 - 50) \div 2$ )
- 7  $40^\circ$  (angles OAP = OBP =  $90^\circ$ ;  $360 - 90 - 90 - 140$ )
- 8  $r = 10$  cm (AM = 8;  $\sqrt{(8^2 + 6^2)}$ )
- 9  $d = 5$  cm (AM = 12;  $\sqrt{(13^2 - 12^2)}$ )

## Unit 9 · Sheet B — Cyclic quadrilaterals, tangents and chords

- 1 (a)  $(3y + 5) + (2y - 15) = 180$  M1 (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ ) B1;  $y = 38$  A1 [3] (b)  $119^\circ$  B1 ft [1]
- 2 (a)  $58^\circ$  B1; alternate segment theorem B1 [2] (b)  $180 - 58 - 64 = 58^\circ$  B1 [1] (c) angle BAC = angle ACB (=  $58^\circ$ ), so triangle ABC is isosceles with AB = BC B1 [1]
- 3 (a)  $90^\circ$  (tangent  $\perp$  radius) B1 [1] (b)  $360 - 90 - 90 - 46$  M1;  $134^\circ$  A1 [2] (c)  $67^\circ$  B1; angle at the centre is twice the angle at the circumference B1 [2]
- 4 (a) perpendicular from the centre bisects the chord: 7.5 cm M1;  $\sqrt{(8.5^2 - 7.5^2)}$  M1; 4 cm A1 [3] (b) 4 cm B1; equal chords are equidistant from the centre B1 [2]
- 5 (a) PA = PB (tangents from an external point are equal) B1;  $(180 - 72) \div 2$  M1;  $54^\circ$  A1 [3] (b)  $90 - 54 = 36^\circ$  B1 ft [1]
- 6 angle DAB =  $84^\circ$  (exterior angle of a cyclic quadrilateral = interior opposite angle) B1; angle ADB =  $180 - 84 - 31$  M1;  $65^\circ$  A1 [3]

### Unit 10 · Sheet A — Units of measure

- 1 470
- 2 36.5
- 3 2300
- 4 0.85
- 5 3200
- 6 0.45
- 7 2500
- 8 1750
- 9 0.6
- 10 30 000
- 11 4.5
- 12 500
- 13 2 000 000
- 14 0.75
- 15 4000
- 16 2.5
- 17 1800
- 18 350 000

### Unit 10 · Sheet B — Units of measure

- 1 (a)  $120 \times 80 \times 50$  M1; 480 000 cm<sup>3</sup> A1 [2] (b) 480 litres B1 ft [1] (c)  $480 \div 12$  M1; 40 minutes A1 [2]
- 2 (a) 13.5 m<sup>2</sup> B1 [1] (b)  $0.3 \times 0.3$  M1; 0.09 m<sup>2</sup> A1 [2] (c)  $13.5 \div 0.09$  (or  $15 \times 10$ ) M1; 150 A1 [2]
- 3 (a)  $250 \div 5$  M1; 50 A1 [2] (b)  $40 \times 250 = 10\,000$  mg M1; 10 g A1 [2]
- 4 (a) 42 000 m<sup>2</sup> B1 [1] (b)  $0.008 \times 1\,000\,000 = 8000$  cm<sup>3</sup> M1;  $\sqrt[3]{8000}$  M1; 20 cm A1 [3]
- 5 (a) 375 000 litres B1 [1] (b)  $375\,000 \div 2.5 = 150\,000$  s M1;  $\div 3600$  M1; 41.7 hours A1 [3]
- 6 (a)  $360 \div 45$  M1; 8 A1 [2] (b) 480 m, 0.49 km, 0.5 km, 52 000 cm B2 (B1 for converting to a common unit) [2]

### Unit 11 · Sheet A — Perimeter and area

- 1 34 cm<sup>2</sup>
- 2 42 cm<sup>2</sup>
- 3 49.5 cm<sup>2</sup>
- 4 54 cm<sup>2</sup>
- 5 22 cm
- 6 24 cm
- 7  $80 - 6 \times 5 = 50$  cm<sup>2</sup> (or split into rectangles)
- 8  $60 + 24 = 84$  cm<sup>2</sup>
- 9 7 cm
- 10 6 cm
- 11 5 cm
- 12 8 cm ( $52 = \frac{1}{2} \times 13 \times h$ )

### Unit 11 · Sheet B — Perimeter and area

- 1 (a) missing sides 2.8 m and 2.5 m M1; 24.8 m A1 [2] (b)  $6.4 \times 6 - 2.8 \times 2.5$  (or split into two rectangles) M1; 31.4 m<sup>2</sup> A1 [2] (c)  $31.4 \times 18.50$  M1; \$580.90 A1 ft [2]
- 2 (a)  $\frac{1}{2} \times (85 + 130) \times 60$  M1; 6450 m<sup>2</sup> A1 [2] (b)  $6450 \times 35 = 225\,750$  g M1;  $\div 1000$  M1; 225.75 kg A1 ft [3]
- 3 (a)  $15 \times 9 = 135$  B1;  $\frac{1}{2} \times 6 \times 5 = 15$  B1; 120 cm<sup>2</sup> B1 [3]
- 4  $\frac{1}{2} \times 8 \times 9 = 36$  M1;  $36 \div 12$  M1; 3 cm A1 [3]
- 5 (a)  $x(x + 5) = 84$  leading to  $x^2 + 5x - 84 = 0$  B1 [1] (b)  $(x + 12)(x - 7) = 0$  M1;  $x = 7$  (reject -12) A1; 7 cm by 12 cm A1 [3] (c) 38 cm B1 ft [1]
- 6 perimeter = 36 cm M1; side = 9 cm M1; 81 cm<sup>2</sup> A1 [3]

### Unit 12 · Sheet A — Circles, arcs and sectors

- 1 31.4 cm
- 2 37.7 cm
- 3 154 cm<sup>2</sup>
- 4 78.5 cm<sup>2</sup>
- 5 25.1 cm<sup>2</sup>
- 6 20.6 cm
- 7 28.3 cm<sup>2</sup>
- 8 9.42 cm
- 9 47.0 cm<sup>2</sup>
- 10 7.96 cm
- 11 7.98 cm
- 12 85.9°

### Unit 12 · Sheet B — Circles, arcs and sectors

- 1 (a)  $80 \times 60 = 4800$  B1;  $\frac{1}{2} \times \pi \times 40^2$  M1; 7310 cm<sup>2</sup> A1 [3] (b)  $\pi \times 80 \div 2 = 126$  M1;  $80 + 60 + 60 + 126$  M1; 326 cm A1 [3]
- 2 (a)  $\pi \times 4.7^2 - \pi \times 3.5^2$  M2 (M1 for one area); 30.9 m<sup>2</sup> A1 [3] (b)  $2 \times \pi \times 4.7$  M1; 29.5 m A1 [2]
- 3 (a)  $72/360 \times 2 \times \pi \times 15$  M1; 18.8 cm A1 [2] (b) 48.8 cm B1 ft [1] (c)  $72/360 \times \pi \times 15^2$  M1; 141 cm<sup>2</sup> A1 [2]
- 4  $\theta/360 \times \pi \times 81 = 50$  M1;  $\theta = 50 \times 360 \div (81\pi)$  M1; 70.7° A1 [3]
- 5 circumference =  $65\pi = 204$  cm M1;  $100\,000 \div 204 = 489.7$  M1; 489 complete turns A1 [3]
- 6 quarter circle  $25\pi = 78.5$  M1; triangle  $\frac{1}{2} \times 10 \times 10 = 50$  M1; 28.5 cm<sup>2</sup> A1 [3]

### Unit 13 · Sheet A — Prisms and cylinders

- 1 60 cm<sup>3</sup>
- 2  $2(20 + 15 + 12) = 94$  cm<sup>2</sup>
- 3  $\frac{1}{2} \times 6 \times 4 \times 10 = 120$  cm<sup>3</sup>
- 4  $90\pi = 283$  cm<sup>3</sup>
- 5  $60\pi = 188$  cm<sup>2</sup>
- 6  $104\pi = 327$  cm<sup>2</sup>
- 7  $22 \times 15 = 330$  cm<sup>3</sup>
- 8  $2 \times 6 + (3 + 4 + 5) \times 10 = 132$  cm<sup>2</sup>
- 9 5 cm
- 10 edge 5 cm; 125 cm<sup>3</sup>
- 11 6.37 cm
- 12 5.64 cm

### Unit 13 · Sheet B — Prisms and cylinders

- 1** (a)  $\frac{1}{2} \times (1 + 2.5) \times 20$  M1; 35 m<sup>2</sup> A1 [2] (b) 280 m<sup>3</sup> B1 ft [1] (c) 280 m<sup>3</sup> = 280 000 litres B1; 280 000 ÷ 400 = 700 minutes M1; 11 hours 40 minutes A1 [3]
- 2** (a)  $\pi \times 4^2 \times 11$  M1; 553 cm<sup>3</sup> A1 [2] (b)  $2 \times \pi \times 4 \times 11$  M1; 276 cm<sup>2</sup> A1 [2] (c) 553 ÷ 80 M1; 6.91 cm A1 [2]
- 3** (a)  $\frac{1}{2} \times 2.4 \times 1.6 = 1.92$  M1;  $\times 3 = 5.76$  m<sup>3</sup> A1 [2] (b)  $2 \times 1.92 = 3.84$  B1;  $2 \times 2 \times 3 + 2.4 \times 3 = 19.2$  M1; 23.04 m<sup>2</sup> A1 [3]
- 4**  $\pi \times 5^2 \times h = 1000$  M1;  $h = 1000 \div (25\pi)$  M1; 12.7 cm A1 [3]
- 5** (a) 5 cm B1 [1] (b)  $2(12 \times 6 + 12 \times 5 + 6 \times 5)$  M1; 324 cm<sup>2</sup> A1 [2] (c)  $6 \times 3 \times 2$  M1 (only 2 layers fit in 5 cm); 36 A1 [2]

### Unit 14 · Sheet A — Pyramids, cones and spheres

- 1**  $\frac{1}{3} \times 36 \times 10 = 120$  cm<sup>3</sup>
- 2**  $48\pi = 151$  cm<sup>3</sup>
- 3**  $65\pi = 204$  cm<sup>2</sup>
- 4**  $36\pi = 113$  cm<sup>3</sup>
- 5**  $144\pi = 452$  cm<sup>2</sup>
- 6**  $(2/3)\pi \times 125 = 262$  cm<sup>3</sup>
- 7**  $2\pi \times 25 + \pi \times 25 = 75\pi = 236$  cm<sup>2</sup>
- 8**  $\pi \times 6 \times 10 + \pi \times 36 = 96\pi = 302$  cm<sup>2</sup>
- 9**  $\frac{1}{3} \times 40 \times 12 = 160$  cm<sup>3</sup>
- 10** 6.20 cm
- 11** 7.96 cm
- 12** 3.99 cm

### Unit 14 · Sheet B — Pyramids, cones and spheres

- 1** cone  $\frac{1}{3} \times \pi \times 3^2 \times 10 = 30\pi$  M1; hemisphere  $\frac{2}{3} \times \pi \times 3^3 = 18\pi$  M1; total  $48\pi$  M1; 151 cm<sup>3</sup> A1 [4]
- 2** (a)  $\frac{2}{3} \times \pi \times 4^3 = 134$  M1;  $\frac{1}{3} \times \pi \times 4^2 \times 6 = 101$  M1; 235 cm<sup>3</sup> A1 [3] (b) slant height  $\sqrt{4^2 + 6^2} = 7.21$  M1; cone  $\pi \times 4 \times 7.21 = 90.6$  M1; hemisphere  $2 \times \pi \times 4^2 = 101$  M1; 191 cm<sup>2</sup> A1 [4]
- 3** volume of sphere =  $(4/3)\pi \times 216 = 288\pi$  M1;  $\pi \times 16 \times h = 288\pi$  M1;  $h = 18$  cm A1 [3]
- 4** (a)  $\frac{1}{3} \times 100 \times 12$  M1; 400 m<sup>3</sup> A1 [2] (b)  $4 \times \frac{1}{2} \times 10 \times 13 = 260$  M1; + 100 M1; 360 m<sup>2</sup> A1 [3]
- 5**  $\frac{1}{3} \times \pi \times 9^2 \times 24 = 648\pi$  M1;  $\frac{1}{3} \times \pi \times 3^2 \times 8 = 24\pi$  M1; 624π M1; 1960 cm<sup>3</sup> A1 [4]
- 6**  $(4/3) \times \pi \times 11^3$  M1; 5580 cm<sup>3</sup> A1 [2]

### End-of-book check

- 1** 68° (alternate angles are equal)
- 2** 52° (angle in a semicircle = 90°)
- 3** 156°
- 4** 15 cm
- 5** 28 cm<sup>2</sup>
- 6** 154 cm<sup>2</sup>
- 7** 6.28 cm
- 8** 88.0 cm<sup>3</sup>
- 9** 268 cm<sup>3</sup>
- 10** 37.7 cm<sup>3</sup>
- 11** 3.6 m<sup>3</sup>
- 12** 108°

### Practice paper

- 1** (a) a = 123° B1; angles on a straight line / co-interior angles B1 [2] (b) b = 57° B1; corresponding angles are equal (or vertically opposite) B1 [2]
- 2** (a) 308° B1 [1] (b)  $14.4 \times 25\,000 = 360\,000$  cm M1; 3.6 km A1 [2]
- 3** (a) 360 ÷ 24 M1; 15 A1 [2] (b)  $(15 - 2) \times 180$  M1; 2340° A1 [2]
- 4** (a)  $k = 10 \div 6$  M1; 12 cm A1 [2] (b)  $13.5 \times 6 \div 10$  M1; 8.1 cm A1 [2]
- 5** (a) 90° B1; angle in a semicircle B1 [2] (b) angle ABC = 59° B1; angle ABT = 90° (tangent ⊥ radius) so angle CBT = 31° B1 [2]
- 6** (a) 94° B1; angle at the centre is twice the angle at the circumference B1 [2] (b) 47° B1; angles in the same segment are equal B1 [2] (c)  $(180 - 94) \div 2 = 43°$  B1 ft [1]
- 7** (a)  $60 \times 35 \times 40 = 84\,000$  cm<sup>3</sup> M1; ÷ 1000 M1; 84 litres A1 [3]
- 8** (a)  $2 \times (11 + 9)$  M1; 40 m A1 [2] (b)  $11 \times 9 - 4 \times 4.5$  M1; 81 m<sup>2</sup> A1 [2]
- 9** (a)  $130/360 \times 2\pi \times 12$  M1; 27.2 cm A1 [2] (b)  $130/360 \times \pi \times 12^2$  M1; 163 cm<sup>2</sup> A1 [2]
- 10** (a)  $\pi \times 6^2 \times 15$  M1; 1700 cm<sup>3</sup> A1 [2] (b)  $2\pi \times 6 \times 15 = 565$  M1;  $2 \times \pi \times 6^2 = 226$  M1; 792 cm<sup>2</sup> A1 [3]
- 11** cone  $\frac{1}{3} \times \pi \times 5^2 \times 12 = 100\pi$  M1; hemisphere  $\frac{2}{3} \times \pi \times 5^3 = 262$  M1; add M1; 576 cm<sup>3</sup> A1 (formulas: V cone =  $\frac{1}{3}\pi r^2 h$ , V sphere =  $(4/3)\pi r^3$ ) [4]
- 12** (a)  $400 \times 1.5^3$  M1; 1350 cm<sup>3</sup> A1 [2] (b)  $594 \div 1.5^2$  M1; 264 cm<sup>2</sup> A1 [2]
- 13** (a) 8 B1 [1] (b) 3 B1 [1] (c) 9 B1 [1] (d) 1 B1 [1]
- 14** (a) arc centre A radius 6 cm B1; arc centre B radius 5 cm B1; triangle completed B1 [3] (b) 39° (± 2°) B1 [1]
- 15**  $(2y + 14) + (y + 37) = 180$  M1 (opposite angles of a cyclic quadrilateral) ;  $3y = 129$  M1;  $y = 43$  A1; angle PSR = 80° A1 [4]