



BRITISH LEARNING CENTRE

CAMBRIDGE IGCSE · 0580

Mathematics

Algebra & Graphs

Fourteen units covering Topic 2 (Algebra and graphs) and Topic 3 (Coordinate geometry) of the Cambridge IGCSE Mathematics 0580 syllabus, Core and Extended, with worked examples, practice, exam-style questions and full mark schemes.

Name:

Key knowledge · Practice · Exam-style questions · Mark schemes

How to use this workbook

1

Key knowledge

Read the notes, worked examples and common mistakes.

2

Practice

Build the skill on Sheet A.

3

Exam-style

Answer exam-style questions on Sheet B. Marks are shown in [].

4

Check & reflect

Mark your work with the mark scheme, then colour the tracker.

My progress tracker

Colour each row: **Red** = not yet · **Amber** = nearly · **Green** = secure.

#	Syllabus ref	Topic · I can...	R	A	G
1	C2.1–2.2, E2.1–2.2 Core & Extended	Expressions, simplifying and substitution I can simplify expressions and substitute numbers into expressions and formulae.			
2	C2.2, E2.2 Core & Extended	Expanding brackets I can expand single, double and (Extended) triple brackets and simplify the result.			
3	C2.2, E2.2 Core & Extended	Factorising I can factorise expressions, including quadratics and the difference of two squares.			
4	C2.4, E2.3–2.4 Core & Extended	Indices and algebraic fractions I can use the laws of indices, including fractional indices, and simplify algebraic fractions.			
5	C2.5, E2.5 Core & Extended	Linear equations and changing the subject I can solve linear equations, including with fractions, and rearrange formulae.			
6	C2.5, E2.5 Core & Extended	Simultaneous equations I can solve simultaneous equations, including pairs where one equation is non-linear.			
7	E2.2, E2.5 Extended	Quadratic equations I can solve quadratics by factorising, the formula and completing the square.			
8	C2.6, E2.6 Core & Extended	Inequalities and regions I can solve linear inequalities, list integer solutions and show regions on a graph.			
9	C2.7, E2.7 Core & Extended	Sequences I can find and use the n th term of linear, quadratic, cubic and geometric sequences.			
10	C2.9, E2.8–2.9 Core & Extended	Proportion and graphs in practical situations I can use direct and inverse proportion and interpret travel graphs.			
11	C2.10–2.11, E2.10–2.11 Core & Extended	Graphs of functions and sketching curves I can draw and sketch curves and use graphs to solve equations.			
12	E2.12 Extended	Differentiation and turning points I can differentiate powers of x and use the derivative to find gradients and turning points.			
13	E2.13 Extended	Functions I can use function notation and find composite and inverse functions.			
14	C3.1–3.6, E3.1–3.7 Core & Extended	Coordinate geometry and straight lines I can find gradients, midpoints, lengths and equations of straight lines.			



CHECK

Starting point check

Starting point check

Name _____ Date _____ Score _____ / 12

Answer these questions before you begin. They show what you already know.

1 Simplify $4a + 7b - a - 3b$

2 Find $5x - 2y$ when $x = 3$ and $y = -4$

3 Expand $4(2x - 3)$

4 Factorise $6x + 9$

5 Solve $4x + 7 = 31$

6 Solve $3(x - 2) = 15$

7 Find the n th term of 6, 10, 14, 18, ...

8 Simplify $x^4 \times x^3$

9 Write down the gradient of $y = 5x - 3$

10 Solve $2x - 1 < 9$

11 Make x the subject of $y = 2x + 5$

12 Solve $x + y = 9$ and $x - y = 3$

Expressions, simplifying and substitution

I can... simplify expressions and substitute numbers into expressions and formulae.

Syllabus ref: C2.1–2.2, E2.1–2.2 · Core & Extended

An **expression** is made of **terms** such as $5a$, $-3xy$ or 7 . **Like terms** have exactly the same letters and powers, so $4x^2$ and $-x^2$ are like terms but $4x^2$ and $4x$ are not. Only like terms can be collected.

To **substitute**, replace each letter with its value in brackets, then follow the order of operations (BIDMAS). Brackets stop sign errors with negative numbers.

Key words

expression

term

like terms

coefficient

formula

substitute

Key points

- $3a$ means $3 \times a$; ab means $a \times b$; a^2 means $a \times a$.
- Collect like terms: $5a + 3b - 2a = 3a + 3b$.
- Multiply numbers first, then letters: $3p \times 4q = 12pq$.
- $(-3)^2 = 9$ but $-3^2 = -9$.

Worked example 1

Simplify $4x^2 + 3x - x^2 - 7x$.

x^2 terms: $4x^2 - x^2 = 3x^2$ · x terms: $3x - 7x = -4x$

Answer: $3x^2 - 4x$

Worked example 2

Find the value of $a^2 - 3ab$ when $a = -2$ and $b = 5$.

$(-2)^2 - 3 \times (-2) \times 5 = 4 - (-30) = 34$

Common mistakes

- Writing $3a + 2b = 5ab$. Unlike terms cannot be combined.
- Treating $2x^2$ as $(2x)^2$. Only the x is squared.

Exam tip: Write the substitution line down with brackets before you calculate; it usually earns the method mark even if the arithmetic goes wrong.

Answer each question. Show your working where it helps.

1 Simplify $5a + 3b - 2a + b$

2 Simplify $4x^2 + 3x - x^2 - 7x$

3 Simplify $3p \times 4q$

4 Simplify $2xy + 5yx - 3xy$

5 Simplify $7m - (-3m)$

6 Simplify $x \times x \times x \times y$

7 Simplify $12a^2b \div 4ab$

8 Write an expression for 6 more than three times n .

9 Find $2a + 5b$ when $a = 4$ and $b = -3$.

10 Find $a^2 - b^2$ when $a = 4$ and $b = -3$.

11 Find $(a + b)/(a - b)$ when $a = 4$ and $b = -3$.

12 Find $3a^2b$ when $a = -2$ and $b = 5$.

13 Find $4x^3$ when $x = -2$.

14 $v = u + at$. Find v when $u = 12$, $a = -3$, $t = 5$.

15 $s = ut + \frac{1}{2}at^2$. Find s when $u = 4$, $t = 6$, $a = -2$.

16 $E = \frac{1}{2}mv^2$. Find E when $m = 8$ and $v = -5$.

17 $P = 2(l + w)$. Find P when $l = 7.5$ and $w = 3.2$.

18 $T = 2\pi\sqrt{l/g}$. Find T when $l = 0.8$ and $g = 9.8$. Give 3 s.f.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

1 (a) Simplify $6x - 2y - 4x + 9y$. [2]

Answer

(b) Simplify $3a \times 5ab$. [1]

Answer

(c) Simplify $20p^3q^2 \div 5pq$. [2]

Answer

2 The area, A , of a trapezium is given by $A = \frac{1}{2}h(a + b)$.

(a) Find A when $h = 6$, $a = 7.5$ and $b = 4.3$. [2]

Answer $A =$

(b) Find h when $A = 72$, $a = 10$ and $b = 8$. [3]

Answer $h =$

3 A rectangle has length $(2x + 3)$ cm and width $(x - 1)$ cm.

(a) Write an expression for the perimeter of the rectangle. Simplify your answer. [2]

Answer cm

(b) The perimeter is 40 cm. Find the value of x . [2]

Answer $x =$

4 $p = -3$, $q = 4$ and $r = \frac{1}{2}$.

(a) Work out p^2q .

[2]

Answer

(b) Work out $(q - p) \div r$.

[2]

Answer

(c) Work out $\sqrt{(q^2 - 4pr)}$. Give your answer correct to 3 significant figures.

[2]

Answer

5 The stopping distance, d metres, of a car travelling at v km/h is given by $d = 0.2v + 0.005v^2$.

(a) Find the stopping distance when $v = 60$.

[2]

Answer m

(b) Show that the stopping distance at 80 km/h is more than double the stopping distance at 40 km/h.

[3]

6 Ali has n sweets. Ben has 5 more sweets than Ali. Chloe has twice as many sweets as Ben.

(a) Write an expression, in terms of n , for the number of sweets Chloe has.

[1]

Answer

(b) Write an expression for the total number of sweets. Simplify your answer.

[2]

Answer

(c) Altogether they have 63 sweets. How many sweets does Chloe have?

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Expanding brackets

I can... expand single, double and (Extended) triple brackets and simplify the result.

Syllabus ref: C2.2, E2.2 · Core & Extended

To **expand** a bracket, multiply every term inside by the term outside: $-4(3 - 2y) = -12 + 8y$.

For two brackets, multiply each term in the first bracket by each term in the second (a grid or FOIL helps), then collect like terms. For three brackets (Extended), expand two of them first, then multiply the answer by the third bracket.

Key words

expand

bracket

product

identity

simplify

Key points

- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + a)^2 = x^2 + 2ax + a^2$, not $x^2 + a^2$
- $(x + a)(x - a) = x^2 - a^2$
- $\equiv$ means 'identical for every value of x '.

Worked example 1

Expand and simplify $(2x + 5)(x - 3)$.

$$2x \times x = 2x^2, 2x \times (-3) = -6x, 5 \times x = 5x, 5 \times (-3) = -15 = 2x^2 - x - 15$$

Worked example 2 (Extended)

Expand and simplify $(x + 1)(x + 2)(x + 3)$.

$$(x + 1)(x + 2) = x^2 + 3x + 2$$

$$(x^2 + 3x + 2)(x + 3) = x^3 + 3x^2 + 3x^2 + 9x + 2x + 6 = x^3 + 6x^2 + 11x + 6$$

Common mistakes

- Forgetting to multiply the second term: $3(2x - 5) = 6x - 5$ is wrong.
- Sign errors with a negative outside: $-3(y - 4) = -3y + 12$.
- Writing $(x + 5)^2 = x^2 + 25$.

Exam tip: In a 'show that' question every line of working must be written; the final line must match the given expression exactly.

Expand, and simplify where possible.

1 Expand $3(2x - 5)$

2 Expand $-4(3 - 2y)$

3 Expand $x(x + 7)$

4 Expand $2a(3a - 4b)$

5 Expand and simplify $3(x + 2) + 2(x - 5)$

6 Expand and simplify $5(2y - 1) - 3(y - 4)$

7 Expand and simplify $(x + 4)(x + 3)$

8 Expand and simplify $(x - 6)(x + 2)$

9 Expand and simplify $(2x + 5)(x - 3)$

10 Expand and simplify $(3x - 2)(2x - 1)$

11 Expand and simplify $(x + 5)^2$

12 Expand and simplify $(2x - 3)^2$

13 Expand and simplify $(x + 7)(x - 7)$

14 Expand and simplify $(4 - x)(3 + 2x)$

15 Expand and simplify $(x + 1)(x + 2)(x + 3)$

16 Expand and simplify $(x - 2)^2(x + 4)$

17 Expand and simplify $(2x + 1)(x - 3)(x + 2)$

18 Expand and simplify $2x(x - 3)^2$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** Expand and simplify.

(a) $4(3x - 2) - 2(x - 5)$

[2]**Answer**

(b) $(x + 8)(x - 3)$

[2]**Answer**

(c) $(3y - 4)^2$

[2]**Answer**

(d) $(x - 5)(x + 5)$

[1]**Answer****2** A rectangle has length $(2x + 3)$ cm and width $(x + 4)$ cm.

(a) Show that the area of the rectangle is $(2x^2 + 11x + 12)$ cm².

[2]

(b) Show that $(x + 5)^2 - (x - 5)^2 = 20x$.

[2]**3** Expand and simplify $(x + 3)(x - 1)(2x + 5)$.**[3]**

- 4 n is an integer. Show that $(2n + 1)^2 - (2n - 1)^2$ is always a multiple of 8. [3]

- 5 (a) $x^2 + 12x + c \equiv (x + a)^2$. Find the values of a and c . [2]

Answer $a = \dots\dots\dots$ $c = \dots\dots\dots$

- (b) $(x + p)(x + q) \equiv x^2 + 2x - 15$, where $p > q$. Find p and q . [2]

Answer $p = \dots\dots\dots$ $q = \dots\dots\dots$

- 6 A cuboid has length $(x + 2)$ cm, width x cm and height $(x - 1)$ cm.

- (a) Show that the volume of the cuboid is $(x^3 + x^2 - 2x)$ cm³. [3]

- (b) Find the volume when $x = 4$. [1]

Answer cm³

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Factorising

I can... factorise expressions, including quadratics and the difference of two squares.

Syllabus ref: C2.2, E2.2 · Core & Extended

Factorising is the reverse of expanding. Always take out the highest common factor (HCF) first: $8a^2 - 12a = 4a(2a - 3)$.

For $x^2 + bx + c$, find two numbers that multiply to c and add to b .

For $ax^2 + bx + c$ (Extended), find two numbers that multiply to ac and add to b , split the middle term, then group.

Key words

factorise

common factor

quadratic

difference of two squares

completely

Key points

- Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$
- 'Completely' means there may be more than one step.
- Grouping: $ax + ay + bx + by = (a + b)(x + y)$
- Check by expanding your answer.

Worked example 1

Factorise $x^2 - 2x - 15$.

Multiply to -15 , add to -2 : -5 and $+3$ · Answer: $(x - 5)(x + 3)$

Worked example 2 (Extended)

Factorise $3x^2 - 10x - 8$.

$ac = -24$; numbers -12 and 2 · $3x^2 - 12x + 2x - 8 = 3x(x - 4) + 2(x - 4)$

Answer: $(x - 4)(3x + 2)$

Common mistakes

- Not taking out the full HCF: $8a^2 - 12a = 2a(4a - 6)$ is not fully factorised.
- Trying to factorise $x^2 + 49$ as a difference of two squares.
- Getting the signs the wrong way round; always expand to check.

Exam tip: If a question says 'Hence', use the answer you have just found; a different method may get no marks.

Factorise each expression. Check by expanding.

1 Factorise $6x + 15$

2 Factorise completely $8a^2 - 12a$

3 Factorise $10x^2 + 5xy$

4 Factorise completely $9p^2q - 6pq^2$

5 Factorise $x^2 + 7x + 12$

6 Factorise $x^2 - 2x - 15$

7 Factorise $x^2 - 9x + 20$

8 Factorise $x^2 - 49$

9 Factorise $4y^2 - 25$

10 Factorise $2x^2 + 7x + 3$

11 Factorise $3x^2 - 10x - 8$

12 Factorise $6x^2 + x - 2$

13 Factorise completely $3x^2 - 75$

14 Factorise completely $x^3 - 4x$

15 Factorise $ax + ay + bx + by$

16 Factorise $x^2 + 14x + 49$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** Factorise completely.

(a) $12x^2y - 18xy^2$ [2]

Answer

(b) $x^2 - x - 42$ [2]

Answer

(c) $16 - 9a^2$ [2]

Answer**2** **(a)** Factorise $2x^2 - 5x - 12$. [2]**Answer**

(b) Hence solve $2x^2 - 5x - 12 = 0$. [1]

Answer $x = \dots\dots\dots$ OR $x = \dots\dots\dots$

(c) Factorise $4x^2 - 12x + 9$. [2]

Answer**3** **(a)** Use factorisation to work out $97^2 - 3^2$ without a calculator. Show your working. [2]**Answer**

(b) n is a positive integer. Show that $n^2 + n$ is always even. [2]

4 Factorise completely.

(a) $5x^2 - 45$ [2]

Answer

(b) $xy - 3x + 2y - 6$ [2]

Answer

5 The area of a rectangle is $(x^2 + 9x + 20) \text{ cm}^2$. The length is $(x + 5) \text{ cm}$.

(a) Find an expression for the width.

[1]

Answer cm

(b) Find an expression for the perimeter. Simplify your answer.

[2]

Answer cm

6

(a) Simplify fully $(x^2 - 16) \div (x^2 + x - 12)$. Write your answer as a fraction.

[3]

Answer

(b) Hence solve $(x^2 - 16) \div (x^2 + x - 12) = 3$.

[2]

Answer $x =$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Indices and algebraic fractions

I can... use the laws of indices, including fractional indices, and simplify algebraic fractions.

Syllabus ref: C2.4, E2.3–2.4 · Core & Extended

The laws of indices work for letters as well as numbers. Negative indices give reciprocals and fractional indices give roots.

Algebraic fractions (Extended) follow the same rules as number fractions. To add or subtract, use a common denominator. To simplify, factorise the top and the bottom fully and then cancel common **factors** (whole brackets), never single terms.

Key words

index

power

reciprocal

root

algebraic fraction

common denominator

Key points

- $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$
- $a^0 = 1$ $a^{-n} = 1/a^n$ $a^{1/n} = \sqrt[n]{a}$ $a^{m/n} = (\sqrt[n]{a})^m$
- $(2x^3)^4 = 2^4x^{12} = 16x^{12}$: the number is raised to the power too.

Worked example 1

Evaluate $27^{-2/3}$.

Cube root first: $27^{1/3} = 3$ · Square: $3^2 = 9$ · Reciprocal: $1/9$

Worked example 2 (Extended)

Simplify $(x^2 + 3x)/(x^2 - 9)$.

$= x(x + 3) / ((x - 3)(x + 3)) = x/(x - 3)$

Common mistakes

- Multiplying the indices when multiplying: $x^5 \times x^3$ is x^8 , not x^{15} .
- Cancelling terms: $(x + 6)/x$ is not 6.

Exam tip: With fractional indices, take the root first; the numbers stay small and you can do it without a calculator.

Answer each question. Show your working where it helps.

1 Simplify $x^5 \times x^3$

2 Simplify $y^9 \div y^4$

3 Simplify $(a^3)^4$

4 Simplify $3x^2 \times 4x^5$

5 Simplify $15a^6 \div 5a^2$

6 Simplify $(2x^3)^4$

7 Evaluate $16^{1/2}$

8 Evaluate $27^{2/3}$

9 Evaluate $8^{-1/3}$

10 Simplify $x^{1/2} \times x^{3/2}$

11 Simplify $(16x^8)^{1/2}$

12 Solve $2^x = 32$

13 Solve $3^{x+1} = 81$

14 Write as a single fraction: $x/3 + x/4$

15 Write as a single fraction: $2/x + 3/(2x)$

16 Write as a single fraction: $(x+1)/3 - (x-2)/4$

17 Write as a single fraction: $3/(x+1) + 2/(x-2)$

18 Simplify $6x^2/(9x)$

19 Simplify $(x^2 + 3x)/(x^2 - 9)$

20 Simplify $(2/x) \times (x^2/6)$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.**1** Simplify.

(a) $(4x^3y^2)^3$ [2]

Answer

(b) $(125x^6)^{1/3}$ [2]

Answer

(c) Find n when $2^n \times 2^5 = 2^{12}$. [1]

Answer $n =$

2 **(a)** Evaluate $(9/4)^{-3/2}$. Give your answer as a fraction. [2]

Answer

(b) Evaluate $64^{-2/3}$. [1]

Answer

(c) Simplify $(x^{1/2})^4 \times x^{-3}$. [2]

Answer

3 Solve $9^x = 27^x - 1$. [3]

4 **(a)** Write $3/(x - 2) - 1/(x + 3)$ as a single fraction. Simplify the numerator. [3]

Answer

(b) Hence solve $3/(x - 2) - 1/(x + 3) = 1/2$. [3]

Answer $x =$ OR $x =$

5 Simplify fully $(2x^2 + 5x - 3)/(4x^2 - 1)$.

[3]

6 Simplify $((x^2 - 4)/(3x + 6)) \div ((x - 2)/9)$.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Linear equations and changing the subject

I can... solve linear equations, including with fractions, and rearrange formulae.

Syllabus ref: C2.5, E2.5 · Core & Extended

To solve an equation, do the same to both sides until the letter is on its own. Expand brackets first; with fractions, multiply every term by the lowest common denominator.

Changing the subject uses the same steps. If the new subject appears twice (Extended), collect those terms on one side and factorise.

Key words

equation

solve

inverse operation

subject

rearrange

Key points

- Undo operations in reverse order: subtract or add first, then divide or multiply.
- Letters on both sides: move the smaller x term.
- Square roots: write $\sqrt{\quad}$ over the whole expression.
- Check by substituting your answer back in.

Worked example 1

Solve $(x + 2)/3 = (x - 1)/2$.

Multiply by 6: $2(x + 2) = 3(x - 1)$ · $2x + 4 = 3x - 3$ · $x = 7$

Worked example 2 (Extended)

Make x the subject of $y = (x + a)/(x - a)$.

$y(x - a) = x + a$ so $xy - ay = x + a$ · $xy - x = a + ay$ so $x(y - 1) = a(1 + y)$

$x = a(y + 1)/(y - 1)$

Common mistakes

- Multiplying only one term by the denominator.
- Sign errors when subtracting a bracket: $-4(x - 2) = -4x + 8$.
- Square-rooting each term separately: $\sqrt{a^2 + b^2}$ is not $a + b$.

Exam tip: Read the question: 'Write an equation' earns a mark of its own, so show the equation before you solve it.

Answer each question. Show your working where it helps.

1 Solve $5x - 7 = 18$

2 Solve $3(2x + 1) = 27$

3 Solve $7x + 4 = 3x - 12$

4 Solve $2(x - 3) = 5(x + 6)$

5 Solve $x/4 + 3 = 8$

6 Solve $(2x - 1)/3 = 5$

7 Solve $(x + 2)/3 = (x - 1)/2$

8 Solve $5 - 2x = 11$

9 Solve $0.4x + 1.2 = 3.6$

10 Solve $3/x = 12$

11 Make x the subject: $y = 3x - 7$

12 Make x the subject: $a = b + cx$

13 Make n the subject: $P = 3(m + n)$

14 Make s the subject: $v^2 = u^2 + 2as$

15 Make r the subject: $A = \pi r^2$ ($r > 0$)

16 Make l the subject: $T = 2\pi\sqrt{l/g}$

17 Make x the subject: $y = (x + a)/(x - a)$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

1 Solve.

(a) $4(x - 3) = 2x + 9$

[3]

Answer $x =$

(b) $(3x + 1)/4 - (x - 2)/3 = 2$

[4]

Answer $x =$

2 The angles of a triangle are $(2x + 10)^\circ$, $(3x - 5)^\circ$ and $(x + 25)^\circ$.

(a) Write an equation in x and solve it.

[3]

Answer $x =$

(b) Find the size of the largest angle.

[1]

Answer $^\circ$

3 Make r the subject of the formula $V = (1/3)\pi r^2 h$, where $r > 0$.

[3]

4 Make x the subject of the formula $y = (2x + 3)/(x - 1)$.

[4]

5 A taxi company works out the cost, \$ C , of a journey of d km using $C = 3.5 + 1.8d$.

(a) Find the cost of a 12 km journey.

[1]

Answer \$

(b) A journey costs \$43.10. Find the length of the journey.

[2]

Answer km

(c) Make d the subject of the formula.

[2]

Answer $d =$

6 Mia is x years old. Her mother is three times as old as Mia. In 12 years' time her mother will be twice as old as Mia. Write an equation and solve it to find Mia's age now.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Simultaneous equations

I can... solve simultaneous equations, including pairs where one equation is non-linear.

Syllabus ref: C2.5, E2.5 · Core & Extended

Simultaneous equations are two equations that are true at the same time. The solution is the pair of values that works in both, which is where their graphs cross.

Elimination: make the coefficients of one letter equal, then add or subtract. **Substitution** (Extended, one non-linear equation): substitute the linear equation into the other. The quadratic usually gives two pairs of answers.

Key words

simultaneous

eliminate

substitute

intersection

non-linear

Key points

- Same signs: subtract. Different signs: add.
- After finding one letter, substitute back to find the other.
- Pair the answers correctly: each x goes with its own y .

Worked example 1

Solve $2x + 3y = 13$ and $3x + 2y = 12$.

$\times 3$ and $\times 2$: $6x + 9y = 39$ and $6x + 4y = 24$ · Subtract: $5y = 15$, so $y = 3$

Substitute: $2x + 9 = 13$, so $x = 2$

Worked example 2 (Extended)

Solve $y = x^2$ and $y = x + 6$.

$x^2 = x + 6$ so $x^2 - x - 6 = 0$ · $(x - 3)(x + 2) = 0$

$x = 3, y = 9$ or $x = -2, y = 4$

Common mistakes

- Subtracting negative terms wrongly: $4y - (-2y) = 6y$.
- Forgetting to multiply every term, including the number on the right-hand side.

Exam tip: Label your equations (1) and (2) and write what you are doing, for example '(1) $\times$ 2'. Examiners can then award method marks.

Solve each pair of simultaneous equations.

1 Solve: $x + y = 10$ and $x - y = 4$

2 Solve: $2x + y = 11$ and $x + y = 7$

3 Solve: $3x + 2y = 16$ and $x - 2y = 0$

4 Solve: $4x - y = 9$ and $2x + y = 9$

5 Solve: $2x + 3y = 13$ and $3x + 2y = 12$

6 Solve: $5x - 2y = 19$ and $3x + 4y = 1$

7 Solve: $y = 2x + 1$ and $3x + y = 16$

8 Solve: $2a + 5b = -1$ and $3a - 2b = 27$

9 Solve: $x + 2y = 7$ and $3x - 4y = 1$

10 Solve: $0.5x + y = 4$ and $x - y = 5$

11 (Extended) Solve: $y = x^2$ and $y = x + 6$

12 (Extended) Solve: $x + y = 5$ and $x^2 + y^2 = 13$

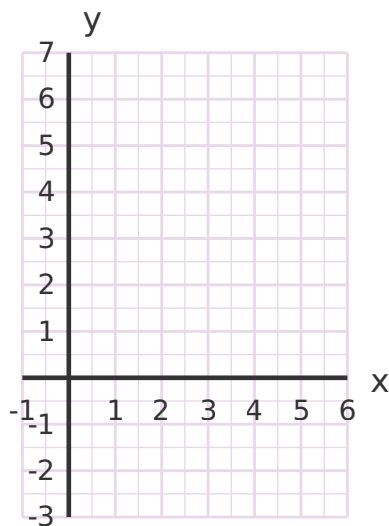
13 (Extended) Solve: $y = 2x - 1$ and $xy = 3$

14 Two numbers have a sum of 31 and a difference of 9. Find the numbers.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 (a) On the grid, draw the lines $y = 2x - 1$ and $x + y = 5$.

[2]

- (b) Use your graph to solve the simultaneous equations $y = 2x - 1$ and $x + y = 5$.

[1]**Answer** $x = \dots\dots\dots$ $y = \dots\dots\dots$

- 2 Solve the simultaneous equations. Show your working.

$$3x + 4y = 5$$

$$5x - 2y = 17$$

[3]

- 3 Three adult tickets and two child tickets for a show cost \$36. Two adult tickets and five child tickets cost \$46. An adult ticket costs \$ a and a child ticket costs \$ c .

(a) Write down two equations in a and c .

[2]

(b) Solve your equations to find the cost of each ticket.

[3]

Answer adult \$ child \$

(c) Find the cost of four adult tickets and three child tickets.

[1]

Answer \$

- 4 Solve the simultaneous equations.

$$y = x^2 - 3x + 1$$

$$y = 2x - 3$$

[4]

- 5 The line $x + 2y = 10$ meets the circle $x^2 + y^2 = 25$ at the points A and B .

(a) Find the coordinates of A and B .

[5]

Answer (.....,) and (.....,)

(b) Find the length of AB . Give your answer correct to 3 significant figures.

[2]

Answer

- 6 The line $y = mx + c$ passes through the points $(1, 5)$ and $(3, 11)$. Write two equations in m and c and solve them.

[3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Quadratic equations

I can... solve quadratics by factorising, the formula and completing the square.

Syllabus ref: E2.2, E2.5 · Extended

A quadratic equation can be written $ax^2 + bx + c = 0$. First make one side zero. If it factorises, set each bracket equal to zero. If it does not, use the quadratic formula. Completing the square gives exact answers and the turning point of the graph.

Key points

- Quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$
- $x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$
- The turning point of $y = (x + p)^2 + q$ is $(-p, q)$.
- '2 decimal places' tells you to use the formula.

Key words

quadratic

root

formula

completing the square

surd

turning point

Worked example 1

Solve $x^2 + 3x - 7 = 0$. Give answers to 2 d.p.

$$a = 1, b = 3, c = -7 \quad \cdot \quad x = (-3 \pm \sqrt{9 + 28}) / 2 = (-3 \pm \sqrt{37}) / 2$$

$$x = -4.54 \text{ or } x = 1.54$$

Worked example 2

Write $x^2 + 6x + 2$ in the form $(x + p)^2 + q$.

$$\text{Halve 6: } (x + 3)^2 = x^2 + 6x + 9$$

$$x^2 + 6x + 2 = (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$$

Common mistakes

- Dividing by x in $x^2 = 9x$ and losing the answer $x = 0$.
- Using $-b^2$ instead of $(-b)^2$, or dividing only the square root by $2a$.
- Giving only the positive root of $x^2 = 64$.

Exam tip: Write the formula with the numbers substituted before you use your calculator; this line earns the method marks.

Answer each question. Show your working where it helps.

1 Solve $x^2 - 5x + 6 = 0$

2 Solve $x^2 + 4x - 21 = 0$

3 Solve $x^2 - 9x = 0$

4 Solve $x^2 - 64 = 0$

5 Solve $2x^2 + 5x - 3 = 0$

6 Solve $3x^2 - 11x + 6 = 0$

7 Solve $x^2 + 3x - 7 = 0$. Give answers to 2 d.p.

8 Solve $2x^2 - 6x + 1 = 0$. Give answers to 2 d.p.

9 Solve $5x^2 + 2x - 4 = 0$. Give answers to 2 d.p.

10 Write $x^2 + 6x + 2$ in the form $(x + p)^2 + q$

11 Write $x^2 - 10x + 31$ in the form $(x + p)^2 + q$

12 Write $2x^2 + 12x + 5$ in the form $a(x + p)^2 + q$

13 Find the turning point of $y = x^2 - 8x + 3$

14 Solve $(x - 4)^2 = 5$. Leave answers in surd form.

15 Solve $x(x + 2) = 35$

16 Solve $x + 6/x = 5$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 (a) Factorise $x^2 - 2x - 24$. [2]

Answer

- (b) Hence solve $x^2 - 2x - 24 = 0$. [1]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- 2 Solve $3x^2 + 4x - 9 = 0$. Show all your working and give your answers correct to 2 decimal places. [3]

- 3 (a) Write $x^2 - 6x - 11$ in the form $(x - a)^2 - b$. [2]

Answer

- (b) Hence solve $x^2 - 6x - 11 = 0$. Give your answers in the form $p \pm q\sqrt{5}$. [2]

Answer $x = \dots\dots\dots$

- (c) Write down the minimum value of $x^2 - 6x - 11$. [1]

Answer

- 4 A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2 .

- (a) Show that $x^2 + 3x - 70 = 0$. [2]

- (b) Solve the equation and find the perimeter of the rectangle. [3]

- 5 Solve $2/(x + 1) + 3/x = 2$. Give your answers correct to 2 decimal places.

[5]

- 6 A ball is thrown upwards. Its height, h metres, after t seconds is $h = 20t - 5t^2$.

- (a) Find the two times when the ball is at a height of 15 m.

[3]

Answer $t = \dots\dots\dots$ and $t = \dots\dots\dots$

- (b) Find the greatest height of the ball.

[2]

Answer m

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Inequalities and regions

I can... solve linear inequalities, list integer solutions and show regions on a graph.

Syllabus ref: C2.6, E2.6 · Core & Extended

Solve a linear inequality like an equation, but if you **multiply or divide by a negative number**, **reverse the inequality sign**.

On a graph, a region is bounded by lines. Draw a **solid** line for $\leq$ or $\geq$ and a **broken** line for $<$ or $>$. Test a point such as $(0, 0)$ to decide which side you need.

Key points

- $<$ less than; $\leq$ less than or equal to; $>$ greater than; $\geq$ greater than or equal to
- Number line: open circle for $<$ or $>$, filled circle for $\leq$ or $\geq$.
- Integers include 0 and negative whole numbers.
- (Extended) For $x^2 < k$: $-\sqrt{k} < x < \sqrt{k}$. For $x^2 > k$: two separate parts.

Key words

inequality

integer

region

boundary

strict

number line

Worked example 1

Solve $5 - 3x > 17$.

$-3x > 12$ · Divide by -3 and reverse the sign: $x < -4$

Worked example 2

List the integers n with $1 \leq 3n - 2 < 13$.

Add 2: $3 \leq 3n < 15$ · Divide by 3: $1 \leq n < 5$ · $n = 1, 2, 3, 4$

Common mistakes

- Not reversing the sign when dividing by a negative.
- Including 5 in the answer to $1 \leq n < 5$.
- Shading the wrong side of a line. Always test a point.

Exam tip: Read whether the question asks you to shade the region you want or the regions you do not want, and label the required region R .

Answer each question. Show your working where it helps.

1 Solve $3x + 5 < 20$

2 Solve $2x - 7 \geq 9$

3 Solve $5 - 3x > 17$

4 Solve $4(x + 2) \leq 2x + 1$

5 Solve $x/3 + 1 > 4$

6 Solve $-3 < 2x + 1 \leq 9$

7 List the integers n such that $-2 < n \leq 3$

8 List the integers n such that $1 \leq 3n - 2 < 13$

9 Find the largest integer x such that $7x - 3 < 40$

10 Find the smallest integer n such that $5n > 37$

11 Does the point $(2, 3)$ satisfy $y > 2x - 2$?

12 (Extended) Solve $x^2 < 25$

13 (Extended) Solve $x^2 \geq 9$

14 (Extended) Solve $x^2 - 4x < 0$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** (a) Solve $7x - 4 > 2x + 21$. [2]

Answer

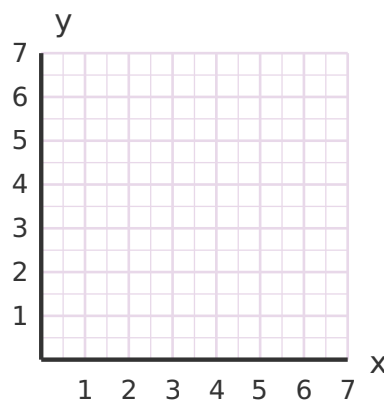
- (b) List all the integers n that satisfy $-5 \leq 2n + 1 < 7$. [3]

Answer

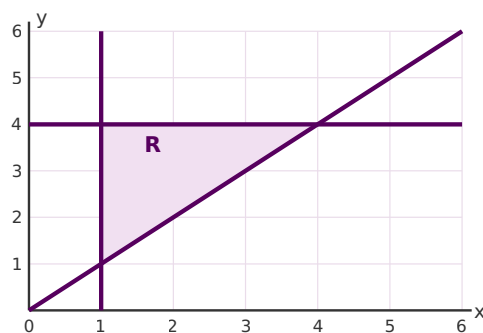
- 2** On the grid, draw the lines $y = 1$, $x + y = 6$ and $y = 2x$. Label the region R that satisfies all three inequalities

$$y \geq 1 \quad x + y \leq 6 \quad y \leq 2x.$$

[4]



- 3** Write down the three inequalities that define the shaded region R . [3]



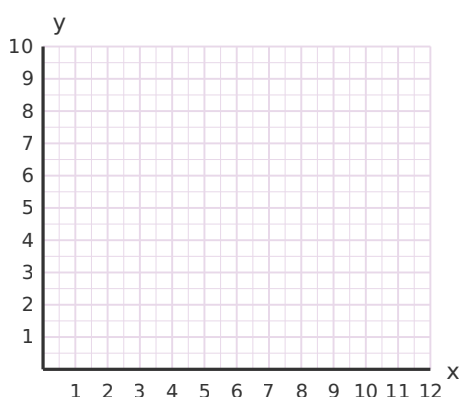
Not to scale

- 4 A school buys x footballs and y basketballs. A football costs \$8 and a basketball costs \$12. The school buys at least 3 footballs and at least 2 basketballs. It buys no more than 10 balls and spends no more than \$96.

(a) Show that $2x + 3y \leq 24$. [1]

(b) Write down three more inequalities. [2]

(c) On the grid, draw lines to show the four inequalities and label the region R that satisfies them. [2]



(d) The school buys the greatest possible number of balls for the least possible cost. How many of each does it buy? [2]

Answer footballs, basketballs

- 5 (a) (Extended) Solve $x^2 - x - 12 < 0$. [3]

Answer

(b) List the integer values of x that satisfy this inequality. [1]

Answer

- 6 A rectangle has length $(x + 3)$ cm and width $(2x - 1)$ cm. The perimeter is less than 40 cm. Find the range of possible values of x . [3]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Sequences

I can... find and use the n th term of linear, quadratic, cubic and geometric sequences.

Syllabus ref: C2.7, E2.7 · Core & Extended

A **linear** sequence has a constant first difference d ; its n th term is $dn + (\text{first term} - d)$.

A **quadratic** sequence has a constant second difference. Half the second difference is the coefficient of n^2 . Subtract that n^2 sequence and find the linear part. (Extended) A cubic has constant third differences; a **geometric** sequence multiplies by a common ratio r .

Key words

term

 n th term

difference

linear

quadratic

geometric

Key points

- Linear: n th term = $dn + c$
- Quadratic: $an^2 + bn + c$, with $2a$ = second difference
- Geometric: n th term = ar^{n-1}
- Special sequences: squares n^2 , cubes n^3 , triangular $n(n + 1)/2$

Worked example 1

Find the n th term of 3, 9, 19, 33, ...

Differences 6, 10, 14; second difference 4, so $2n^2$

$2n^2$: 2, 8, 18, 32. Sequence – $2n^2$: 1, 1, 1, 1 · n th term = $2n^2 + 1$

Worked example 2

Is 100 a term of $3n + 2$?

$3n + 2 = 100$ gives $n = 32.67$, which is not a whole number, so no.

Common mistakes

- Using the first difference as the constant: 5, 8, 11 is $3n + 2$, not $n + 3$.
- Using the whole second difference instead of half of it.
- Forgetting that n must be a positive whole number.

Exam tip: Always check your n th term by substituting $n = 1, 2$ and 3.

Answer each question. Show your working where it helps.

1 Write the next two terms: 3, 7, 11, 15, ...

2 Find the n th term: 5, 8, 11, 14, ...

3 Find the n th term: 20, 16, 12, 8, ...

4 Find the 50th term of the sequence with n th term $7n - 3$

5 Is 100 a term of the sequence $3n + 2$? Give a reason.

6 Which term of $4n + 3$ is 147?

7 Find the n th term: 2, 5, 10, 17, 26, ...

8 Find the n th term: 3, 9, 19, 33, ...

9 (Extended) Find the n th term: 4, 11, 22, 37, ...

10 (Extended) Find the n th term: 2, 9, 28, 65, ...

11 (Extended) Find the n th term: 3, 6, 12, 24, ...

12 (Extended) Find the n th term: 81, 27, 9, 3, ...

13 The first term is 4. The rule is 'multiply by 3 then subtract 1'. Find the next two terms.

14 Write the next term of the triangular numbers 1, 3, 6, 10, ...

15 Find the 5th term of the sequence with n th term $n^2 - 3n$

16 Find the 20th term of the square numbers 1, 4, 9, 16, ...

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 The diagram shows a sequence of patterns made from sticks.



- (a) How many sticks are needed for Pattern 6? [1]

Answer

- (b) Find an expression for the number of sticks in Pattern n . [2]

Answer

- (c) Can a pattern in this sequence be made with exactly 100 sticks? Show how you decide. [2]

Answer

- 2 Here are the first five terms of a sequence: 1, 7, 17, 31, 49

- (a) Write down the next term. [1]

Answer

- (b) Find an expression for the n th term. [3]

Answer

- (c) Find the 20th term. [1]

Answer

- 3 (Extended) The n th term of a sequence is $n^3 - 1$.

- (a) Write down the first four terms. [2]

Answer

- (b) Which term is equal to 999? [1]

Answer

4 (Extended) The first three terms of a geometric sequence are 5, 15, 45.

(a) Find an expression for the n th term.

[2]

Answer

(b) Find the first term that is greater than 10 000. State which term it is.

[2]

Answer

5 Sequence A has n th term $3n + 4$. Sequence B has n th term $44 - 2n$.

(a) Find the value of n for which the n th terms are equal, and the value of this term.

[3]

Answer $n = \dots\dots$ term = $\dots\dots$

(b) Find the first term of sequence A that is greater than 100.

[2]

Answer

6 The n th term of a sequence is $n^2 + 3n$.

(a) Explain why every term of the sequence is even.

[2]

(b) Which term of the sequence is equal to 180?

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Proportion and graphs in practical situations

I can... use direct and inverse proportion and interpret travel graphs.

Syllabus ref: C2.9, E2.8–2.9 · Core & Extended

(Extended) If y is **directly proportional** to x^2 , then $y = kx^2$. If y is **inversely proportional** to x , then $y = k/x$. Use the given pair of values to find k , then write the formula.

On a **distance–time** graph the gradient is the speed and a horizontal line means stopped. On a **speed–time** graph the gradient is the acceleration and the area under the graph is the distance travelled.

Key words

direct proportion

inverse proportion

constant

gradient

speed

acceleration

Key points

- Step 1: write $y = k \times (\dots)$. Step 2: find k . Step 3: use the formula.
- speed = distance $\div$ time; acceleration = change in speed $\div$ time
- Negative gradient on a speed–time graph = deceleration.

Worked example 1

y is inversely proportional to x^2 . $y = 2$ when $x = 3$. Find y when $x = 6$.

$$y = k/x^2, \text{ so } 2 = k/9 \text{ and } k = 18 \cdot y = 18/36 = 0.5$$

Worked example 2

A car speeds up from rest to 20 m/s in 8 s. Find the distance travelled.

$$\text{The graph is a triangle: area} = \frac{1}{2} \times 8 \times 20 = 80 \text{ m}$$

Common mistakes

- Forgetting to square (or square root) when finding k .
- Mixing units, for example km and minutes, when finding speed in km/h.
- Using the gradient instead of the area to find distance on a speed–time graph.

Exam tip: Split the area under a speed–time graph into triangles, rectangles and trapezia, and write each area down.

Answer each question. Show your working where it helps.

- 1 y is proportional to x . $y = 12$ when $x = 3$. Find y when $x = 7$.

- 2 y is proportional to x^2 . $y = 50$ when $x = 5$. Find y when $x = 8$.

- 3 y is inversely proportional to x . $y = 6$ when $x = 4$. Find y when $x = 3$.

- 4 y is inversely proportional to x^2 . $y = 2$ when $x = 3$. Find y when $x = 6$.

- 5 y is proportional to $\sqrt{x}$. $y = 12$ when $x = 9$. Find y when $x = 25$.

- 6 y is proportional to x^3 . x is doubled. What happens to y ?

- 7 y is inversely proportional to x . x is multiplied by 4. What happens to y ?

- 8 What does the gradient of a distance–time graph represent?

- 9 What does the area under a speed–time graph represent?

- 10 Sam walks 6 km in 1.5 hours. Find his average speed.

- 11 Change 72 km/h into m/s.

- 12 A car speeds up uniformly from rest to 20 m/s in 8 s. Find its acceleration.

- 13 For the car in the last question, find the distance travelled in the 8 s.

- 14 A cyclist travels at a constant 15 m/s for 12 s. Find the distance.

- 15 A lorry slows from 24 m/s to rest in 6 s. Find the deceleration.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1 y is directly proportional to the square of x . When $x = 3$, $y = 45$.

(a) Find a formula for y in terms of x .

[2]

Answer $y =$

(b) Find y when $x = 4$.

[1]

Answer $y =$

(c) Find the positive value of x when $y = 245$.

[2]

Answer $x =$

- 2 The force, F , between two magnets is inversely proportional to the square of the distance, d , between them. $F = 20$ when $d = 3$.

(a) Find F when $d = 6$.

[3]

Answer $F =$

(b) Find d when $F = 0.8$.

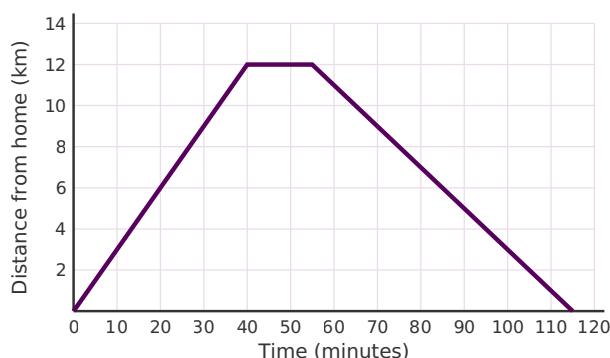
[2]

Answer $d =$

- 3 (Extended) y is proportional to x^2 . x is increased by 50%. Find the percentage increase in y .

[2]

- 4 Kai cycles from home to a park, rests, and then cycles home. The travel graph shows his journey.



- (a) Find Kai's speed, in km/h, on the way to the park. [2]

Answer km/h

- (b) How long does Kai rest? [1]

Answer min

- (c) Find Kai's speed, in km/h, on the way home. [2]

Answer km/h

- (d) Find his average speed for the whole time shown, from leaving home to arriving home. [2]

Answer km/h

- 5 A train starts from rest and accelerates uniformly to 18 m/s in 30 seconds. It travels at 18 m/s for 60 seconds, then decelerates uniformly to rest in 20 seconds.

- (a) Find the acceleration during the first 30 seconds. [2]

Answer m/s^2

- (b) Find the total distance travelled. [3]

Answer m

- (c) Find the average speed for the journey. [1]

Answer m/s

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Graphs of functions and sketching curves

I can... draw and sketch curves and use graphs to solve equations.

Syllabus ref: C2.10–2.11, E2.10–2.11 · Core & Extended

To draw a graph, complete a table of values, plot each point accurately and join the points with a **smooth curve** (not straight lines). To solve $f(x) = k$, read the x -values where the curve meets the line $y = k$.

To solve a different equation, rearrange it so that one side is the curve you have drawn; the other side is the straight line to draw. (Extended) The gradient of a curve at a point is the gradient of the **tangent** there.

Key words

quadratic

cubic

reciprocal

exponential

asymptote

tangent

Key points

- Quadratic $y = x^2$...: U shape, symmetrical. With $-x^2$: n shape.
- Cubic $y = x^3$...: S shape. Reciprocal $y = k/x$: two branches, asymptotes on the axes.
- Exponential $y = a^x$: passes through (0, 1), never below the x -axis.
- A sketch shows the shape and where the curve meets the axes; it is not plotted.

Worked example

The graph of $y = x^2 - 2x - 4$ is drawn. Which line should be drawn to solve $x^2 - 3x - 2 = 0$?

Make the left side match the curve: $x^2 - 2x - 4 = x - 2$

(subtract 2 from both sides of $x^2 - 2x = x + 2$)

Draw $y = x - 2$ and read the x -values of the crossing points.

Common mistakes

- Joining points with a ruler, or giving a quadratic a flat bottom.
- Working out $(-2)^2$ as -4 in a table.

Exam tip: Check any point that does not fit the smooth shape: it is almost always a mistake in the table.

Answer each question. Show your working where it helps.

1 For $y = x^2 - 2x - 3$, find y when $x = -2$.

2 Write down the y -intercept of $y = x^2 + 4x - 5$.

3 Where does $y = (x - 2)(x + 5)$ cross the x -axis?

4 Write down the line of symmetry of $y = x^2 - 6x + 1$.

5 Write down the turning point of $y = (x + 1)^2 - 4$.

6 For $y = x^3 - 4x$, find y when $x = -1$.

7 For $y = 12/x$, find y when $x = -3$.

8 For $y = 2^x$, find y when $x = -1$.

9 For $y = 3 \times 2^x$, find y when $x = 3$.

10 Name the type of graph of $y = 5/x$.

11 Write down the equations of the asymptotes of $y = 1/x$.

12 How many times does $y = x^2 + 1$ cross the x -axis?

13 The graph of $y = x^2 - 3x$ is drawn. Which line should be drawn to solve $x^2 - 3x - 2 = 0$?

14 The graph of $y = x^2 - 3x$ is drawn. Which line should be drawn to solve $x^2 - 5x + 1 = 0$?

(Extended) A tangent to a curve at P passes through $(1, 2)$ and $(3, 8)$. Estimate the gradient of the curve at P .

(Extended) For $y = x^3$, what happens to y when x is negative?

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

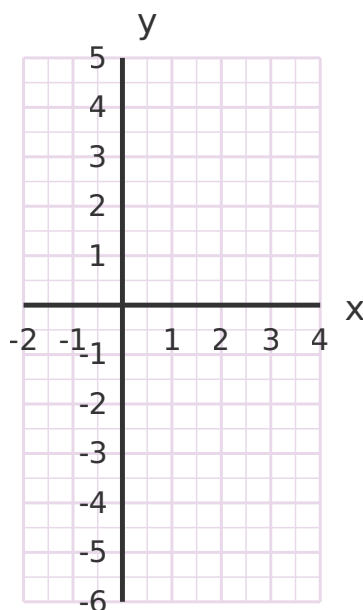
- 1 (a) Complete the table of values for $y = x^2 - 2x - 4$.

x	-2	-1	0	1	2	3	4
y	4		-4		-4		4

[2]

- (b) On the grid, draw the graph of $y = x^2 - 2x - 4$ for $-2 \leq x \leq 4$.

[4]



- (c) Use your graph to solve $x^2 - 2x - 4 = 0$.

[2]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- (d) By drawing a suitable straight line, solve $x^2 - 3x - 2 = 0$.

[3]

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$

- 2 (Extended) The curve $y = x^2 + ax + b$ crosses the x-axis at (2, 0) and (-5, 0).

- (a) Find the values of a and b .

[2]

Answer $a = \dots\dots\dots$ $b = \dots\dots\dots$

- (b) Find the coordinates of the turning point of the curve.

[2]

Answer (.....,

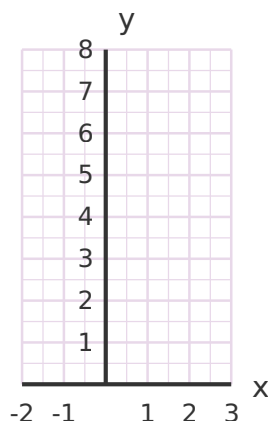
- 3 (a) Complete the table of values for $y = 2^x$.

x	-2	-1	0	1	2	3
y		0.5		2	4	

[2]

- (b) On the grid, draw the graph of $y = 2^x$ for $-2 \leq x \leq 3$.

[3]



- (c) Use your graph to solve $2^x = 5$.

[1]

Answer $x = \dots\dots\dots$

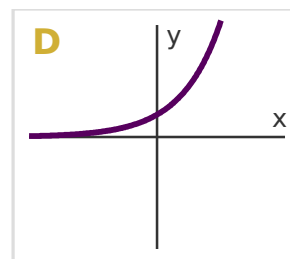
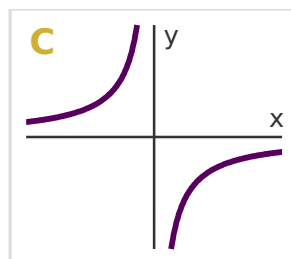
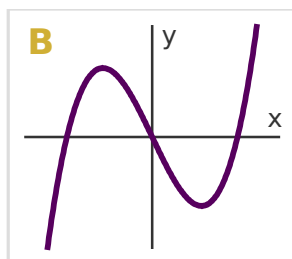
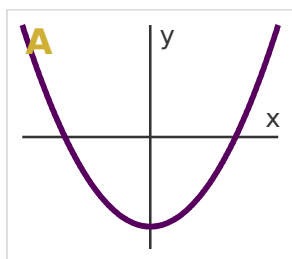
- (d) (Extended) By drawing a tangent, estimate the gradient of the curve at $x = 2$.

[3]

Answer $\dots\dots\dots$

- 4 Each of the sketches below shows one of these equations.

$y = 3^x$ $y = x^3 - 4x$ $y = -2/x$ $y = x^2 - 4$



Write the correct letter next to each equation.

$y = 3^x$ $y = x^3 - 4x$ $y = -2/x$ $y = x^2 - 4$

[4]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Differentiation and turning points

I can... differentiate powers of x and use the derivative to find gradients and turning points.

Syllabus ref: E2.12 · Extended

The **derivative** dy/dx gives the gradient of a curve at any point. Expand brackets and write fractions as negative powers before you differentiate.

At a **turning point** (stationary point) the gradient is zero. Solve $dy/dx = 0$ to find the x -coordinates, then substitute into the equation of the curve to find the y -coordinates.

Key words

derivative

gradient function

dy/dx

stationary point

maximum

minimum

Key points

- If $y = ax^n$, then $dy/dx = nax^{n-1}$.
- The derivative of a constant is 0; of kx it is k .
- Second derivative d^2y/dx^2 : negative $\rightarrow$ maximum, positive $\rightarrow$ minimum.
- Tangent at (a, b) : $y - b = m(x - a)$, with $m = dy/dx$ at $x = a$.

Worked example

Find the stationary points of $y = x^3 - 12x$ and their nature.

$$dy/dx = 3x^2 - 12 = 0, \text{ so } x^2 = 4 \text{ and } x = \pm 2$$

$$x = 2: y = 8 - 24 = -16; x = -2: y = 16$$

$$d^2y/dx^2 = 6x: \text{ at } x = 2 \text{ it is } 12 > 0 \text{ (minimum); at } x = -2 \text{ it is } -12 < 0 \text{ (maximum)}$$

Common mistakes

- Differentiating $4/x$ as 4. Write it as $4x^{-1}$, which gives $-4x^{-2}$.
- Forgetting to find the y -coordinate of a turning point.
- Differentiating a product such as $x(x + 3)$ without expanding first.

Exam tip: When a question asks for the 'nature' of a turning point, show a test: the second derivative or the gradient on each side.

Differentiate, then answer each question.

1 Find dy/dx when $y = x^5$

2 Find dy/dx when $y = 3x^4$

3 Find dy/dx when $y = 7x$

4 Find dy/dx when $y = 9$

5 Find dy/dx when $y = 2x^3 - 5x^2 + 4x - 1$

6 Find dy/dx when $y = 4/x$

7 Find dy/dx when $y = 6/x^2$

8 Find dy/dx when $y = x(x + 3)$

9 Find dy/dx when $y = (x + 2)^2$

10 Find the gradient of $y = x^2 - 4x$ at $x = 5$.

11 Find the gradient of $y = x^3$ at $x = -2$.

12 Find the turning point of $y = x^2 - 6x + 2$.

13 Find the point on $y = 2x^2 + 3x$ where the gradient is 11.

14 Find the stationary points of $y = x^3 - 12x$.

15 Find the equation of the tangent to $y = x^2$ at $(3, 9)$.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** A curve has equation $y = 2x^3 - 9x^2 + 12x + 1$.

(a) Find dy/dx .

[2]

Answer

(b) Find the coordinates of the two turning points.

[4]

Answer (.....,) (.....,)

(c) Determine whether each turning point is a maximum or a minimum. Show how you decide.

[2]

- 2** A curve has equation $y = x^2 - 5x + 4$.

(a) Find the gradient of the curve at the point (4, 0).

[2]

Answer

(b) Find the equation of the tangent to the curve at (4, 0).

[2]

Answer $y =$

- 3** A curve has equation $y = x + 9/x$, for $x > 0$.

(a) Find dy/dx .

[2]

Answer

(b) Find the coordinates of the minimum point.

[3]

- 4 A box with no lid is made from a sheet of card 18 cm by 12 cm. A square of side x cm is cut from each corner and the sides are folded up.

(a) Show that the volume, $V \text{ cm}^3$, of the box is $V = 4x^3 - 60x^2 + 216x$. [2]

(b) Find the value of x that gives the maximum volume. Give your answer correct to 3 significant figures. [3]

Answer $x = \dots\dots\dots$

(c) Find the maximum volume. [1]

Answer cm^3

- 5 Find the coordinates of the points on the curve $y = x^3 - 3x^2$ where the gradient is 9. [4]

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Functions

I can... use function notation and find composite and inverse functions.

Syllabus ref: E2.13 · Extended

A **function** is a rule that maps an input to an output. $f(3)$ means 'put 3 into f '.

In a **composite function** $fg(x)$, do g first, then f : $fg(x) = f(g(x))$. The **inverse** $f^{-1}(x)$ undoes f . To find it, write $y = f(x)$, make x the subject, then swap x and y .

Key words

function

input

output

composite

inverse

domain

Key points

- $fg(x)$ and $gf(x)$ are usually different.
- $ff^{-1}(x) = f^{-1}f(x) = x$
- The domain is the set of allowed inputs: you cannot divide by zero.
- $f^{-1}(x)$ does not mean $1/f(x)$.

Worked example 1

$f(x) = 3x - 2$ and $g(x) = x^2 + 1$. Find $gf(x)$.

$$gf(x) = g(3x - 2) = (3x - 2)^2 + 1 = 9x^2 - 12x + 5$$

Worked example 2

Find $f^{-1}(x)$ when $f(x) = 3x - 2$.

$$y = 3x - 2, \text{ so } x = (y + 2)/3 \cdot f^{-1}(x) = (x + 2)/3$$

Common mistakes

- Working out $fg(x)$ in the wrong order.
- Multiplying: $fg(x)$ is not $f(x) \times g(x)$.
- Forgetting the $\pm$ when solving $(5 - 2x)^2 = 49$.

Exam tip: Check an inverse with a number: $f(4) = 10$ and $f^{-1}(10)$ should give 4.

Use $f(x) = 3x - 2$, $g(x) = x^2 + 1$ and $h(x) = 2/(x + 1)$ unless told otherwise.

1 $f(x) = 3x - 2$. Find $f(4)$.

2 $f(x) = 3x - 2$. Find $f(-2)$.

3 $g(x) = x^2 + 1$. Find $g(-3)$.

4 $h(x) = 2/(x + 1)$. Find $h(3)$.

5 Using f and g above, find $fg(2)$.

6 Using f and g above, find $gf(2)$.

7 Find $fg(x)$. Simplify your answer.

8 Find $gf(x)$. Simplify your answer.

9 Find $ff(x)$.

10 Find $f^{-1}(x)$.

11 Find $f^{-1}(7)$.

12 Solve $f(x) = 13$.

13 Find $h^{-1}(x)$, where $h(x) = 2/(x + 1)$.

14 Which value of x must be excluded from the domain of h ?

15 $k(x) = 2^x$. Find $k(3)$.

16 $k(x) = 2^x$. Find $k(-1)$.

17 Write down the value of $ff^{-1}(5)$.

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

1 $f(x) = 5 - 2x$ $g(x) = x^2$

(a) Find $f(-3)$. [1]**Answer****(b)** Find $gf(1)$. [2]**Answer****(c)** Find $fg(x)$. [1]**Answer****(d)** Find $f^{-1}(x)$. [2]**Answer** $f^{-1}(x) = \dots\dots\dots$ **(e)** Solve $gf(x) = 49$. [3]**Answer** $x = \dots\dots\dots$ Or $x = \dots\dots\dots$

2 $f(x) = (x + 4)/3$ $g(x) = 2x - 1$ $h(x) = x^2 - 3$

(a) Find $hg(2)$. [2]**Answer****(b)** Find $g^{-1}(x)$. [2]**Answer** $g^{-1}(x) = \dots\dots\dots$ **(c)** Solve $f(x) = g(x)$. [3]**Answer** $x = \dots\dots\dots$ **(d)** Show that $gg(x) = 4x - 3$. [2]

3 $f(x) = 3/(x - 2), x \neq 2$

(a) Find $f^{-1}(x)$. [3]

Answer $f^{-1}(x) = \dots\dots\dots$

(b) Find $ff(5)$. [2]

Answer $\dots\dots\dots$

4 $f(x) = 2^x \quad g(x) = x + 3$

(a) Find $fg(2)$. [1]

Answer $\dots\dots\dots$

(b) Solve $fg(x) = 1/4$. [2]

Answer $x = \dots\dots\dots$

(c) Find $gf(x)$. [1]

Answer $\dots\dots\dots$

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Coordinate geometry and straight lines

I can... find gradients, midpoints, lengths and equations of straight lines.

Syllabus ref: C3.1–3.6, E3.1–3.7 · Core & Extended

A straight line has equation $y = mx + c$, where m is the **gradient** and c is the y -intercept. If the equation is not in this form, rearrange it to make y the subject first.

Parallel lines have equal gradients. (Extended) Perpendicular gradients multiply to -1 , so the perpendicular gradient is the negative reciprocal.

Key words

gradient

intercept

midpoint

parallel

perpendicular

Key points

- Gradient = $(y_2 - y_1) \div (x_2 - x_1)$
- Midpoint = $((x_1 + x_2)/2, (y_1 + y_2)/2)$
- (Extended) Length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- To find c , substitute a point on the line into $y = mx + c$.

Worked example

Find the equation of the line through (2, 3) and (4, 11).

$$m = (11 - 3) \div (4 - 2) = 4 \quad \cdot \quad 3 = 4 \times 2 + c, \text{ so } c = -5 \quad \cdot \quad y = 4x - 5$$

Worked example (Extended)

Find the gradient of a line perpendicular to $3x + 4y = 12$.

$$4y = -3x + 12, \text{ so } y = -\frac{3}{4}x + 3 \text{ and } m = -\frac{3}{4} \quad \cdot \quad \text{Perpendicular gradient} = \frac{4}{3}$$

Common mistakes

- Dividing the x -difference by the y -difference.
- Reading the gradient of $2y = 6x + 5$ as 6.
- Subtracting the coordinates in a different order on the top and the bottom.

Exam tip: Sketch the points quickly: a line going up from left to right must have a positive gradient.

Answer each question. Show your working where it helps.

1 Find the gradient of the line through (2, 5) and (6, 13).

2 Find the gradient of the line through (-1, 4) and (3, -8).

3 Find the midpoint of (2, 7) and (8, -1).

4 Find the length of the line from (1, 2) to (7, 10).

5 Find the length of the line from (-3, 5) to (2, -7).

6 Write down the gradient and y-intercept of $y = 4x - 7$.

7 Find the gradient of $2y = 6x + 5$.

8 Find the gradient of $3x + 4y = 12$.

9 Find the equation of the line with gradient -2 through (0, 5).

10 Find the equation of the line through (2, 3) and (4, 11).

11 Find the equation of the line parallel to $y = 3x + 1$ through (2, 9).

12 Find the x-intercept of $y = 2x - 8$.

13 Does (3, 7) lie on $y = 3x - 2$? Explain.

14 (Extended) Write down the gradient of a line perpendicular to $y = 2x - 3$.

15 (Extended) A line has gradient $-3/4$. Write down the gradient of a perpendicular line.

16 (Extended) Find the perpendicular bisector of (0, 0) and (4, 2).

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure

Answer all questions. Show your working clearly.

- 1** A is the point $(-2, 1)$ and B is the point $(4, 9)$.

(a) Find the coordinates of the midpoint of AB.

[2]

Answer (.....,)

(b) (Extended) Find the length of AB.

[3]

Answer

(c) Find the equation of the line AB. Give your answer in the form $ax + by = c$.

[3]

Answer

- 2** The line L has equation $2y - x = 6$.

(a) Find the gradient of L.

[1]

Answer

(b) Find the equation of the line parallel to L that passes through $(4, -1)$.

[2]

Answer $y =$

(c) (Extended) Find the equation of the line perpendicular to L that passes through $(2, 4)$. **[3]**

Answer $y =$

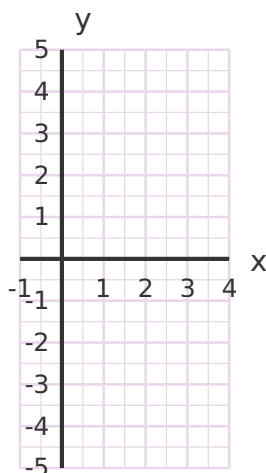
- 3 (a) Complete the table of values for $y = 3 - 2x$.

x	-1	0	1	2	3	4
y	5			-1		

[1]

- (b) Draw the graph of $y = 3 - 2x$ for $-1 \leq x \leq 4$.

[2]



- (c) On the same grid, draw $y = x$ and write down the coordinates of the point where the lines cross.

[1]

Answer (.....,

- 4 (Extended) P is the point $(1, 3)$ and Q is the point $(7, -1)$. Find the equation of the perpendicular bisector of PQ . Give your answer in the form $y = mx + c$.

[4]

- 5 (Extended) $A(1, 1)$, $B(5, 3)$ and $C(3, 7)$ are three vertices of a triangle.

- (a) Show that angle ABC is a right angle.

[3]

- (b) Find the area of triangle ABC .

[2]

Answer

Reflect: ☐ Red — not yet ☐ Amber — nearly ☐ Green — secure



CHECK

End-of-book check

End-of-book check

Name _____ Date _____ Score _____ / 12

Answer these questions to check what you have learned.

1 Expand and simplify $(x - 4)(x + 6)$ 2 Factorise $x^2 - 8x + 15$ 3 Solve $x^2 + x - 20 = 0$ 4 Simplify $(3x^2y)^3$ 5 Solve $5(x + 1) = 2x - 7$ 6 Find the n th term of 4, 9, 16, 25, ...7 Find the gradient of the line through $(-1, 2)$ and $(3, 14)$ 8 $f(x) = 2x + 5$. Find $f^{-1}(x)$ 9 Find dy/dx when $y = 4x^3 - 6x$ 10 Find the midpoint of $(-4, 7)$ and $(6, 1)$ 11 y is inversely proportional to x . $y = 10$ when $x = 2$. Find y when $x = 5$.12 Solve $-1 \leq 3x + 2 < 11$



Practice paper

Name _____ Date _____ Marks _____ / 60

Time: 1 hour 15 minutes
Calculator allowed
Total: 60 marks
Core and Extended content

Answer all questions. Show all your working. Give non-exact answers correct to 3 significant figures unless the question says otherwise.

1 (a) Simplify $5p - 3q + 2p + 7q$. [2]

Answer

(b) Expand and simplify $(2x - 5)(x + 3)$. [2]

Answer

2 Solve.

(a) $7x - 9 = 3x + 11$ [2]

Answer $x =$

(b) $(x + 2)/3 + (x - 1)/2 = 4$ [3]

Answer $x =$

3 Factorise completely.

(a) $15x^2y + 10xy^2$ [2]

Answer

(b) $x^2 - 3x - 28$ [2]

Answer

(c) $9x^2 - 4$ [1]

Answer

- 4 Make t the subject of the formula $v = u + at^2$, where $t > 0$. [3]

- 5 Solve the simultaneous equations.

$$4x + 3y = 6$$

$$2x - 5y = 16$$

[4]

- 6 (a) Find the n th term of the sequence 7, 12, 17, 22, ... [2]

Answer

- (b) (Extended) Find the n th term of the sequence 3, 10, 21, 36, ... [3]

Answer

- 7 (Extended) Solve $2x^2 - 7x + 2 = 0$. Give your answers correct to 2 decimal places. [3]

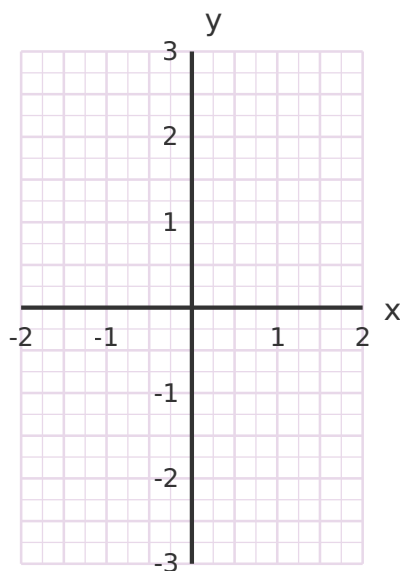
- 8 (a) Complete the table of values for $y = x^3 - 3x$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-2		2	1.375	0		-2		2

[2]

- (b) Draw the graph of $y = x^3 - 3x$ for $-2 \leq x \leq 2$.

[3]



- (c) Use your graph to solve $x^3 - 3x = 1$.

[2]

Answer $x = \dots\dots\dots$

- 9 (Extended) $f(x) = 2x - 3$ and $g(x) = x^2 + 1$.

- (a) Find $fg(2)$.

[2]

Answer $\dots\dots\dots$

- (b) Find $f^{-1}(x)$.

[2]

Answer $f^{-1}(x) = \dots\dots\dots$

- (c) Solve $gf(x) = 10$.

[3]

Answer $x = \dots\dots\dots$ OR $x = \dots\dots\dots$

10 (Extended) A curve has equation $y = x^3 - 6x^2 + 9x + 2$.

(a) Find dy/dx .

[2]

Answer

(b) Find the coordinates of the stationary points.

[4]

Answer (.....,) (.....,)

(c) Determine the nature of each stationary point.

[2]

11 A is the point $(-1, 3)$ and B is the point $(5, -1)$.

(a) (Extended) Find the length of AB.

[3]

Answer

(b) (Extended) Find the equation of the perpendicular bisector of AB.

[4]

Answer $y =$

12 Solve $3 - 2x \leq 11$.

[2]



FOR TEACHERS & PARENTS

Answers & mark schemes

Mark schemes list the points that earn marks. Accept equivalent answers and follow-through where working is correct. M = method mark, A = accuracy mark, B = independent mark.

Starting point check

- 1 $3a + 4b$
- 2 23
- 3 $8x - 12$
- 4 $3(2x + 3)$
- 5 $x = 6$
- 6 $x = 7$
- 7 $4n + 2$
- 8 x^7
- 9 5
- 10 $x < 5$
- 11 $x = (y - 5)/2$
- 12 $x = 6, y = 3$

Unit 1 · Sheet A — Expressions, simplifying and substitution

- 1 $3a + 4b$
- 2 $3x^2 - 4x$
- 3 $12pq$
- 4 $4xy$
- 5 $10m$
- 6 x^3y
- 7 $3a$
- 8 $3n + 6$
- 9 -7
- 10 7
- 11 $1/7$
- 12 60
- 13 -32
- 14 -3
- 15 -12
- 16 100
- 17 21.4
- 18 180

Unit 1 · Sheet B — Expressions, simplifying and substitution

- 1 (a) $2x + 7y$ B2 (B1 for one term correct) [2] (b) $15a^2b$ B1 [1] (c) $4p^2q$ B2 (B1 for two of 4, p^2 , q correct) [2]
- 2 (a) $\frac{1}{2} \times 6 \times (7.5 + 4.3)$ M1; 35.4 A1 [2] (b) $72 = \frac{1}{2}h(18)$ M1; $72 = 9h$ M1; $h = 8$ A1 [3]
- 3 (a) $2(2x + 3) + 2(x - 1)$ M1; $6x + 4$ A1 [2] (b) $6x + 4 = 40$ M1; $x = 6$ A1 [2]
- 4 (a) $(-3)^2 \times 4$ M1; 36 A1 [2] (b) $7 \div \frac{1}{2}$ M1; 14 A1 [2] (c) $16 - 4(-3)$ ($\frac{1}{2}$) = 22 M1; 4.69 A1 [2]
- 5 (a) $0.2 \times 60 + 0.005 \times 60^2$ M1; 30 m A1 [2] (b) $d(80) = 48$ B1; $d(40) = 16$ B1; $48 > 2 \times 16 = 32$ so more than double B1 [3]
- 6 (a) $2(n + 5)$ oe B1 [1] (b) $n + n + 5 + 2n + 10$ M1; $4n + 15$ A1 [2] (c) $4n + 15 = 63$ so $n = 12$ M1; Chloe has 34 A1 [2]

Unit 2 · Sheet A — Expanding brackets

- 1 $6x - 15$
- 2 $8y - 12$
- 3 $x^2 + 7x$
- 4 $6a^2 - 8ab$
- 5 $5x - 4$
- 6 $7y + 7$
- 7 $x^2 + 7x + 12$
- 8 $x^2 - 4x - 12$
- 9 $2x^2 - x - 15$
- 10 $6x^2 - 7x + 2$
- 11 $x^2 + 10x + 25$
- 12 $4x^2 - 12x + 9$
- 13 $x^2 - 49$
- 14 $-2x^2 + 5x + 12$
- 15 $x^3 + 6x^2 + 11x + 6$
- 16 $x^3 - 12x + 16$
- 17 $2x^3 - x^2 - 13x - 6$
- 18 $2x^3 - 12x^2 + 18x$

Unit 2 · Sheet B — Expanding brackets

1 (a) $12x - 8 - 2x + 10$ M1; $10x + 2$ A1 **[2]** **(b)** $x^2 - 3x + 8x - 24$ M1; $x^2 + 5x - 24$ A1 **[2]** **(c)** $(3y - 4)(3y - 4)$ with 4 terms M1; $9y^2 - 24y + 16$ A1 **[2]** **(d)** $x^2 - 25$ B1 **[1]**

2 (a) $(2x + 3)(x + 4)$ M1; $2x^2 + 8x + 3x + 12 = 2x^2 + 11x + 12$ A1 (must see 4 terms) **[2]**

(b) $x^2 + 10x + 25 - (x^2 - 10x + 25)$ M1; $= 20x$ A1 **[2]**

3 $(x + 3)(x - 1) = x^2 + 2x - 3$ M1; multiply by $(2x + 5)$ with 6 terms M1; $2x^3 + 9x^2 + 4x - 15$ A1 **[3]**

4 $4n^2 + 4n + 1$ and $4n^2 - 4n + 1$ M1; difference $= 8n$ A1; $8n = 8 \times n$ so a multiple of 8 A1 **[3]**

5 (a) $a = 6$ B1; $c = 36$ B1 **[2]** **(b)** $p + q = 2$ and $pq = -15$ M1; $p = 5$, $q = -3$ A1 **[2]**

6 (a) $x(x + 2) = x^2 + 2x$ M1; $(x^2 + 2x)(x - 1)$ M1; $= x^3 + x^2 - 2x$ A1 **[3]**

(b) 72 cm^3 B1 **[1]**

Unit 3 · Sheet A — Factorising

1 $3(2x + 5)$

2 $4a(2a - 3)$

3 $5x(2x + y)$

4 $3pq(3p - 2q)$

5 $(x + 3)(x + 4)$

6 $(x - 5)(x + 3)$

7 $(x - 5)(x - 4)$

8 $(x - 7)(x + 7)$

9 $(2y - 5)(2y + 5)$

10 $(x + 3)(2x + 1)$

11 $(x - 4)(3x + 2)$

12 $(2x - 1)(3x + 2)$

13 $3(x - 5)(x + 5)$

14 $x(x - 2)(x + 2)$

15 $(a + b)(x + y)$

16 $(x + 7)^2$

Unit 3 · Sheet B — Factorising

1 (a) $6xy(2x - 3y)$ B2 (B1 for partial factorisation) **[2]** **(b)** $(x - 7)(x + 6)$ B2 (B1 for $(x + a)(x + b)$ with $ab = -42$ or $a + b = -1$) **[2]** **(c)** $(4 - 3a)(4 + 3a)$ B2 **[2]**

2 (a) $(x - 4)(2x + 3)$ B2 **[2]** **(b)** $x = -3/2$ or $x = 4$ B1ft **[1]** **(c)** $(2x - 3)^2$ B2 **[2]**

3 (a) $(97 - 3)(97 + 3)$ M1; $= 94 \times 100 = 9400$ A1 **[2]** **(b)** $n(n + 1)$ M1; product of two consecutive integers, one of which is even, so the product is even A1 **[2]**

4 (a) $5(x^2 - 9)$ M1; $5(x - 3)(x + 3)$ A1 **[2]** **(b)** $x(y - 3) + 2(y - 3)$ M1; $(x + 2)(y - 3)$ A1 **[2]**

5 (a) $x + 4$ B1 **[1]** **(b)** $2(x + 5) + 2(x + 4)$ M1; $4x + 18$ A1 **[2]**

6 (a) $(x - 4)(x + 4)$ B1; $(x + 4)(x - 3)$ B1; $(x - 4)/(x - 3)$ B1 **[3]** **(b)** $x - 4 = 3(x - 3)$ M1; $x = 2.5$ A1 **[2]**

Unit 4 · Sheet A — Indices and algebraic fractions

1 x^8

2 y^5

3 a^{12}

4 $12x^7$

5 $3a^4$

6 $16x^{12}$

7 4

8 9

9 $1/2$

10 x^2

11 $4x^4$

12 $x = 5$

13 $x = 3$

14 $7x/12$

15 $7/(2x)$

16 $(x + 10)/12$

17 $(5x - 4)/((x + 1)(x - 2))$

18 $2x/3$

19 $x/(x - 3)$

20 $x/3$

Unit 4 · Sheet B — Indices and algebraic fractions

1 (a) $64x^9y^6$ B2 (B1 for two of 64 , x^9 , y^6) **[2]** **(b)** $5x^2$ B2 (B1 for 5 or x^2) **[2]** **(c)** $n = 7$ B1 **[1]**

2 (a) $(4/9)^{3/2}$ or $(3/2)^{-3}$ M1; $8/27$ A1 **[2]** **(b)** $1/16$ B1 **[1]** **(c)** $x^2 \times x^{-3}$ M1; x^{-1} or $1/x$ A1 **[2]**

3 $3^{2x} = 3^{3(x-1)}$ M1; $2x = 3x - 3$ M1; $x = 3$ A1 **[3]**

4 (a) common denominator $(x - 2)(x + 3)$ M1; $3(x + 3) - (x - 2)$ M1; $(2x + 11)/((x - 2)(x + 3))$ A1 **[3]** **(b)** $2(2x + 11) = (x - 2)(x + 3)$ M1; $x^2 - 3x - 28 = 0$ A1; $x = 7$ or $x = -4$ A1 **[3]**

5 $(2x - 1)(x + 3)$ B1; $(2x - 1)(2x + 1)$ B1; $(x + 3)/(2x + 1)$ B1 **[3]**

6 $(x - 2)(x + 2)/(3(x + 2))$ B1; $x/9/(x - 2)$ M1; $= 3$ A1 **[3]**

Unit 5 · Sheet A — Linear equations and changing the subject

- 1 $x = 5$
- 2 $x = 4$
- 3 $x = -4$
- 4 $x = -12$
- 5 $x = 20$
- 6 $x = 8$
- 7 $x = 7$
- 8 $x = -3$
- 9 $x = 6$
- 10 $x = 1/4$
- 11 $x = (y + 7)/3$
- 12 $x = (a - b)/c$
- 13 $n = P/3 - m$
- 14 $s = (v^2 - u^2)/(2a)$
- 15 $r = \sqrt{A/\pi}$
- 16 $l = gT^2/(4\pi^2)$
- 17 $x = a(y + 1)/(y - 1)$

Unit 5 · Sheet B — Linear equations and changing the subject

- 1 (a) $4x - 12 = 2x + 9$ M1; $2x = 21$ M1; $x = 10.5$ oe A1 [3] (b) $x \times 12$: $3(3x + 1) - 4(x - 2) = 24$ M1; $9x + 3 - 4x + 8 = 24$ M1; $5x = 13$ M1; $x = 2.6$ oe A1 [4]
- 2 (a) $2x + 10 + 3x - 5 + x + 25 = 180$ M1; $6x + 30 = 180$ A1; $x = 25$ A1 [3] (b) 70° B1ft [1]
- 3 $3V = \pi r^2 h$ M1; $r^2 = 3V/(\pi h)$ M1; $r = \sqrt{3V/(\pi h)}$ A1 [3]
- 4 $y(x - 1) = 2x + 3$ M1; $xy - 2x = y + 3$ M1; $x(y - 2) = y + 3$ M1; $x = (y + 3)/(y - 2)$ A1 [4]
- 5 (a) \$25.10 B1 [1] (b) $(43.10 - 3.5) \div 1.8$ M1; 22 km A1 [2] (c) $C - 3.5 = 1.8d$ M1; $d = (C - 3.5)/1.8$ oe A1 [2]
- 6 $3x + 12 = 2(x + 12)$ M1 A1; $x = 12$ A1 [3]

Unit 6 · Sheet A — Simultaneous equations

- 1 $x = 7, y = 3$
- 2 $x = 4, y = 3$
- 3 $x = 4, y = 2$
- 4 $x = 3, y = 3$
- 5 $x = 2, y = 3$
- 6 $x = 3, y = -2$
- 7 $x = 3, y = 7$
- 8 $a = 7, b = -3$
- 9 $x = 3, y = 2$
- 10 $x = 6, y = 1$
- 11 $x = -2, y = 4; x = 3, y = 9$
- 12 $x = 2, y = 3; x = 3, y = 2$
- 13 $x = -1, y = -3; x = 3/2, y = 2$
- 14 20 and 11

Unit 6 · Sheet B — Simultaneous equations

- 1 (a) $y = 2x - 1$ correct B1; $x + y = 5$ correct B1 [2] (b) $x = 2, y = 3$ B1ft [1]
- 2 correct method to eliminate one variable M1; $x = 3$ A1; $y = -1$ A1 [3]
- 3 (a) $3a + 2c = 36$ B1; $2a + 5c = 46$ B1 [2] (b) eliminate one variable M1; $a = 8$ A1; $c = 6$ A1 [3] (c) \$50 B1ft [1]
- 4 $x^2 - 3x + 1 = 2x - 3$ M1; $x^2 - 5x + 4 = 0$ A1; $(x - 1)(x - 4) = 0$ M1; $x = 1, y = -1; x = 4, y = 5$ A1 [4]
- 5 (a) $x = 10 - 2y$ M1; $(10 - 2y)^2 + y^2 = 25$ M1; $5y^2 - 40y + 75 = 0$ A1; $y = 3$ or $y = 5$ M1; $(4, 3)$ and $(0, 5)$ A1 [5] (b) $\sqrt{(4 - 0)^2 + (3 - 5)^2}$ M1; 4.47 A1 [2]
- 6 $m + c = 5$ and $3m + c = 11$ M1; $m = 3$ A1; $c = 2$ A1 [3]

Unit 7 · Sheet A — Quadratic equations

- 1 $x = 2$ or $x = 3$
- 2 $x = -7$ or $x = 3$
- 3 $x = 0$ or $x = 9$
- 4 $x = -8$ or $x = 8$
- 5 $x = -3$ or $x = 1/2$
- 6 $x = 2/3$ or $x = 3$
- 7 $x = -4.54$ or $x = 1.54$
- 8 $x = 0.18$ or $x = 2.82$
- 9 $x = -1.12$ or $x = 0.72$
- 10 $(x + 3)^2 - 7$
- 11 $(x - 5)^2 + 6$
- 12 $2(x + 3)^2 - 13$
- 13 $(4, -13)$
- 14 $x = 4 + \sqrt{5}$ or $x = 4 - \sqrt{5}$
- 15 $x = -7$ or $x = 5$
- 16 $x = 2$ or $x = 3$

Unit 7 · Sheet B — Quadratic equations

- 1 (a) $(x - 6)(x + 4)$ B2 [2] (b) $x = 6$ or $x = -4$ B1ft [1]
- 2 $(-4 \pm \sqrt{4^2 - 4 \times 3 \times (-9)})/(2 \times 3)$ M2 (M1 for $\sqrt{124}$ or $(-4 \pm \sqrt{q})/6$; $x = 1.19$ and $x = -2.52$ A1 [3]
- 3 (a) $(x - 3)^2 - 20$ B2 (B1 for $a = 3$ or $b = 20$) [2] (b) $(x - 3)^2 = 20$ M1; $x = 3 \pm 2\sqrt{5}$ A1 [2] (c) -20 B1ft [1]
- 4 (a) $(x + 5)(x - 2) = 60$ M1; $x^2 + 3x - 10 = 60$ so $x^2 + 3x - 70 = 0$ A1 [2] (b) $(x + 10)(x - 7) = 0$ M1; $x = 7$ (reject -10) A1; perimeter = $2(12 + 5) = 34 \text{ cm}$ A1 [3]
- 5 $2x + 3(x + 1) = 2x(x + 1)$ M1; $2x^2 - 3x - 3 = 0$ A1; correct use of formula M1; $x = 2.19$ A1; $x = -0.69$ A1 [5]
- 6 (a) $5t^2 - 20t + 15 = 0$ M1; $(t - 1)(t - 3) = 0$ M1; $t = 1 \text{ s}$ and $t = 3 \text{ s}$ A1 [3] (b) $t = 2$ (symmetry) M1; $h = 20 \text{ m}$ A1 [2]

Unit 8 · Sheet A — Inequalities and regions

- 1 $x < 5$
- 2 $x \geq 8$
- 3 $x < -4$
- 4 $x \leq -3.5$
- 5 $x > 9$
- 6 $-2 < x \leq 4$
- 7 $-1, 0, 1, 2, 3$
- 8 $1, 2, 3, 4$
- 9 6
- 10 8
- 11 Yes ($3 > 2$)
- 12 $-5 < x < 5$
- 13 $x \leq -3$ or $x \geq 3$
- 14 $0 < x < 4$

Unit 8 · Sheet B — Inequalities and regions

- 1 (a) $5x > 25$ M1; $x > 5$ A1 [2] (b) $-6 \leq 2n < 6$ M1; $-3 \leq n < 3$ A1; $-3, -2, -1, 0, 1, 2$ A1 [3]
- 2 $y = 1$ B1; $x + y = 6$ B1; $y = 2x$ B1; correct triangle with vertices (0, 5), (5, 1), (2, 4) labelled R B1 [4]
- 3 $x \geq 1$ B1; $y \leq 4$ B1; $y \geq x$ B1 [3]
- 4 (a) $8x + 12y \leq 96$, divide by 4 B1 [1] (b) $x \geq 3, y \geq 2, x + y \leq 10$ B2 (B1 for two correct) [2] (c) four correct lines B1; correct region R B1 [2] (d) $x + y = 10$ considered M1; 8 footballs and 2 basketballs (cost \$88) A1 [2]
- 5 (a) $(x - 4)(x + 3) \leq 0$ M1; critical values -3 and 4 A1; $-3 < x < 4$ A1 [3] (b) $-2, -1, 0, 1, 2, 3$ B1ft [1]
- 6 $2(x + 3) + 2(2x - 1) < 40$ M1; $x < 6$ A1; width > 0 so $x > 0.5$; $0.5 < x < 6$ A1 [3]

Unit 9 · Sheet A — Sequences

- 1 19, 23
- 2 $3n + 2$
- 3 $24 - 4n$
- 4 347
- 5 No; $3n = 98$ has no whole-number solution
- 6 36th term
- 7 $n^2 + 1$
- 8 $2n^2 + 1$
- 9 $2n^2 + n + 1$
- 10 $n^3 + 1$
- 11 $3 \times 2^{n-1}$
- 12 $81 \times (1/3)^{n-1}$ or 3^{5-n}
- 13 11, 32
- 14 15
- 15 10
- 16 400

Unit 9 · Sheet B — Sequences

- 1 (a) 19 B1 [1] (b) $3n + 1$ B2 (B1 for $3n + k$) [2] (c) $3n + 1 = 100$ M1; $n = 33$, yes (Pattern 33) A1 [2]
- 2 (a) 71 B1 [1] (b) second difference 4 M1; $2n^2$ seen A1; $2n^2 - 1$ A1 [3] (c) 799 B1ft [1]
- 3 (a) 0, 7, 26, 63 B2 (B1 for 3 correct) [2] (b) 10th B1 [1]
- 4 (a) $5 \times 3^{n-1}$ B2 (B1 for ratio 3 or $k \times 3^{n-1}$) [2] (b) trial of terms M1; 10935 (8th term) A1 [2]
- 5 (a) $3n + 4 = 44 - 2n$ M1; $n = 8$ A1; term = 28 A1 [3] (b) $3n + 4 > 100$ so $n > 32$ M1; 103 A1 [2]
- 6 (a) $n(n + 3)$ M1; if n is odd, $n + 3$ is even; if n is even, n is even, so the product is even A1 [2] (b) $n^2 + 3n - 180 = 0$ M1; $n = 12$ A1 [2]

Unit 10 · Sheet A — Proportion and graphs in practical situations

- 1 28
- 2 128
- 3 8
- 4 $1/2$
- 5 20
- 6 multiplied by 8
- 7 divided by 4
- 8 speed
- 9 distance travelled
- 10 4 km/h
- 11 20 m/s
- 12 2.5 m/s^2
- 13 80 m
- 14 180 m
- 15 4 m/s^2

Unit 10 · Sheet B — Proportion and graphs in practical situations

- 1 (a) $y = kx^2$ and $45 = 9k$ M1; $y = 5x^2$ A1 [2] (b) 80 B1ft [1] (c) $x^2 = 49$ M1; $x = 7$ A1 [2]
- 2 (a) $F = k/d^2$ M1; $k = 180$ A1; $F = 5$ A1 [3] (b) $d^2 = 180 \div 0.8 = 225$ M1; $d = 15$ A1 [2]
- 3 $1.5^2 = 2.25$ M1; 125% A1 [2]
- 4 (a) $12 \div (40/60)$ M1; 18 km/h A1 [2] (b) 15 minutes B1 [1] (c) $12 \div 1$ M1; 12 km/h A1 [2] (d) $24 \div (115/60)$ M1; 12.5 km/h A1 [2]
- 5 (a) $18 \div 30$ M1; 0.6 m/s^2 A1 [2] (b) area method M1; $270 + 1080 + 180$ M1; 1530 m A1 [3] (c) 13.9 m/s B1ft [1]

Unit 11 · Sheet A — Graphs of functions and sketching curves

- 1 5
- 2 (0, -5)
- 3 (2, 0) and (-5, 0)
- 4 $x = 3$
- 5 (-1, -4)
- 6 3
- 7 -4
- 8 $1/2$
- 9 24
- 10 reciprocal (hyperbola)
- 11 $x = 0$ and $y = 0$
- 12 0 (none)
- 13 $y = 2$
- 14 $y = 2x - 1$
- 15 3
- 16 y is negative

Unit 11 · Sheet B — Graphs of functions and sketching curves

- 1 -1, -5, -1 B2 (B1 for 2 correct) [2] (b) all 7 points plotted correctly B2ft (B1 for 5 or 6); smooth curve through all points B2 (B1 for a curve with minor errors) [4] (c) $x = -1.2$ B1; $x = 3.2$ B1 (accept ± 0.1) [2] (d) $y = x - 2$ seen B1; line drawn B1; $x = -0.6$ and $x = 3.6$ B1 (accept ± 0.1) [3]
- 2 (a) $(x - 2)(x + 5)$ M1; $x^2 + 3x - 10$ so $a = 3$, $b = -10$ A1 [2] (b) $x = -1.5$ M1; (-1.5, -12.25) A1 [2]
- 3 0.25, 1, 8 B2 (B1 for 2 correct) [2] (b) points plotted B2ft (B1 for 4 correct); smooth curve B1 [3] (c) $x = 2.3$ B1 (accept 2.2 to 2.4) [1] (d) tangent drawn at $x = 2$ B1; rise $\div$ run M1; 2.8 A1 (accept 2.3 to 3.3) [3]
- 4 $y = 3^x$: D; $y = x^3 - 4x$: B; $y = -2/x$: C; $y = x^2 - 4$: A B1 each [4]

Unit 12 · Sheet A — Differentiation and turning points

- 1 $5x^4$
- 2 $12x^3$
- 3 7
- 4 0
- 5 $6x^2 - 10x + 4$
- 6 $-4/x^2$
- 7 $-12/x^3$
- 8 $2x + 3$
- 9 $2x + 4$
- 10 6
- 11 12
- 12 (3, -7)
- 13 (2, 14)
- 14 (2, -16) and (-2, 16)
- 15 $y = 6x - 9$

Unit 12 · Sheet B — Differentiation and turning points

- 1 (a) $6x^2 - 18x + 12$ B2 (B1 for two terms correct) [2] (b) $6x^2 - 18x + 12 = 0$ M1; $x = 1$, $x = 2$ A1; (1, 6) A1; (2, 5) A1 [4] (c) $d^2y/dx^2 = 12x - 18$ or gradient test M1; (1, 6) maximum, (2, 5) minimum A1 [2]
- 2 (a) $2x - 5$ M1; 3 A1 [2] (b) $y - 0 = 3(x - 4)$ M1; $y = 3x - 12$ A1 [2]
- 3 (a) 1 B1; $-9x^{-2}$ or $-9/x^2$ B1 [2] (b) $1 - 9/x^2 = 0$ M1; $x = 3$ A1; (3, 6) A1 [3]
- 4 (a) $V = x(18 - 2x)(12 - 2x)$ M1; correct expansion to given answer A1 [2] (b) $dV/dx = 12x^2 - 120x + 216$ M1; $= 0$ and solve M1; $x = 2.35$ A1 (reject 7.65) [3] (c) 228 cm^3 B1 [1]
- 5 $3x^2 - 6x = 9$ M1; $x^2 - 2x - 3 = 0$ M1; $x = 3$, $x = -1$ A1; (3, 0) and (-1, -4) A1 [4]

Unit 13 · Sheet A — Functions

- 1 10
- 2 -8
- 3 10
- 4 $1/2$
- 5 13
- 6 17
- 7 $3x^2 + 1$
- 8 $9x^2 - 12x + 5$
- 9 $9x - 8$
- 10 $(x + 2)/3$
- 11 3
- 12 $x = 5$
- 13 $(2 - x)/x$ or $2/x - 1$
- 14 $x = -1$
- 15 8
- 16 $1/2$
- 17 5

Unit 13 · Sheet B — Functions

- 1 (a) 11 B1 [1] (b) $f(1) = 3$ M1; 9 A1 [2] (c) $5 - 2x^2$ B1 [1] (d) $x = 5 - 2y$ or $y = 5 - 2x$ rearranged M1; $(5 - x)/2$ A1 [2] (e) $(5 - 2x)^2 = 49$ M1; $5 - 2x = \pm 7$ M1; $x = -1$ or $x = 6$ A1 [3]
- 2 (a) $g(2) = 3$ M1; 6 A1 [2] (b) $x + 1 = 2y$ M1; $(x + 1)/2$ A1 [2] (c) $x + 4 = 3(2x - 1)$ M1; $5x = 7$ M1; $x = 1.4$ A1 [3] (d) $2(2x - 1) - 1$ M1; $4x - 2 - 1 = 4x - 3$ A1 [2]
- 3 (a) $x = 3/(y - 2)$ M1; $y - 2 = 3/x$ M1; $f^{-1}(x) = 3/x + 2$ or $(2x + 3)/x$ A1 [3] (b) $f(5) = 1$ M1; $f(1) = -3$ A1 [2]
- 4 (a) 32 B1 [1] (b) $2^x + 3 = 2^{-2}$ M1; $x = -5$ A1 [2] (c) $2^x + 3$ B1 [1]

Unit 14 · Sheet A — Coordinate geometry and straight lines

- 1 2
- 2 -3
- 3 (5, 3)
- 4 10
- 5 13
- 6 4 and (0, -7)
- 7 3
- 8 -3/4
- 9 $y = -2x + 5$
- 10 $y = 4x - 5$
- 11 $y = 3x + 3$
- 12 (4, 0)
- 13 Yes, $3 \times 3 - 2 = 7$
- 14 -1/2
- 15 4/3
- 16 $y = -2x + 5$

Unit 14 · Sheet B — Coordinate geometry and straight lines

- 1 (a) (1, 5) B2 (B1 for one coordinate) [2] (b) $6^2 + 8^2$ M1; $\sqrt{100}$ M1; 10 A1 [3] (c) gradient $8/6 = 4/3$ B1; $y - 1 = 4/3(x + 2)$ M1; $4x - 3y = -11$ oe A1 [3]
- 2 (a) $1/2$ B1 [1] (b) $-1 = \frac{1}{2}(4) + c$ M1; $y = \frac{1}{2}x - 3$ A1 [2] (c) gradient -2 B1; $4 = -2(2) + c$ M1; $y = -2x + 8$ A1 [3]
- 3 3, 1, -3, -5 B1 [1] (b) correct straight line through at least 4 correct points B2 (B1 for 3 points) [2] (c) (1, 1) B1 [1]
- 4 midpoint (4, 1) B1; gradient PQ = $-2/3$ B1; perpendicular gradient $3/2$ M1; $y = 1.5x - 5$ A1 [4]
- 5 (a) gradient AB = $1/2$ B1; gradient BC = -2 B1; $\frac{1}{2} \times (-2) = -1$ so perpendicular A1 [3] (b) AB = BC = $\sqrt{20}$ M1; area = $\frac{1}{2} \times \sqrt{20} \times \sqrt{20} = 10$ A1 [2]

End-of-book check

- 1 $x^2 + 2x - 24$
- 2 $(x - 5)(x - 3)$
- 3 $x = 4$ or $x = -5$
- 4 $27x^6y^3$
- 5 $x = -4$
- 6 $n^2 + 2n + 1$
- 7 3
- 8 $(x - 5)/2$
- 9 $12x^2 - 6$
- 10 (1, 4)
- 11 4
- 12 $-1 \leq x < 3$

Practice paper

- 1 (a) $7p + 4q$ B2 (B1 for one term) [2] (b) 4 terms with 3 correct M1; $2x^2 + x - 15$ A1 [2]
- 2 (a) $4x = 20$ M1; $x = 5$ A1 [2] (b) $2(x + 2) + 3(x - 1) = 24$ M1; $5x + 1 = 24$ M1; $x = 4.6$ A1 [3]
- 3 (a) $5xy(3x + 2y)$ B2 [2] (b) $(x - 7)(x + 4)$ B2 [2] (c) $(3x - 2)(3x + 2)$ B1 [1]
- 4 $v - u = at^2$ M1; $t^2 = (v - u)/a$ M1; $t = \sqrt{(v - u)/a}$ A1 [3]
- 5 correct method to eliminate M1; $13y = -26$ A1; $y = -2$ A1; $x = 3$ A1 [4]
- 6 (a) $5n + 2$ B2 (B1 for $5n + k$) [2] (b) second difference 4 M1; $2n^2$ A1; $2n^2 + n$ A1 [3]
- 7 $(7 \pm \sqrt{49 - 16})/4$ M2 (M1 for one error); 3.19 and 0.31 A1 [3]
- 8 1.125, -1.375, -1.125 B2 (B1 for 2 correct) [2] (b) points plotted B2ft (B1 for 7 correct); smooth curve B1 [3] (c) -1.5, -0.3, 1.9 B2 (B1 for 2 correct; accept ± 0.1) [2]
- 9 (a) $g(2) = 5$ M1; 7 A1 [2] (b) $x + 3 = 2y$ M1; $(x + 3)/2$ A1 [2] (c) $(2x - 3)^2 + 1 = 10$ M1; $2x - 3 = \pm 3$ M1; $x = 0$ or $x = 3$ A1 [3]
- 10 (a) $3x^2 - 12x + 9$ B2 (B1 for 2 terms) [2] (b) $3x^2 - 12x + 9 = 0$ M1; $x = 1$, $x = 3$ A1; (1, 6) A1; (3, 2) A1 [4] (c) $6x - 12$ or gradient test M1; (1, 6) maximum, (3, 2) minimum A1 [2]
- 11 (a) $6^2 + 4^2$ M1; $\sqrt{52}$ M1; 7.21 A1 [3] (b) midpoint (2, 1) B1; gradient AB = $-2/3$ B1; gradient $3/2$ M1; $y = 1.5x - 2$ A1 [4]
- 12 $-2x \leq 8$ M1; $x \geq -4$ A1 [2]